Sci. J. Fac. Monoufia Univ. vol. IV (1990) 273 - 283

THE EFFECT OF A MOVING LOAD IN AN ELASTIC LAYER

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ABSTRACT

The problem of propagation of an elastic wave in a layer occupied by an elastic medium has been investigated. It is assumed that the external boundary of the elastic layer ($y = 0$) is subjected to an inclined load which propagates with constant speed D. The lateral and vertical displacements produced by the load are taken into consideration. The stresses distribution have been calculated.

1 - INTRODUCTION

The problem of elastic wave propagation in a layer has been studeid by several authors such as Rackhmatolin (1975), [7] Von, Karman and Duwez (1950), [6]. Various generalization of these problems were studied by Shapiro (1946), Sokolovski (1948), [9, 10] and Rackhmatolin (1960), [8].

The three-dimensional problem of elastic wave propagation has been solved by El-Dewik (1975), [1]. The problem has been studied when an instantaneous constant load acts on the boundary of elastic halfspace. The load is taken to act perpendicularly to the boundary and the lateral displacements were neglected. A similar study of this problem was carried out in the case where the load is the time dependent (El-Dewik, 1975 a) [2].

The two-dimensional problem was treated under the assumption that propagation load acts perpendicularly to the boundary (El-Dewik, 1977) [3]. The solution was obtained taking into account the vertical displacements, whereas the lateral displacements were neglected. A similar problem was solved (El-Dewik, 1981 [4]. the solution was obtained taking into consideration the lateral displacement as well as the vertical one. The solution was carried out when the load propagates with velocity equal the velocity of longitudinal wave.

Nevertheless,. the problem of propagation of elastic-plastic wave in the half-space has been recently solved (El-Dewk 1982) [5], under the assumption that the propagation load acts perpendicularly to the boundary.

In the present work the dynamic problem of elastic wave propagation in a layer has been studied when inclined loads act on the boundary of elastic layer (y = 0). This load is assumed to propagate with a constant speed D on the boundary. The problem is solved taking into account the lateral displacement as well as the vertical one.

2 - SOLUTION OF BASIC EQUATION

Let us consider an elastic infinite layer occupies the domain between y = 0 and y = h in the Cartesian coordinates. Consider that there exists an inclined load acts on the boundary which propagates with a constant speed D. The relevant equations of motion for the lateral and vertical displacements u and v respectively, are :

$$(\lambda + \mu)\frac{\partial \Theta}{\partial x} + \mu \nabla^2 u = p\frac{\partial^2 u}{\partial t^2}$$

$$(\lambda + \mu)\frac{\partial \Theta}{\partial y} + \mu \nabla^2 v = p\frac{\partial^2 v}{\partial t^2}$$

Where (1, 1)

$$\Theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

and the surface conditions are :

at $y = 0$, $-\infty < x < \infty$

$$\sigma_{yy} = P_o \ \delta\ (x + Dt)\ \sin \alpha \qquad (1, 2)$$

$$\sigma_{xy} = p_o \, \delta \, (x + Dt) \, \cos \alpha$$

Where α is the angle between the load and the boundary $y = 0$ and :

$$\delta \, (x) = \frac{1}{\pi} \int_o^\infty \cos (kx) \cdot dk \qquad (1, 3)$$

at $y = h$, $-\infty < x < \infty$,

$$u = v = o$$

The components of lateral and vertical displacements can be expressed in terms of potential function φ and ψ as follows

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y},$$

$$v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial y}, \qquad (1, 4)$$

By substituting (1, 4) in (1, 1) we have ;

$$\nabla^2 \varphi = \frac{1}{a^2} \frac{\partial^2 \varphi}{\partial t^2}$$

$$\nabla^2 \psi = \frac{1}{b^2} \frac{\partial^2 \psi}{\partial t^2} \qquad (1, 5)$$

Where $a = \sqrt{\dfrac{\lambda + 2\mu}{\rho}}$ Is the velocity of longitudinal wave and

$b = \sqrt{\dfrac{\mu}{\rho}}$ the velocity of transverse wave.

From (1, 2), (1, 3), (1, 4) and Hook's law, the surface conditions can be written :

$$\left(\frac{b^2}{a^2} - 2\right)\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) + 2\frac{\partial^2\psi}{\partial y^2} - 2\frac{\partial^2\psi}{\partial x\,\partial y} = p_o\,\delta\,(x + Dy)\ \sin\alpha,$$

$$2\frac{\partial^2\varphi}{\partial x\,\partial y} + \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial x^2} = p_o\,\delta\,(x + Dy)\ \cos\alpha, \qquad (1, 6)$$

$$\frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial y} = 0$$

and $\frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial y} = 0$

The initial condition at $t = 0$ is

$$\varphi = \frac{\partial\varphi}{\partial t} = \psi = \frac{\partial\psi}{\partial t} = 0 \qquad (1.7)$$

If we use the transformation $x = x' + Dt$, $y = y'$ then the equations (1, 5) will take the form

$$\frac{\partial^2\varphi}{\partial y^2} - \left[\left(\frac{a}{D}\right)^2 - 1\right]\frac{\partial^2\varphi}{\partial x^2} = 0$$

$$\frac{\partial^2\psi}{\partial y^2} - \left[\left(\frac{b}{D}\right)^2 - 1\right]\frac{\partial^2\psi}{\partial x^2} = 0 \qquad (1, 8)$$

The solution of the problem may be written as follows :

$$\varphi = A\ e^{-kqy}\ \sin kx \qquad (1.9)$$

$$\psi = B\ e^{-kqy}\ \cos kx$$

Substituting from (1.9) in (1.8) we get

$$q_{1,2} = \pm \left(\frac{a}{D}\right)^2 - 1 \quad \text{and} \quad q_{3,4} = \pm \left(\frac{b}{D}\right)^2 - 1$$

where (q_1, q_3) and (q_2, q_4) correspond to the positive and negative values, respectively, in the above relation.

Therefore, the general solution of the problem may be written as :

$$\varphi = \Sigma^4_{n=1} \int_o^\infty A_n \; e^{-kq_n y} \sin kx \, dk,$$

$$\psi = \Sigma^4_{n=1} \int_o^\infty A_n \; e^{-kq_n y} \cos kx \, dk \qquad (1, 10)$$

Where A_1, A_2, A_3 and A_4 are unknowns which may be obtained from the boundary condition (1, 6).

Using condition (1, 6) :

$$\Sigma^4_{n=1} \; (\alpha_n - 2k^2) \; A_n = \frac{P_o \sin \alpha}{\pi},$$

$$\Sigma^4_{n=1} \; (q_n - 1)^2 \; A_n = \frac{P_o \cos \alpha}{\pi}, \qquad (1, 11)$$

$$\Sigma^4_{n=1} \; (q_n + 1) \; e^{kq_n h} \; A_n = 0$$

$$\Sigma^{4}_{n=1} \; (1 - q_n) \, e^{kq_n h} \; A_n = 0$$

$$\text{Where } \alpha_n = \left(\frac{b^2}{a^2} - 2\right)(q^2_n - 1)\, k^2$$

Solving the above equations we get :

$$A_1 = \frac{2p_o \cos\alpha}{\pi\Delta}\Big[(\alpha_2 - 2k^2)(q_3 - q_4)\, e^{(q_3+q_4)kh} + (\alpha_3 - 2k^2)(q_4 - q_2)\, e^{(q_2+q_4)kh}$$

$$+ (\alpha_4 - 2k^2)(q_2 - q_3)\, e^{(q_2+q_3)kh}\Big] - \frac{2p_o \sin\alpha}{\pi\Delta}\,(q_2 - 1)^2 (q_3 - q_4)\, e^{(q_3+q_4)kh}$$

$$+ (q_4 - 1)^2 (q_4 - q_2)\, e^{(q_2+q_4)kh} + (q_4 - 1)^2 (q_2 - q_3)\, e^{(q_2+q_3)kh}\Big]$$

$$A_2 = \frac{2p_o \cos\alpha}{\pi\Delta}\Big[(\alpha_1 - 2k^2)(q_3 - q_4)\, e^{(q_3+q_4)kh} + (\alpha_3 - 2k^2)(q_4 - q_1)\, e^{(q_4+q_1)kh}$$

$$+ (\alpha_4 - 2k^2)(q_1 - q_3)\, e^{(q_1+q_3)kh}\Big] - \frac{2p_o \sin\alpha}{\pi\Delta}\Big[(q_1 - 1)^2 (q_3 - q_4)\, e^{(q_3+q_4)kh}$$

$$+ (q_3 - 1)^2 (q_4 - q_1)\, e^{(q_4+q_1)kh} + (q_4 - 1)^2 (q_1 - q_3)\, e^{(q_1+q_3)kh}\Big] \qquad (1, 12)$$

$$A_3 = \frac{2p_o \cos\alpha}{\pi\Delta}\Big[(\alpha_1 - 2k^2)(q_2 - q_4)\, e^{(q_2+q_4)kh} + (\alpha_2 - 2k^2)(q_3 - q_4)\, e^{(q_3+q_4)kh}$$

$$+ (q_4 - 1)^2 (q_1 - q_3)\, e^{(q_1+q_3)kh}\Big] - \frac{2p_o \sin\alpha}{\pi\Delta}\,(q_1 - 1)^2 (q_2 - q_4)\, e^{(q_2+q_4)kh}$$

$$+ (q_2 - 1)^2 (q_3 - q_4)\, e^{(q_3+q_4)\,kh} \;] + (q_4 - 1)^2 (q_2 - q_3)\, e^{(q_2+q_3)\,kh} \;]$$

and

$$A_4 = \frac{2p_o \cos\alpha}{\pi\Delta} \; (\alpha_1 - 2k^2)(q_2 - q_4)\, e^{(q_2+q_4)\,kh} + (\alpha_2 - 2k^2)(q_3 - q_4)\, e^{(q_3+q_4)\,kh}$$

$$+ (\alpha_4 - 2k^2)(q_2 - q_3)\, e^{(q_2+q_3)kh} \;] - \frac{2p_o \sin\alpha}{\pi\Delta} [\, (q_1 - 1)^2 (q_2 - q_4)\, e^{(q_2+q_4)\,kh}$$

$$+ (\alpha_2 - 2k^2)(q_3 - q_4)\, e^{(q_3+q_4)\,kh} \;] + [\, (\alpha_3 - 2k^2)(q_4 - q_1)\, e^{(q_1+q_4)\,Kh}$$

Where,

$$\Delta = \Sigma^4_{n=1} \; (\alpha_n - 2k^2)(q_n - 1)(q_n + 1)\, e^{Khq_n}$$

From (1, 4) and (1, 12) the components of the displacements will take the form :

$$u = \Sigma^{\infty}_{n=1} \int_0^{\infty} (1 + q_n)\, KA_n\, e^{-kq_n y} \cos kx \; dk \qquad (1, 13)$$

$$v = \Sigma^{\infty}_{n=1} \int_0^{\infty} (\sin kx - \cos kx)\, A_n\, kq_n\, e^{-kq_n y} \; dk$$

From Hook's law the components of stresses will take the form

$$\sigma_{xx} = \mu \left(\frac{b}{a^2} - 2 \right) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2 \frac{\partial u}{\partial x} ,$$

Using (1, 12) and (1, 13) we get :

$$\sigma_{xx} = \frac{2\mu p_o \sin\alpha}{\pi}\left[\frac{a^4}{D^4} + \frac{b^4}{a^4} + 2\frac{a^2}{b^2} - 1\right] I_1 - \frac{2\mu p_o \cos\alpha}{\pi}\left(\frac{b}{D}\right)^2 - 1\,] I_2$$

Similarly:

$$\sigma_{yy} = \frac{2\mu p_o \sin\alpha}{\pi}\left[\frac{b^4}{D^4} + \frac{b^4}{a^4} + 2\frac{a^2}{b^2} - 1\right] I_1 - \frac{2\mu p_o \cos\alpha}{\pi}\left[\left(\frac{b}{D}\right)^2 - 1\right] I_2, \qquad (1,14)$$

and

$$\zeta_{xy} = \frac{2\mu p_o \sin\alpha}{\pi}\left[\left(\frac{a}{D}\right)^2 + 1\right] I_1 - \frac{2\mu p_o \cos\alpha}{\pi}\left[\left(\frac{b}{D}\right)^2 - 1\right] I_2$$

Where:

$$I_1 = \int_0^\infty \frac{\sin[\alpha k(h+y)]}{\sin(\alpha kh)} \cos kx \, dk ,$$

and (, 15)

$$I_2 = \int_0^\infty \frac{\sin[\alpha k(h+y)]}{\sin(\alpha kh)} \sin kx \, dk ,$$

The integrals I_1 and I_2 are calculated as follows :

$$\frac{\sin[\alpha k(h+y)]}{\sin \alpha kh} = \cos\alpha ky + \sin(\alpha ky)\cos(\alpha kh)$$

$$= \cos\alpha ky - \frac{1}{2}[\cos\{\alpha k(2h+y)\}]$$

$$- \cos[\alpha k(2h-y)]\;[1 + \cos(2\alpha kh + \cos^2(2\alpha kh) + \ldots\ldots + \cos^n(2\alpha kh) + \ldots\ldots]$$

$$= \cos(\alpha ky) - \Sigma^{\infty}_{n=0} \frac{1}{2^{n+1}} \Sigma^{n}_{n=0} C^m{}_n (\cos\alpha k(2h+y)$$

$$+ 2(n-2m)kh] - \cos[\alpha k(-2h+y) + 2(n-2m)kh]\}$$

Where:

C^m_n are the coefficients of the Newtonian polynomials

Therefore,

$$I_1 = \frac{1}{2}[\delta(x-\alpha y) + \delta(x+\alpha y)] - \Sigma^{\infty}_{n=0} \frac{1}{2^{n+2}} \Sigma^{n}_{m=0} C^m n \{[\delta\{\alpha(h+y) + x + 2\alpha h(n-2m)] - \delta[\alpha(h+y) - x + 2\alpha h(n-2m)] - \delta[\alpha(h+y) - x + 2\alpha h(n-2m)]\}.$$

Also we have

$$I_2 = \Sigma^{\infty}_{n=0} \frac{(2n-1)}{n!\, 2^{2n+5}} \Sigma^{n}_{m=0} C^m_n [\delta\ \alpha(h+y) + x + 2\alpha h(n-2m)] - \delta\{\alpha(h+y) - x + 2\alpha h(n-2m\}].$$

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تأثير حمل متحرك على طبقه مرنه

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ملخص البحث

يدرس هذا البحث انتشار موجه مرنه فى شريحة لانهائية المستوى من وسط مرن واقعه تحت تأثير تحميل خارجى يميل على الشريحه بزاوية معينه ويتحرك بسرعة منتظمة . وقد أمكن ايجاد الازاحات والاجهارات عند أى نقطه داخل هذه الشريحة .