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MODELLING AND LINEARISED ANALYSIS OF RELUCTANCE MOTORS WITH STATIC BALANCERS

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ABSTRACT

In many instances, reluctance motors become unstable due to loss of damping the natural oscillatory mode that occures when the machine is subjected to small changes in load, supply voltage, ..., etc.This is undesirable since the application of reluctance motors is aimed at automated and continous flow processes which require complete synchronism between the various functions of the system .

This paper descibes modelling and linearised analysis of reluctance motors when working with static balancers in the presence of feeder impedance using the state-space approach. The non linear mathematical model based on d-q axis equations, is perturbed about an operating point and then arranged as a multi-input multi-output time - invarient linear model.

Dynamic performance of the motor is predicted and compared with the releavant experimental values as a function of supply condition. Also the results of increasing the load torque as well as the balancers capacitive reactance are demonstrated to enable physical inter pretation of the instability causes .

LIST OF MAIN SYMBOLS :

C = balancer capacitance

f = frequency

J	= moment of inertia
K_d	= friction constant
L_d, L_D	= stator and rotor d-axis self inductance
L_q, L_Q	= stator and rotor q-axis self inductance
L_f	= feeder inductance
M_{AD}	= mutual coupling coefficient for d-axis
M_{AQ}	= mutual coupling coefficient for q-axis
N	= speed
p, P	= number of pair poles , operator $(\frac{d}{d_t})$
R_a	= stator resistance
R_D, R_Q	= Rotor D & Q axis resistance
R_f	= feeder resistance
T_e	= electromagnetic torque
T_L	= load torque
$\hat{V}$	= maximum voltage of supply input
θ	= elect. angle between stator and rotor
ω	= angular velocity (rad./sec.)
δ	= load angle

Suffixes	a , b and c denote phases a , b and c
Suffixes	d , q denote stator d and q axis
Suffixes	D , Q denote rotor d and q axis
Suffixes	0 denote steady state quantities
Suffixes	s , m source and motor respectively
Suffixe	t denote transpose

INTRODUCTION

The problem of instability exists even for an ideal machine under ideal conditions [1,2]. Parallel aperation brings special problems but the present study is confined to an isolated machine. The complete generalised transient equations of a reluctance motor are nonlinear, and hence extremely difficult to solve analyically. Several analogue-computer solutions have been obtained for particular systems, but it is difficult to draw general conclusions from those particular resuls. In

many cases, the transient performance is only required for small oscillations about a steady -state condition, and the machine equations may be linearised, allowing more normal techniques of solution to be applied .
Instability problem of reluctance motors has had considerable practical importance and has given a great attention in many previous work. Lawrenson [1] applied the D-decomposition technique to study the effects of all motor parameters on machine stability. Honsinger [2] investigate motor instability in a simpler manner. Hoft [3] used the first lyapunov method to asses reluctance motor instability with only a single coil in the d-axis. This has been followed by lipo and krause [4] who employed the Nyquist stability criterion and study the effect of electrical parameters on motor instability. Stability boundaries of reluctance motor without damper circuits is given in reference [5].

In some instances, it may be convenient to modify the machine parameters so as to avoid machine instability. However, any significant change in most machine parameters is not normally possible in practice because an existing induction motor stator has normally to be employed. Therefore, in this paper, the investigation are confined to study the influence of load torque, supply voltage and balancers capacitive reactance on machine stability. A simple mathematical model for fast digital computation of reluctance motor transient behaviour is presented. This model avoids numerical inversion of the machine inductance matrix and in turn allows a large saving in computation time. The validity of the computer model, has been confirmed by the results obtained experimentally . The d-q axis non-linear differential equations which describes the system under consideration (machine, balancers and feeder) are arranged in state-space form. The equations are then perturbed about an operating point, leading to the time-inveriant state-space model. Dynamic stability of the machine is then predicted and the results are presented .

NONLINEAR MATHEMATICAL MODEL :

Prediction of transient behaviour of a polyphase reluctance motor, fed from a balanced power supply, can be obtained by solving machine equations along with mechanical system equations. The general non-linear equations of the machine are reported in reference [6] and they given here for completeness.

The transformed equations describing the electrical behaviour of the machine are :

$$\left.\begin{aligned} V_d &= R_a i_d + P\Psi_d - \Psi_q p\theta \\ V_q &= R_a i_q + P\Psi_q + \Psi_d p\theta \\ 0 &= R_D i_D + p\Psi_D \\ 0 &= R_Q i_Q + p\Psi_Q \end{aligned}\right\} \quad (1)$$

The flux linkages equations are given by

$$\left.\begin{aligned} \Psi_d &= L_d i_d + M_{AD} i_D \\ \Psi_q &= L_q i_q + M_{AQ} i_Q \\ \Psi_D &= L_D i_D + M_{AD} i_d \\ \Psi_Q &= L_Q i_Q + M_{AQ} i_q \end{aligned}\right\} \quad (2)$$

The axes voltages can be written as follows :

$$\left.\begin{aligned} V_d &= \hat{V} \sin\delta \\ V_q &= -\hat{V} \cos\delta \end{aligned}\right\} \quad (3)$$

The electromagnetic torque and mechanical system equations are given by :

$$T_E = \Psi_d i_q - \Psi_q i_d \quad (4)$$

$$P\omega_m = \frac{1}{J}(T_E - T_L - K_d \omega_m) \quad (5)$$

where $\omega_m = P\theta$ (6)

To avoid matrix inversion of the machine inductance, equations (1-6) were arranged in state-variable form as follows :

$$P\begin{bmatrix} i_d \\ i_q \\ i_D \\ i_Q \end{bmatrix} = \left[\begin{bmatrix} L_D/K_1 & O & -M_{AD}/K_1 & O \\ O & L_Q/K_2 & O & -M_{AQ}/K_2 \\ -M_{AD}/K_1 & O & L_d/K_1 & O \\ O & -M_{AQ}/K_2 & O & L_q/K_2 \end{bmatrix} \left[\begin{bmatrix} v_d \\ v_q \\ v_D \\ v_Q \end{bmatrix} - \begin{bmatrix} R_a & -L_q\dot{\theta} & O & -M_{AQ}\dot{\theta} \\ L_d\dot{\theta} & R_a & M_{AD}\dot{\theta} & O \\ O & O & R_D & O \\ O & O & O & R_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_D \\ i_Q \end{bmatrix} \right] \right] \quad (7)$$

where : $K_1 = L_d L_D - M^2_{AD}$

$K_2 = L_q L_Q - M^2_{AQ}$

The system under investigation is shown in Fig. (1) and the machine voltages are evaluated from :

$$Pv_{ma} = \frac{1}{C_a}(i_{sa} - i_{ma}) \quad (8)$$

$$Pv_{mb} = \frac{1}{C_b}(i_{sb} - i_{mb}) \quad (9)$$

$$Pv_{mc} = \frac{1}{C_c}(i_{sc} - i_{mc}) \quad (10)$$

The d and q axes supply currents are calculated from :

$$P\,i_{ds} = (v_{ds} - v_d - R_f i_{ds}) / L_f - \dot{\theta}\, i_q \quad (11)$$

$$P\,i_{qs} = (v_{qs} - v_q - R_f i_{qs}) / L_f - \dot{\theta}\, i_d \quad (12)$$

where :

$$\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = [G] \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix}$$

$$\begin{bmatrix} i_{d} \\ i_{q} \end{bmatrix} = [G] \begin{bmatrix} i_{ma} \\ i_{mb} \\ i_{mc} \end{bmatrix}$$

where G is the transformation matrix

v_{ds} , v_{qs} are the d and q axes voltages

i_{ds} , i_{qs} are the d and q axes currents.

A digital computer program has been developed to solve the above nonlinear equations, using a standard numerical integration technique.

VERIFICATION OF THE SIMULATION

A computer programe has been developed to simulate reluctance motors. The resulting differential equation describing the transient behaviour of the machine are inevitably nonlinear even when simplified by the assumption of constant parameters (i.e. neglection of saturation and skin effects). Equations (1-6) were then put in a state variable form , and conveniently solved numerically using the Runge-Kutta numerical integration method .

In order to confirm the validity of the simulation results , a series of tests are carried out on an available laboratory 3-phase reluctance motor having the following partieulars : $^1/_3$ hp, 4 -pole , 50 Hz, 220 / 380 v. The motor parameters are given in Appendix (1) . A balanced disturbance in the input voltage (10 percent reduction) is performed. Fig. (2) shows good agreement between simulated and experimental results of motor voltage and current for a load torque of 0.5 N.m. The above results are recorded for illustration and to confirm the validity of the simulation before embarking on any large scale of the linearisation study .

LINEARISED MODEL

The solutions of the complete nonlinear model give actual values for currents, voltages, electromagnetic torque, etc. in transient and steady state modes, but it does not readily give any indication of whether the machine is stable at a particular operating point nor does it give any quantitative information on the damping and the natural frequencies of the various modes of oscillation. On the other hand, the roots of the characteristic equation describing the system do provide the above information directly. A powerful method of finding the roots of the characteristic equation is by the analysis of the linearised system equations and this is computationally very efficient.

The nonlinear system equations are perturbed about an operating point and a general time-invariant system state-space model is obtained in the form[7].

$$\Delta \dot{\underset{\sim}{X}} = [A] \cdot \Delta \underset{\sim}{X} \tag{13}$$

where,

$$\Delta \underset{\sim}{X} = [\Delta i_{ds} \;\; \Delta i_{qs} \;\; \Delta i_D \;\; \Delta i_Q \;\; \Delta i_d \;\; \Delta i_q \;\; \Delta v_{ds} \;\; \Delta v_{qs} \;\; \Delta \omega_m \;\; \Delta \delta \;]^t$$

Matrix A is called the system matrix and describe the dynamic characteristic of the system including the reluctance motor under specific loading and parameters. Dynamic response of the machine can be evaluated by examining the roots of the characteristic equation :

$$\big[\, [\lambda] \; [I] \; - \; [A] \,\big] = O \tag{14}$$

The performed analysis to find the system matrix is tedious and not given here for simplicity . The final result of matrix A can be summarised in the following form, where the constants appear in Applendix (2) .

$$
[A] = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & 0 & 0 & A_{17} & 0 & A_{19} & 0 \\
A_{21} & A_{22} & A_{23} & A_{24} & 0 & 0 & 0 & A_{28} & A_{29} & 0 \\
A_{31} & A_{32} & A_{33} & A_{34} & 0 & 0 & A_{37} & 0 & A_{39} & 0 \\
A_{41} & A_{42} & A_{43} & A_{44} & 0 & 0 & 0 & A_{48} & A_{49} & 0 \\
0 & 0 & 0 & 0 & A_{55} & A_{56} & A_{57} & 0 & A_{59} & A_{510} \\
0 & 0 & 0 & 0 & A_{65} & A_{66} & 0 & A_{68} & A_{69} & A_{610} \\
A_{71} & 0 & 0 & 0 & A_{75} & 0 & 0 & A_{78} & A_{79} & 0 \\
0 & A_{82} & 0 & 0 & 0 & A_{86} & A_{87} & 0 & A_{89} & 0 \\
A_{91} & A_{92} & A_{93} & A_{94} & 0 & 0 & 0 & 0 & A_{99} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{109} & 0
\end{bmatrix}
$$

RESULTS

A computer program has been proposed to pridict the oscillatory modes from the characteristic equation (14). The system matrix is computed and then the solution of the characteristic equation can be obtained . The eigenvalues of matrix [A] are computed for different values of load torque and capacitive reactance of the balancers.

Fig.(3) shows the effect of increasing load torque on the oscillatory dominant mode at an input voltage and frequency equal to the rated values. The Figure reveals that , the eigen - values of the system initially decreases for light loads (region no. 1), then increases with increasing load torque (region no. 2), followed by another decreasing when the load torque is further increased . This means instability, stability and instability for the three indicated regions respectively according to the applied load torque.

The effect of increasing the static balancer capacitance is illustrated in fig. (4) . The response related to the rated values of input voltage, frequency and load torque (full-load). It is clear that, when the balancer capacitance inereases to a certain limit (region no.1), the eigen-values are increased ; and consequently the reliability of the system is increased. Increasing the balancer capacitance more than the above limit reduces the eigen - values (region no. 2) and hance the reliability of the system is reduced .

CONCLUSIONS

This paper presents a simple mathematical model for fast computation time of the transient performance of polyphase reluctance motor with terminal balancers in the presence of feeder impedance . Simulation results for starting behaviour is given and proved to be satisfactory when compared with the experimental results. The implementation of the state-space approach has been used to predict the eigen values associated with the system under investigation .

The study is restricted to establish the range of operating conditions of load torque and balancer capacitance under which the machine will be stable . No attempt has been made to define precise parameter boundaries which divide satisfactory operation from unsatisfactory operation , since these boundaries are fully reported in the literature .

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Appendix (1)

The parameters of the employed motor are :

$R_a = 27\ \Omega$	$R_D = 78.3\ \Omega$
$R_Q = 89\ \Omega$	$J = 92 \times 10^{-5}$ Kg. m^2
$L_d = 0.8216$ H	$L_q = 0.3248$ H
$L_D = 1.0732$ H	$L_Q = 0.6051$ H
$M_{AD} = 0.7516$ H	$M_{AQ} = 0.2548$ H

The feeder and shunt balancer parameters are :

$R_f = 4\ \Omega$, $L_f = 0.07$ H and $C = 7\mu$ F

Applenidx (2)

The following quantities are used for evaluating the system matrix

$A_{11} = - R_a L_D / K_1$;	$A_{12} = L_q \theta L_D / K_1$
$A_{13} = M_{AD} R_D / K_1$;	$A_{14} = L_D M_{AQ} \dot{\theta} / K_1$
$A_{17} = L_D / K_1$;	$A_{19} = L_D L_q I_{qso} / K_1$
$A_{21} = L_d \dot{\theta} L_Q / K_2$;	$A_{22} = - R_a L_Q / K_2$
$A_{23} = - M_{AD} \dot{\theta} L_Q / K_2$;	$A_{24} = M_{AQ} R_Q / K_2$
$A_{28} = L_Q / K_2$;	$A_{29} = -L_d I_{dso} L_Q / K_2$
$A_{31} = R_a M_{AD} / K_1$;	$A_{32} = - L_q \dot{\theta} M_{AD} / K_1$
$A_{33} = - R_D L_d / K_1$;	$A_{34} = - M_{AQ} \dot{\theta}_0 M_{AD} / K_1$
$A_{37} = - M_{AD} / K_1$;	$A_{39} = - M_{AD} L_q I_{qso} / K_1$
$A_{41} = M_{AQ} L_d \dot{\theta}_0 / K_2$;	$A_{42} = R_a M_{AQ} / K_2$
$A_{43} = M_{AD} M_{AQ} \dot{\theta} / K_2$;	$A_{44} = -R_Q L_q / K_2$
$A_{48} = M_{AQ} / K_2$;	$A_{49} = M_{AQ} L_d I_{dso} / K_2$
$A_{55} = - R_f / L_f$;	$A_{56} = \dot{\theta}_0$
$A_{57} = - 1/L_f$;	$A_{59} = I_{qo}$
$A_{510} = -\sqrt{\frac{3}{2}} \; \frac{\hat{V}}{L_f} \cos \delta_o$;	$A_{65} = - \dot{\theta}_O$
$A_{66} = - R_f / L_f$;	$A_{68} = - 1/L_f$

$$A_{69} = -I_{do} \quad ; \quad A_{610} = -\sqrt{\frac{3}{2}}\;\frac{\hat{V}}{L_f}\cos\;\delta_o$$

$$A_{71} = -1/C \quad ; \quad A_{75} = 1/C$$

$$A_{78} = \theta_0 \quad ; \quad A_{79} = V_{qso}$$

$$A_{82} = -1/C \quad ; \quad A_{86} = 1/C$$

$$A_{87} = \theta_0 \quad ; \quad A_{89} = -V_{dso}$$

$$A_{91} = \frac{3}{2}\frac{P}{J} I_{qso}(L_d - L_q) \; ; \; A_{92} = \frac{3}{2}\frac{P}{J} Idso(L_d - L_q)$$

$$A_{93} = \frac{3}{2}\frac{P}{J} M_{AD} I_{qso} \quad ; \quad A_{94} = -\frac{3}{2}\frac{P}{J} M_{AQ} I_{dso}$$

$$A_{99} = -K_d/J \quad ; \quad A_{109} = -1/p$$

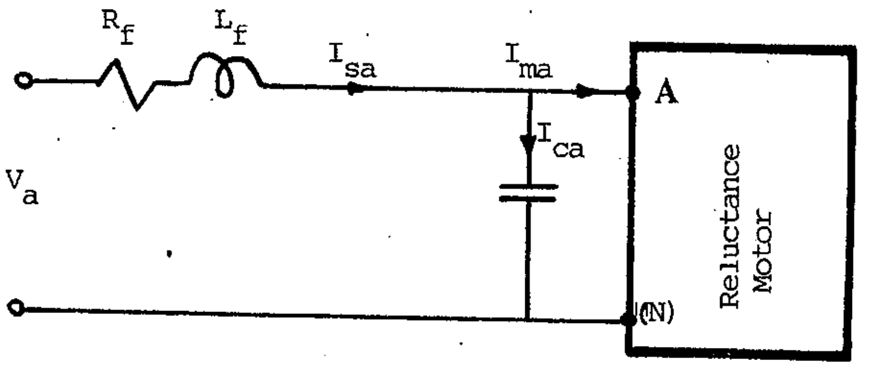
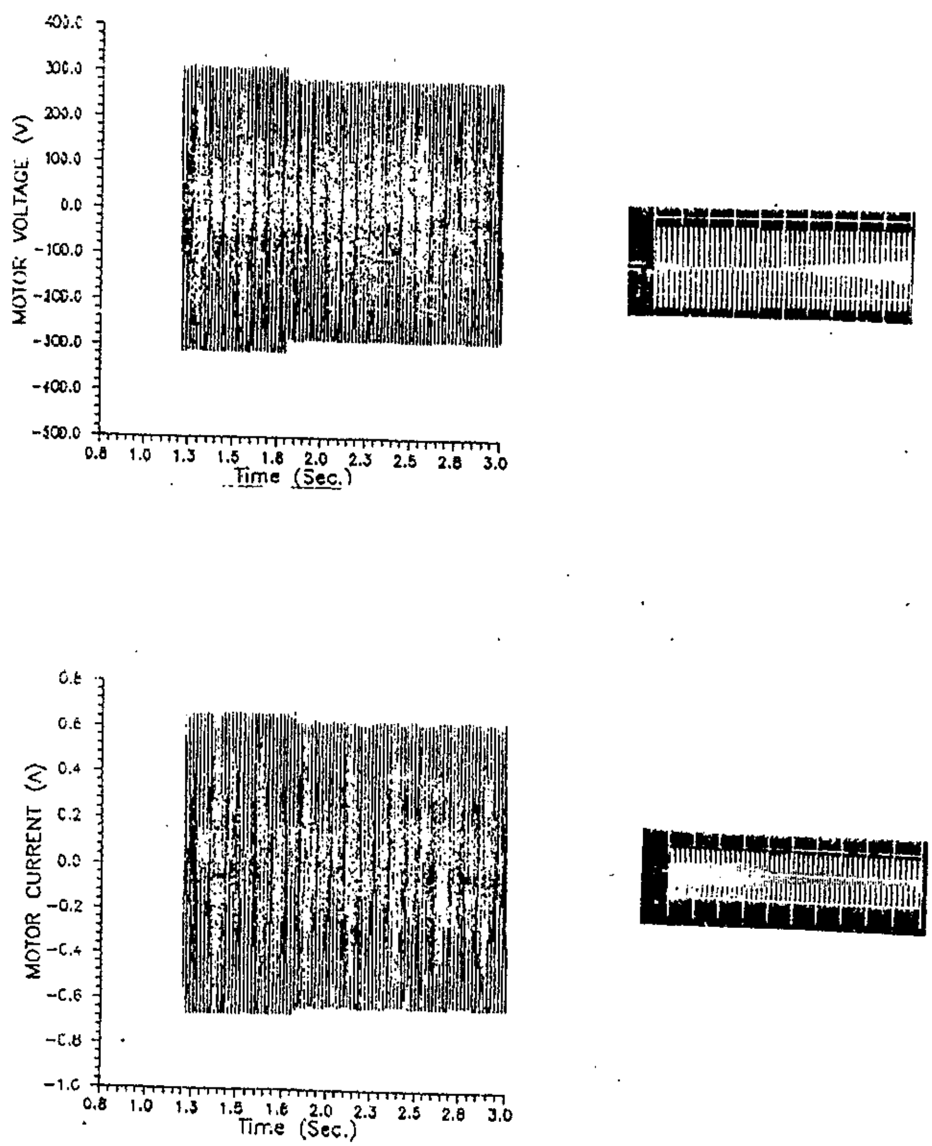


Fig.(1) Reluctance motor with shunt static balancer



(i) Simulated (ii) Experimental

Fig. (2) Simulated and experimental results of motor voltage and current due to balanced disturbance in input voltage (10% reduction)

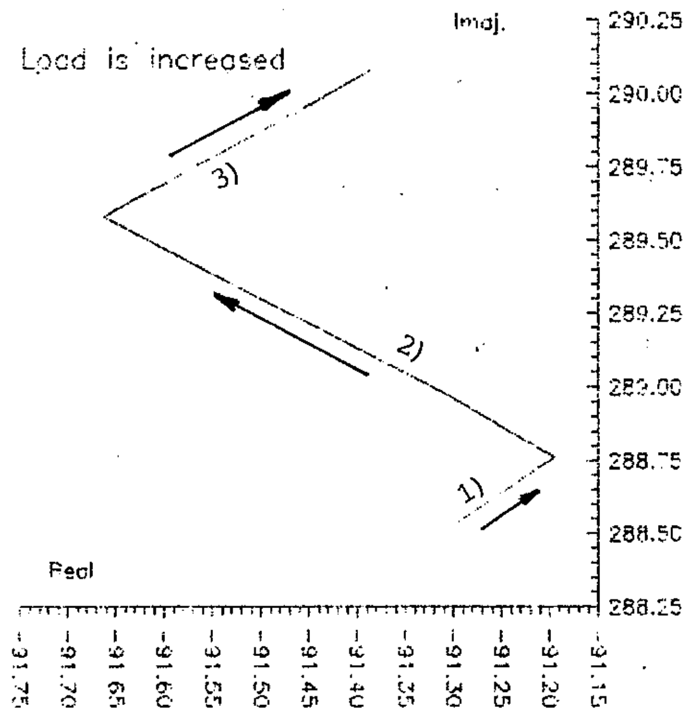


Fig. 3) Effect of load torque on system stability
rated V and F)

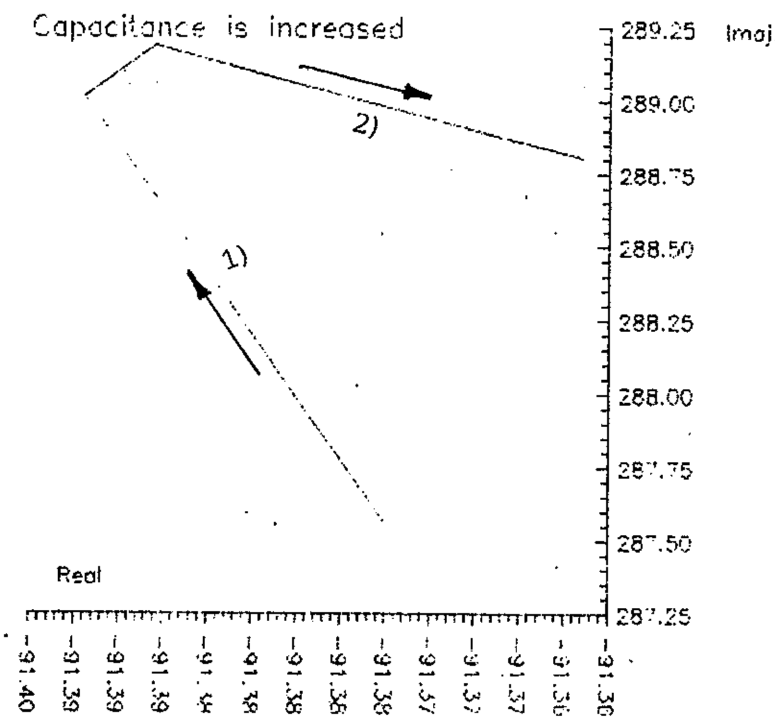


Fig. 4) Effect of balancer capacitive reactance on system stability rated V,F and T_L)

تحليلات النمزجه الخطيه لمحركات الممانعه المغناطيسية في وجود الموازن الاستاتيكي

ملخص البحث :

فى كثير من الحالات تعمل محركات الممانعة المغناطيسية فى حالة عدم اتزان ، ويرجع هذا الى عدم امكانية أخماد التذبذب الطبيعى إذا تعرضت الآله إلى تغير مفاجىء فى الحمل ، جهد المنبع ، .... ، الخ .

وهذا باطلبع غير مرغوب فيه ، حيث ان التطبيقات التى تستخدم فيها هذه النوعيه من الآلات تتطلب اتوماتيكيه واستمرار التوافق التام بين الدخل والخرج . لذا قد يكون من المناسب تعديل بارامترات المحرك اثناء مراحل التصميم لتجنب حالات عدم الاستقرار ، وهذا بالطبع يكون غير مناسب من الناحية العملية فى معظم الحالات التى يستخدم فيها نفس العضو الثابت لآلة تأثيريه كعضو ثابت لمحرك الممانعة المغناطيسية .

يقدم هذا البحث تحليلات النمزجه الخطيه لمحركات الممانعة المغناطيسية فى وجود الموازن الأستاتيكى ومعاوقة المغذى باستخدام تحليل المتغيرات الفراغية (State space approach) . وتم عمل إضطراب لمعادلات النموذج الخطى متعدد الدخل والخرج.

تم التنبوء بالأداء الديناميكى للآله من خلال عمل اضطراب فى جهد المنبع (١٠٪ إقلال من القيمه المقننه) وسجلت النتائج النظرية وتم مقارنتها بمثيلتها المعمليه ووجد توافق بينهما مما يؤكد صحة النموذج الرياضى . وتم ايضا بحث تأثير كل من عزم الحمل وسعة الموازن الاستاتيكى على اتزان الآله باستخدام تحليل المتغيرات الفراغية لتعيين القيم المميزه (eigen values) المصاحبه للمنظومة تحت البحث وتم تحديد مدى حالات التشغيل التى تعمل عندها الآله فى حالة استقرار.