CONTROL ALGORITHM USING SLIDING MODE AND ITS APPLICATIONS

Sabry F. Saraya
Faculty of Engineering
Mansoura University

طريقة للتحكم باستخدام الحركة الانزلاقية وتطبيقاتها

ملخص البحث :

يدور موضوع هذا البحث حول دراسة أنظمة التحكم المتقطع باستخدام أسلوب الحركة الانزلاقية بالاضافة الى تقديم مقترح لتجزئة المنظومة المرادالتحكم فيها. ويناقش البحث تأثير استخدام الحركة الانزلاقية للتحكم فى المنظومة والتى تجعلها لا تتأثر بالاضطرابات الداخلية والخارجية وكذلك أى تغير فى المتغيرات .

وهذه المنظومة عبارة عن مولد تيار متردد متصل بخط نقل للطاقة الكهربية. وقد أجريت محاكاة لكل من المنظومة والحاكم على جهاز الحاسب لتصميم كفاءة الطريقة المقترحة ، وقد أسفرت النتائج عن تحسين اتزان النظام والتخلص من الظواهر العابرة التى تنتج عن التعويض المتالى عبر خط النقل تحت ظروف تشغيل مختلفة.

1 - ABSTRACT :

This paper studies the principle structure of discontinous control system using sliding mode technique. An approach of using decomposition in the design process is studied also.

A Sliding mode excitation controller for a synchronous generator connected to an A-C transmission line is designed. This type of controller depends on the principle of discontinous control.

Results obtained showed that this type of control improves system stability & damps out transient torques appearing due to series compensation on Ac lines, under different system operating conditions.

2 - INTRODUCTION

The optimal effect of the control action in discontinous systems depends on the method used in control process. In these systems the use of sliding modes is very promising in the design of control process. The use of sliding modes requires special mathematical treatment, hence, the use of conventional theory of differential equations is not applicable [3].

The use of sliding mode approach avoids the effects of non-measured external disturbances and gives effective method to obtain information about states and parameters of control system [2].

Power system stability represents one of the important problems for power engineers and planners. These, are however, due to the large and complex power networks now-a-days. Several

methods are used to improve stability of single and complex power systems [6]

The excitation control is an effective and low cost means for improving system stability. Different methods of control are also used for achieving this purpose. these include applying linear optimal control theory to the excitation control.

In this paper a sliding mode excitation controller is suggested. This type of control depends on the principle of discontinous control.

The controller is used to control a synchronous generator connected to long O.H. transmission line with series compensation and the far end is an infinite bus.

A simulation program used to obtain results for the system under study. The results show that, such controller can be effectively used to improve system stability. Results show also that this controller is simple and easy to use.

3 - MATHEMATICAL FORMULATION.

Non linear systems are modeled by nonlinear differential equation in the form;

$$x = f(x,t) + B(x,t).u\ (t) \qquad (1)$$

where;

$x \in R^n$, is the state vector and $u \in R^m$ is the control vector. It has the following discontinuity on some surface $S(x)$,

$$u_i = \begin{cases} u_i^+ (x,t) & S_i(x) > 0 \\ u_i^- (x,t) & S_i(x) < 0 \end{cases} \qquad (2)$$

where u_i^+, u_i^-, are time functions, while S_i is a continuous function and differentialble.

In The above two equations the motions in the sliding mode are not only possible along each discontinuity surface but along their intersection and on $S(x) = 0$.

From the practical point of view, it has been found that, [2] some special intersecting lines when using the sliding mode on the intersection of the discontinuity surface are requires to compose the control systems with the desired property. This is unique feature and is really true, when eq. (1) and (2) are used. This is because the right hand side of equation (1) is discontinous.

To overcome this problem in the sliding mode, the equivalent control concept is used [3]. Therefore, the S vector time derivative is taken zero for system trajectories. Then, the solution with respect to the equivalent control is substituted in the initial equation as follows;

Define $G = \frac{\partial s}{\partial x}$

Then

$$u_{eq} = -(GB)^{-1}Gf \qquad (3)$$

$$\dot{x} = f - B(GB)^{-1}Gf \qquad (4)$$

In case of equivalent control method S = 0 has only one solution [3], i.e. matrix $[GB]^{-1}$ exists.

Supposing that system of eq. (1) is exposed to the control law (3) instead of control law (2), and the control vector has all nonlinearities considered, then function u has such character that the solution of (1) exists, and the motion of the sliding mode tak e s place in the neighborhood of discontinuity surface intersection

$$\| S \| \le \Delta, \qquad \Delta \text{ small}$$

Hence, for any finite time interval, it will be quite possible to show that

$$\lim_{\Delta \to 0} x(t) = x_f(t)$$

Where $x_f(t)$ is the solution of eq. (4).

From which it may be concluded that the sliding mode representation is true.

If a condition of the controlled system is shifted leading to the initiation of sliding motions, it may consider the condition of multi sliding modes, which are either closely nearer to (s = 0) or to the origin coordinates of m dimensional vector $S = (s_1, .. s_m)$

Consequently, the solution of the problem can be obtained by any method used for solving system stability, for the following equation of subsurface

$$\dot{s} = G\,f + G\,B\,u \qquad (5)$$

Solution of eq. (5) using standard methods, used for stability solutions is not effective, therefore, it is useful to consider particular conditions. These conditions depend on the matrix GB in eq.(5) according to its nature which may be, diagonal one with dominant diagonal and symmetrical one.

4 - DECOMPOSITION OF THE SYSTEM:

As mentioned in the previous section, the nature of the sliding mode does not depend on control mechanism, but it depends on the elements of the matrix G. These however, contain the gradient of, S_i. The motion of the sliding mode can be affected by changing the position of the switching surface within the state of the system.

The sliding mode can be described by (n - m)equations instead of n. That is because, any system satisfying $S(x) = 0$, and have m alternates, can be expressed by the remaining (n - m).

Hence, the order of the system is reduced using sliding mode concept and the system is decomposed into two independent subsystems of smaller dimensions. This simplifies analytical study and reduces computer computations.

The decomposition methodology can be summarized in the following steps;

1 - The switching surfaces are chosen to provide the motion in the sliding mode with the desired properties.

2 - Function u_i^+ and u_i^- are chosen such that, they insure the existence of the mode, through the switching surfaces $S(x)= 0$,

3 - The conditions which will allow the representative point to slide on the switching surfaces are provided. These points must belong to analytical position to hit this surface.

To illustrate this method, the second order system described by the following equations is considered:

$$\dot{x} = A\,x + B\,u \tag{6}$$

and $U = F\,x$, F = const.

To compose this system within the sliding mode the equation (6) is first expanded to;

$$\begin{aligned} \dot{x}_1 &= A_{11}\,x_1 + A_{12}\,x_2 \\ \dot{x}_2 &= A_{21}\,x_1 + A_{22}\,x_2 + B_2\,u \end{aligned} \tag{7}$$

where $x_1 \in R^{n-m}$, $x_2 \in R^m$ and $|B_2| \neq 0$

Transformation of coordinates for the case of rank B = m is always possible [2], if

$S = cx$ or in this example $S = C_1 x_1 + x_2$

For sliding mode in the vicinity of $S = 0$ and substituting we have

$$\dot{x}_1 = (A_{11} - A_{12} C_1) x_1 \tag{8}$$

For controlled system (6), the roots of equation (8) can be arranged in the desired way [2].

The second step is to choose the type of control to provide the decomposition of the sliding mode in the vicinity of $S = 0$ as follows:

(a) Design u as a discontinous function of the state vector

$$u = -\,a\,|x|\,B_2^{-1}\,\text{sign}\,S$$

Where; a is constant, $|x|$ is the sum of modules of states x_1 & x_2 and sign S is the vector with the components sign S_i (S_i components of vector S).

(b) Compute the time derivative of vector $S = C_1x_1 + x_2$ w.r.to time trajectories of the system (8), so, the following equation is deduced;

$$\dot{S} = (C_1A_{11} + A_{21})x_1 + (C_1A_{12} + A_{22})x_2 - a\ |x|\ \text{sign}\ S \qquad (9)$$

5 - INVARIANCE OF SYSTEMS WITH SLIDING MODE.

Invariance means that, the system, behavior is independent on external disturbances, and the equation describing this system is;

$$\dot{x} = Ax + Bu + Df \qquad (10)$$

Where f is the external disturbance

In this case, it is necessary to choose the control u such that, the solution of eq.(10) becomes independent on vector disturbance f(t), where $f \in R^l$. Assuming that, the space of the disturbance belongs to the space of control. i.e. D = B Z, where Z is a certain (m x 1) matrix. Then, the equation of the sliding mode for S = Cx = 0 is independent on disturbances. Using the equivalent control concept U_{eq} can be obtained from the equation

$$\dot{S} = C\ A\ x + C\ B\ u_{eq} + C\ D\ f\ = 0$$

And therefore;

$$U_{eq} = -\ (CB)^{-1}\ C\ A\ x\ -\ (CB)^{-1}\ C\ D\ f. \qquad (11)$$

Substituting u_{eq} into eq.(10) with regard to

D = B z yields:

$$\dot{x} = [A\ -\ B\ (CB)^{-1}C\ A\ ]x. \qquad (12)$$

which provides the sliding mode invariance towards disturbances. The spaces must be be chosen carefully.

6 - RESTORATION OF THE STATES AND THE PLANT PARAMETERS

In the above control algorithms the whole states are considered available, and this is not true. Therefore, in this section an algorithm in which some states are only available is deduced. The measured states are in the form;

$y = cx$, $\quad y \in R^k$ and the system in the sliding mode takes the form

$$\dot{\bar{x}} = A\bar{x} + Bu + L\ \text{sign}\ (C\ \bar{x}\ -\ y) \qquad (13)$$

where $\bar{x} \in R^n$, $\text{Sign}\ (C\bar{x} - y) \in R^k$

considering the error equation in the form

$$\bar{e} = \bar{x} - x$$

Solving equations (6) and (13) yields;

$$\dot{e} = A\ e + L\ \text{sign}\ S, \qquad S = C\ e \qquad (14)$$

As far as the right side in (14) undergoes discontinuities in the vicinity of $s = 0$, the sliding mode may appear if the pair A and C is an observed one [3]. If such L matrix exists it would be possible to secure the homogeneous system stability.

As it follows from the stability factor $\lim \bar{e} = 0$ as $t \longrightarrow \infty$ then, $\lim \bar{x} = x$ at $t \longrightarrow \infty$, this solves the problem. It is important that, if the disturbances appear in the observations, filter is almost optimal (14), as well as, the filtration of their statistic characteristics, even, of their variation is unknown.

Considering another restriction, related to the realization of the synthesis procedure, described in [2]. When some parameters of the plant object (or matrices A and B) in eq. (6) are unknown and will fail to obtain the control invariance for these parameters, it is necessary to make their identification either to select the controlling impacts or to organize the adaptation process.

The solution of the identification problem most often supposes the use of models with continuous algorithms, whose states and parameters converges with the states and parameters of the plant [4].

constructing the model;

$\dot{x}_m = A_m x + B_m u + v$, $v = \Psi_x x + \Psi_u u$ is an additional control signal With the coefficients of matrices Ψ_x and Ψ_u which undergoes discontinuities on the planes, if the vicinity of $S = x - x_m = 0$, is formed, the sliding mode allows to obtain the identical equality of the model and the object states vectors and thus, to simplify to a major extent, the algorithms of matrices A_m and B_m alternation.

According to [3] the procedures of the model parameters reconstruction.

$A_m = -\lambda\, v\, x^T$, $B_m = -\lambda\, v\, u^T$, λ is constant

This allows to fulfill identification of all parameters of the linear object [4].

7 - APPLICATION EXAMPLE

To check the suggested method, a synchronous generator connected to an infinite bus through a long series compensated transmission line is considered. The transients in the synchronous machine stator are neglected for simplification.

The following equations describing the generator in d, q reference frame are taken [1,9]

$$P\delta = \omega_0 S \qquad (15.a)$$

$$M\,P\,S = -K_d S + T_m - T_e \qquad (15.b)$$

$$\bar{T}_{d0}\, P\, \bar{e}_q = V_f - (x_d - \bar{x}_d) I_d - \bar{e}_q \qquad (15.c)$$

$$\bar{e}_d = V_d + r_a I_d - x_q I_q \tag{15.d}$$

$$\bar{e}_q = V_q + r_a I_q + \bar{x}_d I_d \tag{15.e}$$

$$T_e = \bar{e}_q I_q - (\bar{x}_d - x_q) I_d I_q \tag{15.f}$$

$$V_t^2 = V_d^2 + V_q^2 \tag{15.g}$$

The equation describing the transmission system is

$$V_d = V_b \sin\delta + r_e I_d - x_e I_q \tag{15.h}$$

$$V_q = V_b \cos\delta + r_e I_q + x_e I_d \tag{15.i}$$

Where, V_b is the bus voltage and r_e, x_e are transmission line resistance and reactance. The equations of the excitation control is;

$$T_e P V_f = u - V_f \tag{15.j}$$

Linearizing equations [15.a] to [15.j] about an initial operating point yields;

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u \tag{15.k}$$

$$\Delta v_t = \bar{c}\, x_2$$

Where x_1 and x_2 represent the state vector and non state respectively, which is;

$$x_1 = [\Delta\delta \quad \Delta S \quad \Delta\bar{e}q \quad \Delta V_f]^T, \qquad x_2 = [\Delta i_d \; \Delta i_q \; \Delta v_d \; \Delta v_q]^T$$

The values of A_{11}, A_{12}, A_{21}, and A_{22} are found to be.

$$A_{11} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ 0 & -K_d/M & -I_{q_0}/M & 0 \\ 0 & 0 & -1/\bar{T}_{d0} & 1/\bar{T}_{d0} \\ 0 & 0 & 0 & -1/T_e \end{bmatrix} \qquad A_{22} = \begin{bmatrix} r_a & -x_q & 1 & 0 \\ \bar{x}_d & r_a & 0 & 1 \\ r_e & -x_e & -1 & 0 \\ x_e & r_e & 0 & -1 \end{bmatrix}$$

$$A_{21}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ V_b\cos\delta_0 & 0 & 0 & 0 \\ -V_b\sin\delta_0 & 0 & 0 & 0 \end{bmatrix} \quad A_{12}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ \dfrac{(\bar{x}_d - x_q)Iq_0}{M} & -eq_0/M & 0 & 0 \\ \dfrac{\bar{x}_d - x_d}{\bar{T}d_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The values of B and C are

$$B = [0 \quad 0 \quad 0 \quad 1/T_e]^T$$

$$\bar{C} = [0 \quad 0 \quad Vd_0/Vt_0 \quad Vq_0/Vt_0]$$

and

$$eq_0 = \bar{e}q_0 - (\bar{x}_d - x_d)\, Id_0$$

using the elementary matrix reduction

$$\dot{x}_1 = A\, x_1 + B\, \Delta u \qquad (15.l)$$

$$\Delta V_t = C\, x_1 \qquad (15.m)$$

where $A = [A_{11} - A_{12}A_{22}^{-1}A_{21}]$ (15.n)

$$C = -\bar{C}\, A_{22}^{-1}\, A_{21} \qquad (15.o)$$

It is interesting to note that if no resistance exists in the stator circuit ($r_a = r_e = 0$) and no active power is sent by the generator, then $i_{q_0} = \delta_0 = V_{d0} = 0$, $V_{q_0} = V_{t0}$, and matrices A,B,C IN Eq. (15.n, 15.o) become.

$$A = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ \dfrac{-e_{q_0} V_b}{M(x_q + x_e)} & \dfrac{-K_d}{M} & 0 & 0 \\ 0 & 0 & \dfrac{-(x_d + x_e)}{\bar{T}_{d0}(\bar{x}_d + \bar{x}_e)} & -1/\bar{T}_{d0} \\ 0 & 0 & 0 & -1/T_a \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 1/T_e \end{bmatrix}^T, \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus the 4th order system is neither completely controllable nor completely observable under this condition.

The results of fig. (1) show that, the terminal voltage effectively follows the change of set point and only low amplitude oscillations occur, but it soon damped.

The system in this case is suffering from the problem of SSR (system stability under subsynchronous resonance effect)due to the series compensation. These are possible by increasing the compensation percentage. It is seen from fig. (1) that,the system transients are well improved to a large extent.

8 - CONCLUSIONS

The analysis and principles of discontinous control using sliding mode technique are studied. An approach of using decomposition in the design process is also shown.

We have presented a new approach to the design of composite controller for synchronous generator based on sliding mode technique.

It is demonstrated by computer simulation that for the change of generator power, terminal voltage and torque angle, the controller has acceptable performance. By using controller the generator can be operated at the transmission power limit, thus the power system steady state stability can be improved to the maximum.

9 - REFERENCES

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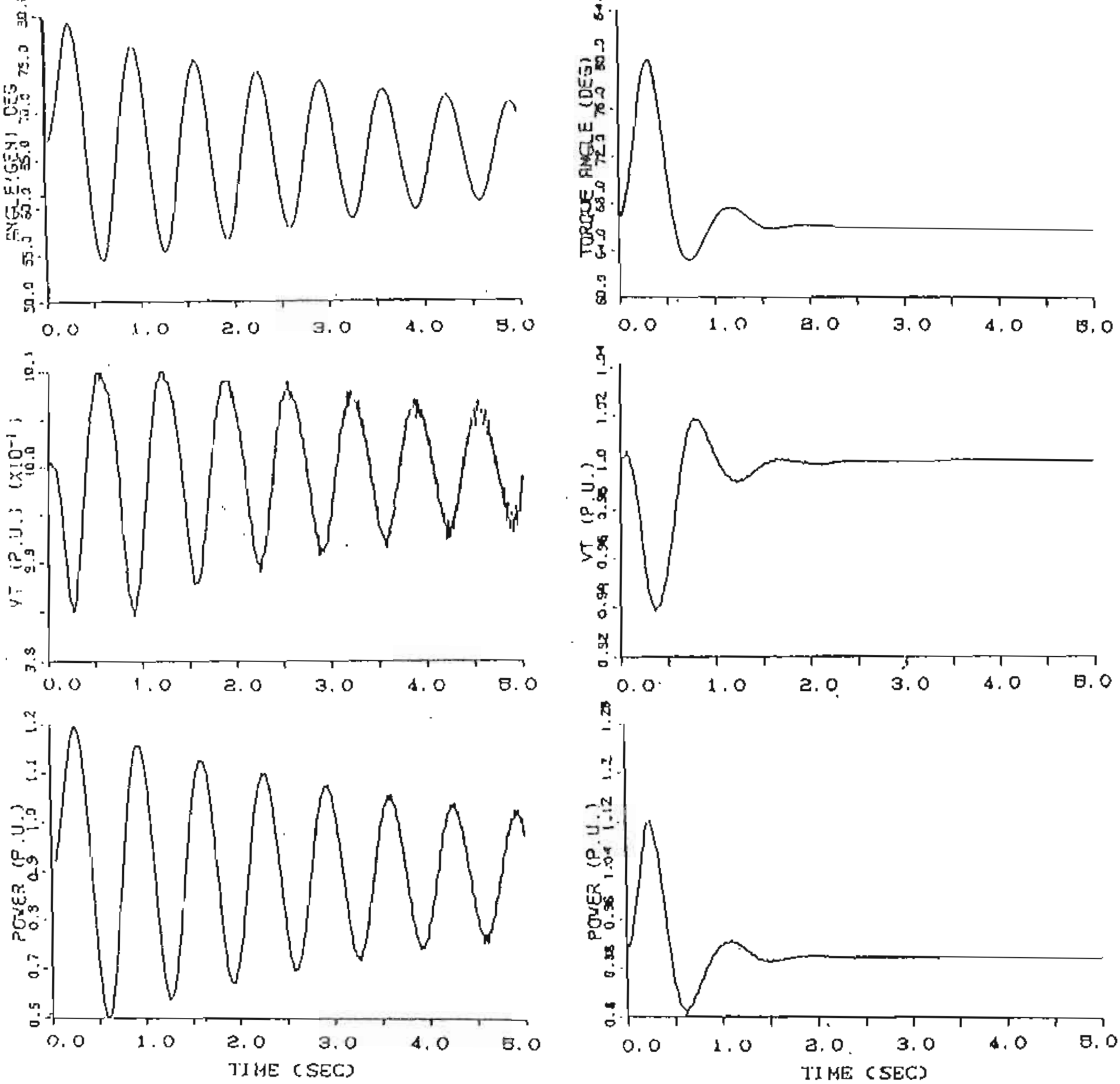

Figure 1 Dynamic response of the generator
(left) without control; (right) with control

DEFINITIONS

δ = Torque angle

p = $\frac{d}{dt}$

ω_0 = Synchronous frequency

M = Inertia constant

K_d = Damping coefficient

T_m, T_e = Mechanical and electric torque

S = Slip

$\bar{e}_d$, $\bar{e}_q$ = Direct and quadrature transient voltages

v_f = Field voltage

x_d, x_q = Direct and quadrature axis reactances

$\bar{x}_d$, $\bar{x}_q$ = Transient reactances

I_d, I_q = direct and quadrature stator currents

V_d, V_q = Direct and quadrature voltages

V_b = Bus voltage

V_t = Terminal voltage

r_a, r_e = Armature and transmission resistances

System data

$r_a = 0$	$K_d = 3$
$x_d = 1.904$	$T_{d0} = 5.66$ sec
$\bar{x}_d = 0.312$	$T_{q0} = 0.065$
$x_q = 1.881$	$M = 5.529$
$\bar{x}_q = 0.26$	