



Answer the following questions

- 1) State the necessary and sufficient conditions for the minimum of a function $f(\mathbf{x})$.
- 2) Write the Taylor's series expansion of a function $f(\mathbf{x})$.
- 3) Discuss the Lagrange multiplier method.
- 4) Determine whether each of the following quadratic forms is positive definite, negative definite, or neither:

i) $f = -x_1^2 + 4x_1x_2 + 4x_2^2$

ii) $f = -x_1^2 + 4x_1x_2 - 9x_2^2 + 2x_1x_3 + 8x_2x_3 - 4x_3^2$

- 5) Find the maximum and minimum values of the function

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5$$

- 6) Find the second-order Taylor's series approximation of the function

$$f(x_1, x_2, x_3) = x_2^2x_3 + x_1e^{x_3} \quad \text{about the point } x^0 = \{1, 0, -2\}^T$$

- 7) Find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius.

$$\text{Minimize } f(x, y) = kx^{-1}y^{-2}$$

$$\text{Subject to } g(x, y) = x^2 + y^2 - a^2 = 0$$

Using the Lagrange multiplier method

- 8) Minimize $f(x_1, x_2) = x_1^2 + x_2^2 - 2x_1 - 4x_2$

$$\text{Subject to } g_1(x_1, x_2) = x_1 + 4x_2 - 5 \leq 0$$

$$g_2(x_1, x_2) = 2x_1 + 3x_2 - 6 \leq 0$$

$$g_3(x_1, x_2) = -x_1 \leq 0$$

$$g_4(x_1, x_2) = -x_2 \leq 0$$

Starting from the point $x = \begin{Bmatrix} 1.0 \\ 1.0 \end{Bmatrix}$.

- 9) Minimize $z = 10x_1 + 5x_2 + 4x_3$

$$\text{Subject to } 3x_1 + 2x_2 - 3x_3 \geq 3$$

$$4x_1 + 2x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

Solve the primal problem by applying the dual simplex algorithm