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Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Science Dep. Post Graduate Examination, 2016-2017 Date of Exam : 07 / 06 / 2017	Minoufia University	Subject: Introduction to Ordinary Differential Equations Code: BES 506 Time Allowed : 3 hrs Total Marks: 100 Marks

Answer all the following questions الامتحان في صفحتان

Question 1 [25 Marks]

(A) Find the general solution of the following first order first degree ordinary differential equations:

1) $\frac{dy}{dx} = -\left(\frac{x}{y}\right)$ by all available different methods 2) $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

(B) Find the general and particular solution of the following first order first degree ordinary differential equations, thin explain the different between the general and particular solutions.

1) $(x\,y^2 - y)\,dx + (x + x^2\,y)\,dy = 0$ 2) $\frac{dy}{dx} + y = \sin x\ ,\ \ y(\pi) = 1$

(C) Find the general solution of the first order first degree ordinary differential equation:

$$x\frac{dy}{dx} + y = y^2 \ln x$$

Question 2 [25 Marks]

(A) Explain all cases of the integrating factor to reduce the first order first degree ordinary differential equation to an exact equation. Solve this equation as an example.

$$(x^2 - y^2)\,dx + x\,y\,dy = 0$$

(B) Find the general solution of the first order but not of first degree ordinary differential equations:

1) $\left(\frac{dy}{dx}\right)^2 - (x+y)\frac{dy}{dx} + xy = 0$ 2) $\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} + y = 0$

(C) Find the general solution of the second order first degree ordinary differential equations:

1- 1) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 4x$ 2) $y\,(y-1)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Question 3 [**25 Marks**]

(A) Prove that if $y_1 = e^x$, $y_2 = e^{2x}$, and $y_3 = e^{3x}$are linearly independent functions. Find the homogeneous differential equation which the complement solution is :
$y_c = c_1 y_1 + c_2 y_2 + c_3 y_3 where\ c_1, c_2, and\ c_3\ are\ constants$.

(B) Find the general solution of the non-homogenous system of differential equations:

$$\frac{d^2x}{dt^2} - y = e^{2t} \qquad \text{and} \qquad \frac{dy}{dt} - x = 20$$

(C) Find the total solution of the following non-homogenous differential equation by the linear differential operator method

$$\frac{d^4x}{dt^4} - 16\,x = cos^3(t)$$

Question 4 [**25 Marks**

(A) Find the total solution of the following non-homogenous differential equation by the undetermined coefficients method.

$$x^2 \frac{d^2y}{dx^2} - 16\,x \frac{dy}{dx} = x^2 + \ln x$$

(B) Find the total solution of the following non-homogenous differential equation by the undetermined coefficients method.

$$[(D)(D-1)(D-2)]\,x = sin(t) + e^{2x} + 15, D = \frac{d}{dt}$$

(C) Show that the power series solution of the differential equation :

$(x+1)\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} - 2y = 0$, using the Leibniz-Maclaurin method is given by :

$y = 1 + x^2 + e^x$, given the boundary conditions that at $\mathrm{x} = 0, \mathrm{y} = \frac{\mathrm{dy}}{\mathrm{dx}} = 1$.

With my best wishes *Dr. Mohamady Bassiony*