CRACK FORMATION

Cracking caused by the development of low melting point liquid films at the grain boundaries, by the presence of brittle segregates at the boundaries that are unable to withstand the restraint imposed during welding. In this case the resulting cracks are intergranular [4,5].

The welding operation causes internal stresses in the immediate vicinity of the weld. These stresses, acting on the base of a sharp notch, are sufficient to cause a crack to be formed spontaneously or under low load. This crack then either arrests or propagates catastrophically through the structure, depending on the inherent crack sensitivity of the metal and or stress level.

This work deals with investigating the nature of brittle fracture in welded pressure gas cylinders and into the contribution of welding to the failure of the cylinders.

PROPOSED GUIDELINES

To guarantee the performance of such welded joints in service conditions, it is necessary to establish certain guidelines at the manufacturing stage for preventing crack initiation leading to an unstable fracture. The implementation of the fracture-mechanics concept as a fracture design procedure then consists of five essential steps.

1-Considering the problem developed on a curvature plate for the calculation of stress intensity factor K_I in terms of various combinations of crack geometry, curvature shape parameter and type of applied loading.

2-Evaluation of the effect of the residual stress developed in the weldments, since it is necessary to determine relevant limits for cracks in residual stress field.

3-Determination of the critical stress intensity factor (K_{IC}) obtained from the value of K_I under conditions where a rapidly propagating crack is arrested.

4-Determination of the crack arrest size that will keep the value of K_I in the component less than K_{IC}. A safety range may apply to the stress, and a safety margin may also incorporate in the crack size.

5-The final step is to determine the total value of the stress intensity factor $\sum K_I$ including the effect of the stress intensity factor due to the residual stress and then compare this value with the determined value of critical stress intensity factor (K_{IC}).

1. FORMULATION

Considering the problem developed on a curvature plane as shown in Fig 1, a point q on the middle surface of the shell can be defined by a set of coordinates (r,θ). The polar coordinates (r,θ) with crack tip as the origin related to (X, Y) by:

$X = l + r\cos\theta$

$Y = r\sin\theta$

The governing differential equation for a thin cylindrical shell, can be written as follows:

$$\nabla^4 \phi = 0$$

where

$$\phi^{(n)} = R_n(r)\{\cos n\theta, \sin n\theta\}$$

It now remains to transform the solution to polar coordinates (r,θ) with crack tip as origin in order to recover the singular stress in the usual form and determine the stress intensity factors. Procedures for determination of expression for membrane and bending stresses at the crack tip are given in appendix-A

1- Membrane stresses:

The stress function obtained can be expressed in the following form:

$$\phi_1 = m_0 + m_1(r.\cos\theta) + m_2(r^{1/2}.\cos\tfrac{\theta}{2}) + m_3(r^{1/2}\cos\tfrac{3\theta}{2}) + m_4(r\sin\theta) + m_5(r^{1/2}\sin\tfrac{\theta}{2}) + m_6(r^{1/2}\sin\tfrac{3\theta}{2})$$

$$\sigma_x^m = \frac{F_I^m}{\sqrt{2r}}[\tfrac{1}{4}\cos\tfrac{5\theta}{2} + \tfrac{3}{4}\cos\tfrac{\theta}{2}]$$

$$\sigma_y^m = \frac{F_I^m}{\sqrt{2r}}[\tfrac{5}{4}\cos\tfrac{\theta}{2} - \tfrac{1}{4}\cos\tfrac{5\theta}{2}] \qquad (1)$$

$$\tau_{xy}^m = \frac{F_I^m}{\sqrt{2r}}[\tfrac{1}{4}\sin\tfrac{5\theta}{2} - \tfrac{1}{4}\sin\tfrac{5\theta}{2}]$$

where

$$F_I^m = \sigma_{app}(1 + \frac{\pi}{8}\beta^2)$$

2- Bending stresses:

The stress function obtained can be expressed in the following form:

$$\phi_2 = b_0 + b_1(r.\cos\theta) + b_2(r^{1/2}.\cos\tfrac{\theta}{2}) + b_3(r^{1/2}.\cos\tfrac{3\theta}{2}) + b_4(r.\sin\theta) + b_5(r^{1/2}.\sin\tfrac{\theta}{2}) + b_6(r^{1/2}\sin\tfrac{3\theta}{2})$$

$$\sigma_x^b = \pm\frac{F_I^b}{\sqrt{2r}}[\{\tfrac{3}{4}(1-\mu) + (1+\mu)\}\cos\tfrac{\theta}{2} - \tfrac{1}{4}(1-\mu)\cos\tfrac{5\theta}{2}$$

$$\sigma_y^b = \pm\frac{F_I^b}{\sqrt{2r}}[\{(1-\mu) - \tfrac{3}{4}(1+\mu)\}\cos\tfrac{\theta}{2} + \tfrac{1}{4}(1-\mu)\cos\tfrac{5\theta}{2}]$$

$$\tau_{xy}^b = \pm\frac{F_I^b}{\sqrt{2r}}[\tfrac{3}{4}(1-\mu)\sin\tfrac{\theta}{2} - (1-\mu)\sin\tfrac{5\theta}{2}] \qquad (2)$$

where

$$F_I^b = \sigma_{app}\frac{1+3\beta^2}{t\sqrt{12(1-\mu^2)}}$$

The state of stress at the tip of an arbitrarily oriented crack characterized as opening mode (Mod-I) Following the convention for plates, the membrane and bending stress intensity factors K_I^m and K_I^b for the welded pressure gas cylinder may be defined as follows [6,7]:

$$K_I^m = \sqrt{2\pi ar}\ \sigma_\theta^m(r,\theta)$$
$$K_I^b = \pm\sqrt{2\pi ar}\ \sigma_\theta^b(r,\theta) \qquad (3)$$

where + and - signs refer to the outer and inner surfaces of the container respectively.

The perturbation parameter being a curvature parameter β defining the size of the crack with respect to the shell dimension and given by

$$\beta^2 \doteq \sqrt{12(1-\mu^2)}\ \frac{t}{4d_i} \qquad (4)$$

where t, d_i thickness and diameter of the cylinder.

For opening-mode situation, the above equations are then modified where K_I, the mod-I stress intensity factor can be evaluated as follows:

$$K_I^m = \sqrt{2\pi a}\ \sigma_{app}\left(1+\frac{5\pi}{8}\beta^2\right)\left[\tfrac{3}{4}\cos\left(\tfrac{\pi}{2}-\tfrac{\theta}{2}\right)-\tfrac{1}{4}\cos\left(\tfrac{\pi}{2}-\tfrac{5\theta}{2}\right)\right] \tag{5}$$

and

$$K_I^b = \pm\sqrt{2\pi a}\ \sigma_{app}\left(\frac{1+3\beta^2}{\sqrt{12(1-\mu^2)}}\right)\{[(1+\mu)-\tfrac{3}{4}(1-\mu)]\cos\left(\tfrac{\pi}{2}-\tfrac{\theta}{2}\right)+\tfrac{1}{4}(1-\mu)\cos\left(\tfrac{\pi}{2}-\tfrac{5\theta}{2}\right)\} \tag{6}$$

Plots of membrane and bending components of stress intensity factors for general crack angles and dimensionless parameter of crack length to shell radius ratios are given in Figs. (2-5)

2. RESIDUAL STRESS ANALYSIS

Weldments are often the most sensitive part of a structure with regard to crack growth and failure. There are several reasons for this. The most important cause is that the temperature cycling of the material during welding process that sets up residual stress field in the weldment. These stresses are superposed on the mechanical stresses and thus affect the fracture behavior of the weldment [8-10].
To take residual stress into account in fracture analysis it is necessary to determine relevant parameters for cracks in residual stress field i.e. stress intensity factor.

$$K_I^r = 0.25 . p_r . d^{1/2} \sqrt{2\pi\beta}\left\{\sqrt{\left(\frac{t}{d_i}\right)}\left[1+\left(\frac{t}{d_i}\right)\right]\right\}^{-1} \tag{7}$$

Details of the derivation of equation (7) are indicated in Appendix-B.

3. CRITICAL STRESS INTENSITY FACTOR

It is to be expected that in the LEFM regime the criterion for crack growth initiation is

$$\sum K_I \le K_{IC} \tag{8}$$

For brittle behavior where crack growth initiation and unstable crack growth occur almost simultaneously it would be relevant to use the maximum value of $\sum K_I$ along the crack front. This interpretation of the criterion has also found experimental support in several studies as reported in [9,11].

Investigations were carried out to evaluate the value of critical stress intensity factor, that can be considered as the material fracture toughness. An attempt was made to correlate the critical crack size at the design stress level with critical stress intensity factor. However, a simplified approach has been made based on certain assumptions in the present analysis. On the basis of above, the best arrived correlation and the arrived empirical relation for determination of K_{IC} can be given by using the following equation.

$$K_{IC}^w = \sqrt{2\pi\ b}\ \sigma_{yield}\left[1+\beta\left(\frac{t}{d_i}\right)\right]^{-1} \tag{9}$$

Details of the of equation (9) are indicated in Appendix-C

Where b is an independent variable that represents the critical crack size.

The leak-before-break criterion that was proposed by Irwin as a means of estimating the necessary toughness of pressure-vessel so that a surface crack could grow through the wall thickness and the vessel leak before fracturing. That is, the critical crack size at the design stress level of a material meeting this criterion would be greater than the wall thickness of vessel. Thus the leak-before-break criterion assumes that a crack of twice the wall thickness in length should be stable at a stress equal to nominal design stress.

4. CRACK ARREST

Considering the fracture mechanics criteria it may be ascertained that conditions for crack arrest are still maintained at an applied stress σ_{app} with ratio of $\sigma_{app}/\sigma_{yield}$ as for design stress, provided that the arrest fracture toughness of the weld metal is at least given as $\sigma_y (\pi b)^{1/2}$ [12-14].

The first approach to crack arrest was to postulate that it was essentially a reveres process of crack initiation. If static conditions, in particular a static stress intensity factors, is responsible for crack initiation, then crack arrest should occur at a stress intensity that is calculated from the arrest geometry and stress loading condition

An attempt was made to correlate the critical crack size at the design stress level with yield strength property It is generally accepted that yield stress loading may occur in the vicinity of structure discontinuities, the critical crack size ,a_c, is proportional to $(K_{IC}/\sigma_y)^2$. Thus, the maximum crack size a structure member can tolerate at a particular stress level is

$$2a_c = c\left(\frac{K_{IC}}{\sigma_y}\right)^2 \tag{10}$$

$$c = \left(\frac{1+3\beta^2}{2\pi}\right)$$

where c is a geometry factor

By knowing the critical value of stress inteusity factor K_{IC} for a particular thickness and loading rate, thus it is possible to determine the crack size that can be tolerated for a given design stress level. It is also possible to determine the design stress level that can be tolerated for an existing crack that may be present in a structure.

5. TOTAL APPLIED STRESS INTENSITY FACTOR

This value of critical stress intensity factor K_{IC} may compare with the total values of stress intensity factors obtained. Hence, the simple analytical expression for crack growth in finite plates $\Delta a = a - a_o$ as a function of K_I in the process of monotonic loading given as [1,5]:

$$\frac{\Delta a}{a_0} = -\left(\frac{\Sigma K_I}{K_{IC}}\right)^2 - \ln\left[1-\left(\frac{\Sigma K_I}{K_{IC}}\right)^2\right]$$

This equation can be modified by taking the curvature parameter into account to satisfy the original problem of crack growth in cylindrical shells by the following expression.

$$\frac{\Delta a}{a_0} = \left(1+\beta^2\right)\left\{-\left(\frac{\Sigma K_I}{K_{IC}}\right)^2 - \ln\left[1-\left(\frac{\Sigma K_I}{K_{IC}}\right)^2\right]\right\} \tag{11}$$

Equation (11) in the dimensionless variables describes the available data quite well as shown in Fig. 7

DISCUSSION

It is essential to emphasize that the quantity K_{IC} which corresponds to the transition on the unstable region and, therefore, the determined failure point is less than the maximum value reached as $\Delta a \rightarrow \infty$. The difference between the failure point and the maximum value of K_I disappears only during brittle fracture, when there is no slow growth or plastic zone. If the crack is stable, that is $dK_I/da \cong 0$.

Examination of equation (10) and Fig. 7, indicates that, using a lower strength level material with higher toughness (a larger K_{IC}/σ_y), even at a design stress which is of a higher percentage of the yield stress, would have been better. However (K_{IC}/σ_y) ratio is one of the primary controling design parameters that can be used to define the relative safety of a structure against brittle fracture

In applying conventional crack propagation theories in the presence of combined stretching and bending, as well as residual stress, a so-called effective stress intensity factor is derived based on an extension of Dugdales theory to cylindrical bending of plates. In mode-I type, the effective stress intensity factor is obtained as

$$K_I^{eff} = \Sigma K$$
$$= K_I^m + K_I^b + K_I^r$$

which is found to give good correlation with K_{IC} in predicting crack propagation rates in cylindrical shells.

Such a numerical analysis presented for solving the problems of existing cracks in welded cylindrical pressure gas cylinder. from which it may invent the condition for crack arrests.

TABLE. 1. Crack arrest conditions.

β	(t/d$_i$)	t mm.	2a$_c$ mm.	$\frac{\Sigma K = K_I^{eff}}{K_{IC}}$	$\frac{\Delta a}{a_o}$
0.1	0.012	3.52	7.04	0.54	0.05
0.2	0.048	3.63	7.26	0.60	0.09
0.4	0.190	4.05	8.10	0.70	0.21
0.6	0.430	4.75	9.50	0.80	0.52
0.8	0.780	5.72	11.44	0.90	1.40

The results of the calculations may be simply expressed in that the possibility of practicable crack arrest conditions determined at an applied stress up to a certain ratio of yield stress. Thereupon the value of crack arrest fracture toughness can be determined by using equation (9).

CONCLUSIONS

1-Simple analytical expressions have been developed to analyze and provide a simple method for fracture control in welded pressure gas cylinders.

2-Analytical results have been obtained by using the developed equations for the determination of singular stresses that characterize the stress field at the crack tip. The stress intensity factor for the effects of residual stress has been evaluated. Thereupon, charts are also presented from which the stress intensity factors can be determined..

3- An attempt have been made to correlate the critical crack size at the design stress level with yield strength, in that the possibility of practicale crack arrest conditions determined at an applied stress up to a certain ratio of yield stress.on the other hand, it is possible to determine the crack size that can be tolerated for a given design stress level. It is also possible to determine the design stress level that can be tolerated for an existing crack that may be present in a structure.

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APPINDEX-A
DETERMINATION OF STRESS INTENSITY FACTOR K_I

In discussing stresses in cylindrical shell of circular cross section, it is advantageous to use polar coordinates. The stress equations of equilibrium in polar coordinates can be derived from the free body diagram of the polar elements. Summing all forces in the radial r and tangential θ Directions and considering the body-force intensity F, gives the simplified equations of equilibrium. The two equations of stresses in radial and tangential directions take the final form

A-in the radial direction.

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)+F_r=0$$

B-in the tangential direction.

$$\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}+F_\theta=0$$

When the body force is zero they are satisfy by putting

$$\sigma_{rr}=\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}$$

$$\sigma_{\theta}=\frac{\partial^2\phi}{\partial r^2}$$

$$\tau_{r\theta}=\frac{1}{r^2}\frac{\partial\phi}{\partial\theta}-\frac{1}{r}\frac{\partial^2\phi}{\partial r.\partial\theta}$$

where Φ is the stress function as a function of r and θ .

The membrane and bending stresses at the crack tip are obtained from the stress function Φ using the following relations:

$$\sigma_r^m=\frac{1}{ta^2}\left[\frac{1}{r}\frac{\partial\phi_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi_1}{\partial\theta^2}\right]$$

$$\sigma_\theta^m=\frac{1}{ta^2}\left[\frac{\partial^2\phi_1}{\partial r^2}\right]$$

$$\tau_{r\theta}^m=-\frac{1}{ta^2}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\phi_1}{\partial\theta}\right]$$

$$\sigma_r^b=\pm\frac{Et}{2a^2\left(1-\mu^2\right)}\left[\frac{\partial^2\phi_2}{\partial r^2}+\mu\left(\frac{1}{r}\frac{\partial\phi_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi_2}{\partial\theta^2}\right)\right]$$

$$\sigma_\theta^b=\pm\frac{Et}{2a^2\left(1-\mu^2\right)}\left[\left(\frac{1}{r}\frac{\partial\phi_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi_2}{\partial\theta^2}\right)+\mu\frac{\partial^2\phi_2}{\partial r^2}\right]$$

$$\tau_{r\theta}^b=\pm\frac{Et}{2a^2\left(1+\mu\ \right)}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\phi_2}{\partial\theta}\right]$$

APPINDEX-B

DETERMINATION OF RESIDUAL STRESS INTENSITY FACTOR K_I^r.

$$\sigma_r = \frac{p_i \cdot d_i^2}{\left(d_o^2 - d_i^2\right)} = \frac{p_i \cdot d_i^2}{\left[\left(d_i + 2t\right)^2 - d_i^2\right]}$$

$$\sigma_r = \frac{p_i \cdot d_i^2}{4d_i^2\left(\frac{t}{d_i}\right)\left[1+\left(\frac{t}{d_i}\right)\right]}$$

$$\sigma_r = \frac{p_i}{4.\left(\frac{t}{d_i}\right)\left[1+\left(\frac{t}{d_i}\right)\right]}$$

$$\sigma_r = 0.25\ p_i \cdot \left\{\left(\frac{t}{d_i}\right)\cdot\left[1+\left(\frac{t}{d_i}\right)\right]\right\}^{-1}$$

$$K_I^r = \sigma_r \sqrt{\pi\ b\beta}$$

Where b is independent variable that represents the critical crack size in the residual stress zone of total length twice the cylinder thickness t, and β is the curvature parameter defining the size of the crack with respect to the shell dimension. The residual stress intensity factor can be evaluated as follows.

$$K_I^r = 0.25 . p_i \left\{\left(\frac{t}{d_i}\right)\cdot\left[1+\left(\frac{t}{d_i}\right)\right]\right\}^{-1} . \sqrt{2t\pi\beta}$$

$$K_I^r = 0.25 . p_i . d^{1/2} \sqrt{2\pi\beta} \left\{\sqrt{\left(\frac{t}{d_i}\right)}\left[1+\left(\frac{t}{d_i}\right)\right]\right\}^{-1} \qquad (7)$$

APPINDEX-C

EMPIRICAL RELATION FOR DETERMINATION OF K_{IC}.

Fracture mechanics assumes that fracture occurs when the near tip stress field, described by K_I reaches a critical value. In other words in mode-I, fracture occurs when K_I reaches K_{IC}.

The interaction of material properties, such as the fracture toughness with yield stress the crack size or the crack effected zone size and the geometry factor of the component, controls the conditions for crack extension in that component. Thus the critical stress intensity factor is proportional to $\sigma_y \sqrt{2\pi b}$ and is represented by a letter K_{IC}.

$$K_{IC} = K_{max}$$

$$K_{IC}^{w} = \sqrt{2\pi b}\ \ \sigma_{yield}\ f(\beta)$$

where $f(\beta)$ is a geometry factor. $f(\beta) = \left[1+\beta\left(\frac{t}{d_i}\right)\right]^{-1}$

$$K_{IC}^{w} = \sqrt{2\pi b}\ \ \sigma_{yield}\left[1+\beta\left(\frac{t}{d_i}\right)\right]^{-1} \qquad (9)$$

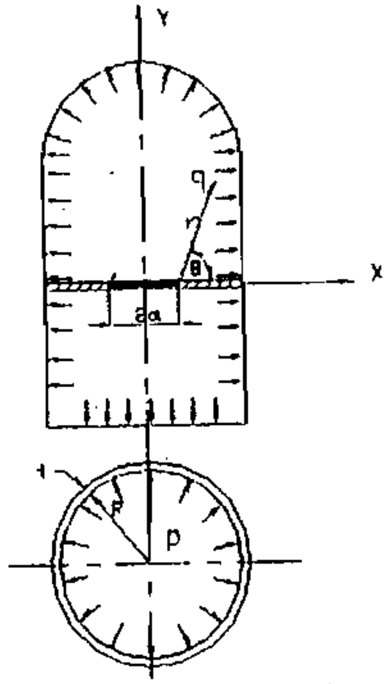

Fig. 1. An arbitrarily oriented crack.

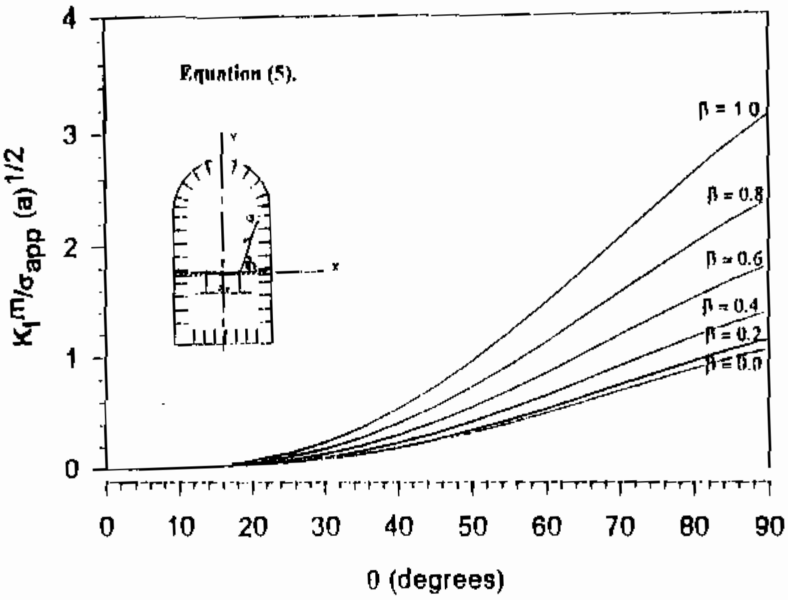

Fig. 2. Variation of membrane stress intensity factor with crack angle for different value of curvature parameter β.

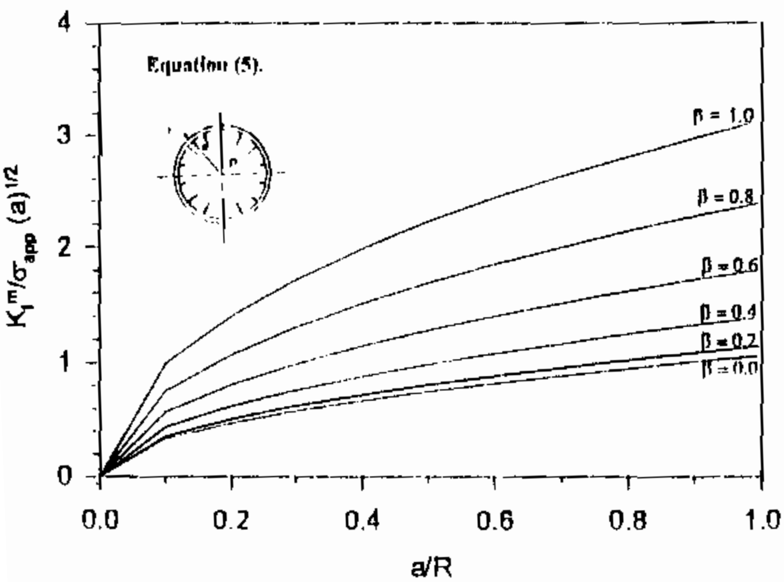

Fig. 3. Variation of membrane stress intensity factor with crack length to cylinder radius ratio.

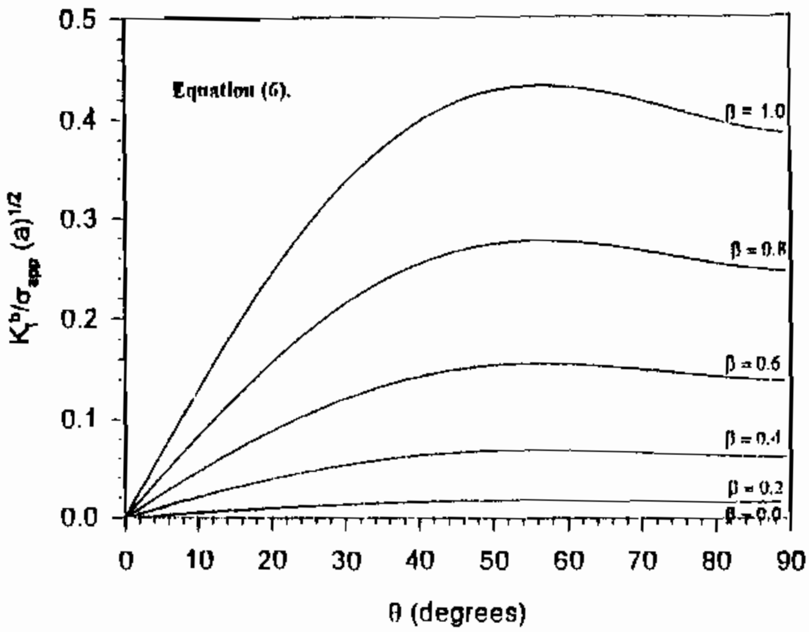

Fig. 4. Variation of bending stress intensity factor with crack angle for different value of curvature parameter β.

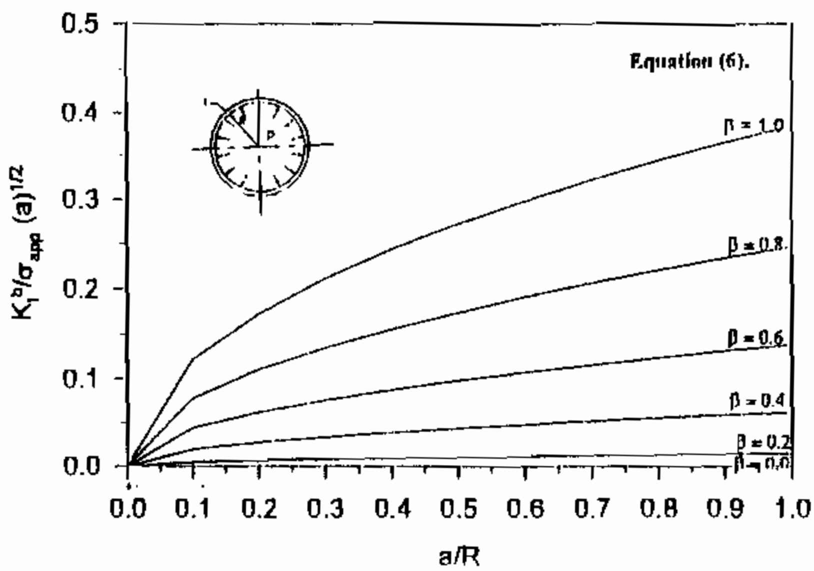

Fig. 5. Variation of bending stress intensity factor with crack length to cylinder radius ratio.

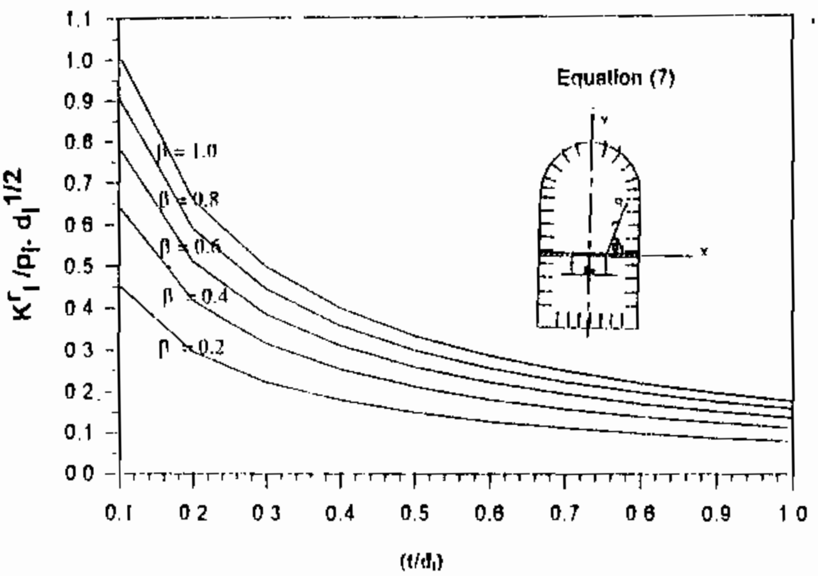

Fig. 6. Variation of residual stress intensity factor in the filed of residual stress zone.

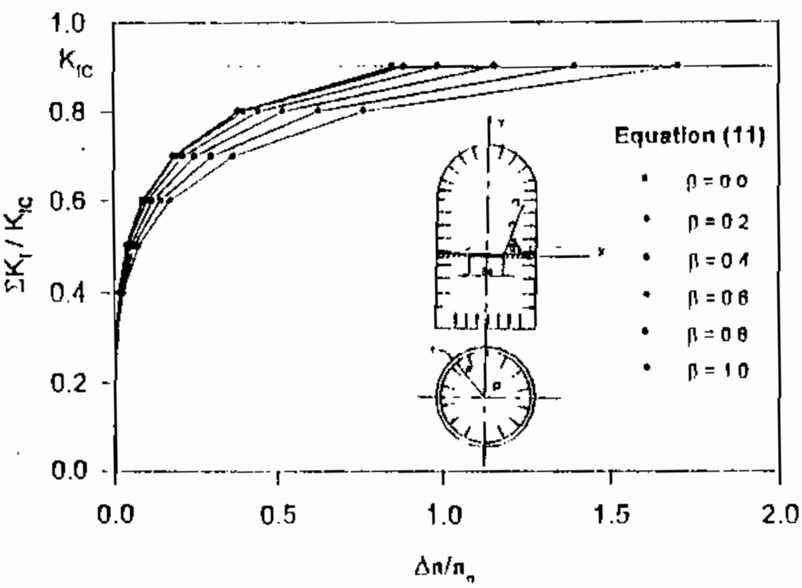

Fig. 7. Crack arrest conditions for fracture influenced by residual stress.