A NON-LINEAR ADAPTIVE CONTROLLER VIA SLIDING MODES

F.F.ZADA , F.F.G. AREED , E.H.EL-KONYALLY

Department of Control and Computers, El Mansoura University

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ملخص البحث:

يعرض البحث فكره جديده لتصميم جهاز تحكم مهايئ باستخدام ظاهره المنظومات المنزلقة • وقد استعمل في البحث نموذج قياسي للمتابعةوظاهرة المنظومات المنزلقة عباره عن خاصية تظهر في المنظومات ذات الهيكل المتغير – ويؤدى وجودها الى أن تصبح المنظومة أقل حساسيه لكل من الاضطرابات الخارجية وتغير معاملات المنظومه وذلك بالاضافة الى تقليل تأثير العناصر اللاخطيه على خواص المنظومة – مثل المناطق الميتـه أو عدم احكام الاتصال الميكانيكي في المنظومات الكهروميكانيكه • وقد أعطى مثـال لطريقة التصميم المقترحة لمنظومة تحكم كهروميكانيكة ذات عناصر لاخطيه وذلك لتوضيح مدى فاعليـة الطريقة المقترحة •

Abstract

A new design concept for adaptive model-following control systems has been developed. The design procedure is based on the sliding mode property which exists in variable stucture systems. When the variable stucture system is operated in the sliding mode, the system response becomes less sensitive to changes in the plant parameters, external disturbance and the effect of nonlinearities commonly occuring in control systems. The paper describes the application of the designed variable structure controller to an electromechanical system overcoming the effect of nonlinearities inherently present in the system. A systematic approach based on a pole assignment technique is developed for specifying the elements of the switching vector. The control problem is formulated and the results of a simulation study are presented.

1-Introduction

The presence of nonlinearities in control systems may lead to many dangerous problems especially concerning the stability of the whole system. The stability of a nonlinear system is very much dependent on the input and also the initial state. For example, a nonlinear system giving its best response for a certain step input may exhibit highly unsatisfactory behaviour when the input amplitude is changed. Further, the nonlinear systems may exhibit limit cycles where the determination of their existence depends on both the type and amplitude of the excitation signal. In control systems nonlinearities are inherently present in the system [1]. The designer goal is to design the system so as to limit the adverse effects of these nonlinearities. Common examples of these nonlinearities are saturation, dead-zone, coulomb friction, backlash, etc..

One of the most important nonlinearities commonly occuring in electromechanical systems is backlash or mechanical hysterisis. The source of backlash that usually receives the most attention is the "looseness" inherent in mechanical gearing. Although attempts have been made to design gears and other mechanical transmission devices so as to fit their mating members very tightly, it is practically impossible to eliminate backlash entirely. The goal of this paper is to design an adaptive controller capable of overcoming the effect of such nonlinearities in electromechanical systems.

Model following control scheme is a suitable method to avoid the difficulty of specifying the design objectives of the control system. A model, which specifies these objectives, is used as a part of the control system. AMFC systems can be designed utilizing the theory of variable structure [2]. Variable structure systems possess a very important property, called sliding mode [3], in which the performance of the system is less sensitive to changes of the plant parameters, noise disturbances as well as the effect of nonlinearities. Whenever a sliding mode is realized, asymptotic stability of the whole system is assured.

An adaptive model-following control scheme [4], developed using the theory of variable structure and sliding mode is used for the design of the controller. The nonlinear electromechanical system is linearized around an operating point and the system model is formulated. The designed controller is implemented for the contol of the performance of a separately excited loaded d.c motor with gear backlash.

2-System Model

The system considered in this paper, as shown in Fig (1), consists of a separately excited d.c motor with the load connected to the motor's shaft through a gear box having backlash.

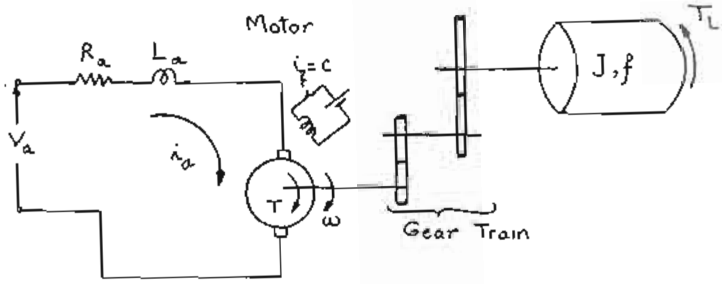


Fig (1) Schematic diagram of system

From the describing function for backlash [Appendix I], it is clear that the presence of backlash in the system affects the output speed in magnitude and inherent a phase lag which may introduce problems in the feedback system.

The effect of backlash is therefore represented by a first order transfer function as shown in block diagram Fig (2)

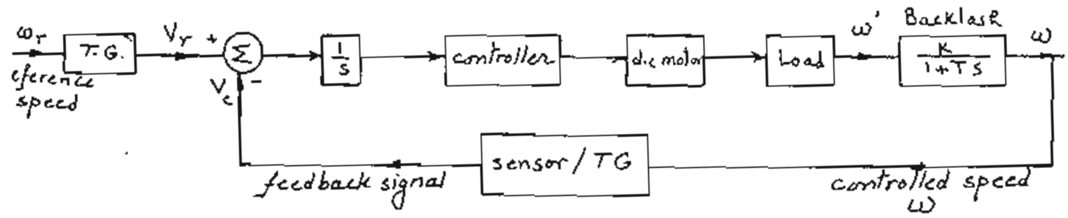


Fig (2) Block diagram of system

The equations describing the dynamic behaviour of the motor are as follows:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + K_f i_f w \qquad (1)$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \qquad (2)$$

$$\frac{dw'}{dt} = \frac{K_f i_f i_a}{J} - \frac{f}{J} w' \qquad (3)$$

$$\frac{dw}{dt} = \frac{K}{T} w' - \frac{w}{T} \qquad (4)$$

Where K_f is a constant. K and T depend on amount of backlash (Appendix I).

Linearizing eqns. 1-4 about the operating point X_o, we obtain the linearized state equation of the d.c motor drive system as

$$\dot{x} = Ax + bu + DF$$

$$x(0) = 0$$

where

$x = [x_1\ x_2\ x_3\ x_4\ x_5]^T$, state vector

$x_1 = (\Delta w_{ref} - \Delta w)dt$

$x_2 = \Delta w$

$x_3 = \Delta w'$

$x_4 = \Delta i_a$

$x_5 = \Delta i_f$

$u = \Delta v_a$, for armature-controlled DC motors, v_f = constant

Δv_f, for field-controlled DC motors, v_a = constant

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -1/T & K/T & 0 & 0 \\ 0 & -f/J & 0 & K_f I_{fo}/J & K_f I_{ao}/J \\ 0 & -K_f i_f/L_a & 0 & -R_a/L_a & -K_f w/L_a \\ 0 & 0 & 0 & 0 & -R_f/L_f \end{bmatrix}$$

$$b = [\ 0 \quad 0 \quad 0 \quad 1/L_a \quad 0\ ]^T$$

for armature controlled DC motors

$$[\ 0 \quad 0 \quad 0 \quad 0 \quad 1/L_f\ ]^T$$

for field controlled DC motors

$$D = [\ 1 \quad 0 \quad 0 \quad 0 \quad 0\ ]^T$$

3-Statement of the problem

It is required to solve the problem of the presence of nonlinearities in eletromechanical systems, which affects seriously both the performance and stability. The application of the designed variable structure model following controller [4], to a nonlinear system, completely represented by a linearized model, is described. The effectiveness of the control scheme is also required to be verified under the effect of external disturbances.

4- The variable structure model following control system

The plant and model are described by the equations:

$$\dot{x}_p(t) = A_p(t)x_p(t)+B_p(t)u(t)+D(t)F(t)$$
$$\dot{x}_m(t) = A_m x_m(t)+B_m r(t)$$

where x_p, $x_m \in R^n$, $u \in R^m$ and $r \in R^l$. r is the input and u is the control. The error vector is $e = x_m - x_p$.

We shall assume that the pairs (A_p, B_p) and (A_m, B_m) are stabilizable. The plant marices A_p and B_p may be uncertain and time varying. The upper and lower bounds of the elements of these matrices are assumed to be known to the designer.

Perfect model-following conditions [5,6] are assumed to hold through this paper.

4-1 Control Law

The variable-structure controller switches from one structure to another according to the sign of the switching hyperplane $S_i(e)$, the control signal is given [4] by

$$U_i = U_i - \zeta_i \operatorname{sign}(S_i) \quad (5)$$

Where ζ_i represents the additional controlling input. If a sliding mode is realised on the hyperplane $S_i = C^{iT} e$, ζ_1 is given by

$$\zeta_1 = (C^{1T} b^1_p)^{-1} C^{1T} \{(A_m - A_p(t)x_p(t) + B_m r(t) - b^2_p(t)u_2 \ldots\ldots - b^m_p(t) - D(t)F(t)\} \quad (6)$$

where

$b^1_p(t), b^2_p(t), \ldots\ldots, b^m_p(t)$ are the columns of matirx $B_p(t)$.

4-2 Selection of the Switching Vector Using the Pole Assignement Technique

Following the concepts developed by Utkin and Yang (1978) [7] the design procedure for selecting the switching vector C is described below

Consider the linear system

$$\dot{X} = AX + BU \quad (7)$$

where X is the state vector of dimension n*1, U is the control vector of dimension m*1 and A and B are constant matrices of dimensions n*n and n*m, respectively.

step 1 Define the coordinate transformation

$$Y = MX \quad (8)$$

such that

$$MB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (9)$$

where M is a non-singular n*n matrix and B_2 is a non-singular m*m matrix.

From eqns.7 and 8 we have

$$\dot{Y} = MAM^{-1}Y + MBU \quad (10)$$

Utilizing eqn.9. eqn.10 can be written in the form

$$\begin{bmatrix} \dot{Y}_1 \\ \\ \dot{Y}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ \\ Y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \\ B_2 \end{bmatrix} U \qquad (11)$$

where A_{11}, A_{12}, A_{21}, and A_{22} are respectively $(n-m)*(n-m)$, $(n-m)*m$, $m*(n-m)$ and $(m*m)$ submatrices.

The first equation of eqn.11 together with

$$S_i(X) = C_i^T X = 0 \quad i=1,2,...,m \qquad (12)$$

specifies the motion of the system in the sliding mode, that is

$$\dot{Y}_1 = A_{11}Y_1 + A_{12}Y_2 \qquad (13)$$

$$S(Y) = C_{11}Y_1 + C_{12}Y_2 = 0 \qquad (14)$$

where C_{11} and C_{12} are $m*(n-m)$ and $(m*m)$ matrices respectively, satisfying the relation

$$[\,C_{11} \quad C_{12}\,] = C^T M^{-1} \qquad (15)$$

eqns.13 and 14 uniquely, determine the dynamics in the sliding mode over the intersection of the switching hyperplanes .

The subsystem described by eqn.13 may be regarded as an open-loop control system with state vector Y_1 and control vector Y_2, the form of control Y_2 being determined by eqn.14 that is :

$$Y_2 = -C_{12}^{-1} C_{11} Y_1 \qquad (16)$$

It is clear that the problem of designing a system with desirable properties in the sliding mode can be regarded as a linear state feedback disign problem. It can be assumed, without loss of generality, that $C_{12} = I$ = Identity matrix.

Step 2 Eqns.7 and 16 can be combined to obtain

$$\dot{Y}_1 = [\,A_{11} - A_{12}C_{11}\,]\, Y_1 \qquad (17)$$

Utkin and Yang (1978) have shown that if the pair (A,B) is controllable, then the pair (A_{11}, A_{12}) is also controllable. If the pair (A_{11}, A_{22}) is controllable, then the eign values of the matrix $(A_{11}-A_{12}C_{11})$ in the sliding mode can be placed arbitrary, by a suitable choice of the matrix C_{11}.

Step 3 With $C_{i2} = I$, eqn.15 reduces to

$$C^{r} = [\ C_{i1} \quad I\ ]\ M \qquad (18)$$

therfore,the switching hyperplanes are given by

$$S_i(X) = C_i^{T}(X) = 0, \quad i=1,2,\ldots,m.$$

5- Illustrative example

Consider the system shown in fig.1. taking the aramture controlled case as an example .

The system matrices are given by

$$A_p = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -11.76 & 11.3 & 0 & 0 \\ 0 & -0.355 & 0 & 6.197 & 27.268 \\ 0 & -120 & 0 & -133.33 & -6666.66 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_p = [\ 0 \quad 0 \quad 0 \quad 111.11 \quad 0\ ]$$

The model matrices arew given by

$$A_M = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & -0.5336 & 0 & 5.769 & 24.46 \\ 0 & -0.5336 & 0 & 5.769 & 24.46 \\ 0 & -120 & 0 & -120 & -5497.8 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$B_M = [\ 0 \quad 0 \quad 0 \quad 100 \quad 0\ ]$$

Taking the matrix M as

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

the matrix product MAM^{-1} turns out to be

$$MAM^{-1} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & -0.5336 & 24.46 & 5.769 \\ 0 & 0 & -1 & 0 \\ \hline 0 & -120 & -5497.8 & -120 \end{array}\right]$$

choosing the poles of the matrix $[A_{11}-A_{12}C]$ arbitrary at -4 and -6 and following the procedure described previously, the switching vector is obtained as

$$C = [\ -0.545 \quad -0.398 \quad -0.398 \quad 1 \quad 0\ ]$$

6- simulation results

Fig.3a shows the transient response of the system for R=1p.u and zero initial conditions

Fig.3b shows the simulation results of Δw_p when the system is subjected to ± 0.2 pu (or ± 36 rad/sec) step change in reference speed Δw_{ref} which is equivalent to a load change of about $\mp 10\%$ off nominal value.

From the results shown in fig.3 , it is clear that the designed variable structure controller is capable of overcoming any deterioration in plant performance due to the presence of backlash. The dynamic response Δw_p also show a fast response insuring the invariance performance of the system inspite of the nonlinearity.

7- Conclusion

Variable structure control of nonlinaer model following control systems has been dicussed. By ensuring sliding mode on the switching hyperplanes insensitivity to plant variations, external disturbances and the effect of nonlinearities is achieved. The desigend variable structure

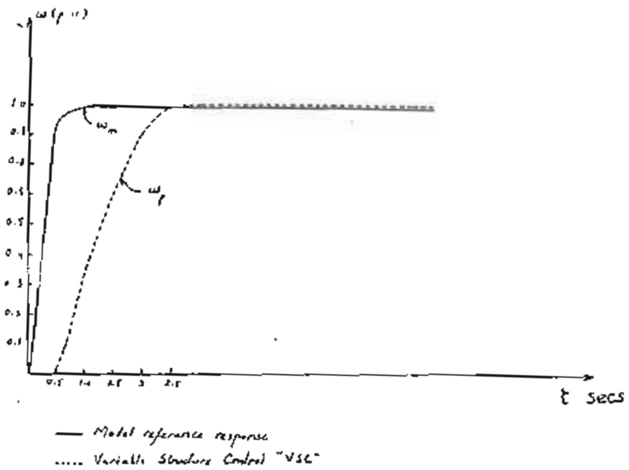
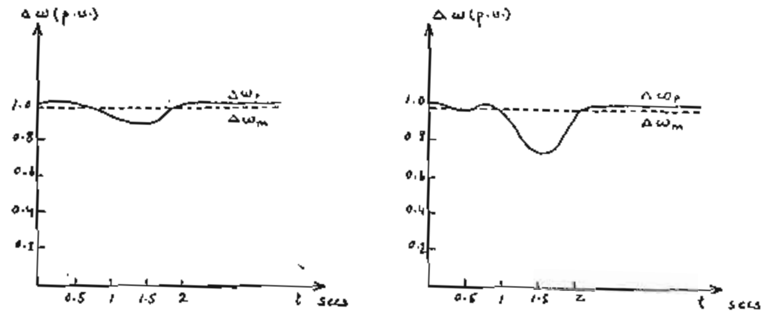


Fig. (3.a)



Change in ref. speed $\Delta\omega_{ref}$ = -0.2 pu

Change in reference speed $\Delta\omega_{ref}$ = 0.2 pu

Fig. (3.b)

controller insures the adaptive control of the system and its invariance performance.

Appendix

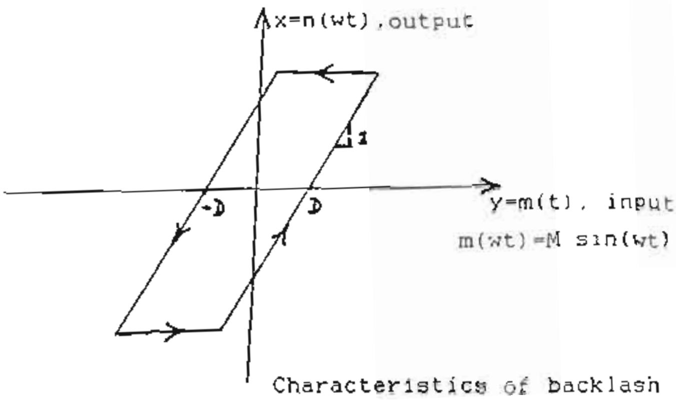
The describing function for backlash [1], shown in fig.4, is given by

$$N_{backlash}(M) = 1/M \sqrt{A_1^2+B_1^2} \;\angle \tan^{-1} A_1/B_1$$

where

$$A_1 = \frac{2D}{\pi M}\left(\frac{2D}{M} - 2\right) M$$

$$B_1 = \frac{1}{\pi}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{2D}{M} - 1\right) - \left(\frac{2D}{M} - 1\right)\cos \sin^{-1}\left(\frac{2D}{M} - 1\right)\right] M$$



Characteristics of backlash

Fig.4

The amount of backlash in the system depends on the ratio D/M. The presence of backlash in a system is represented by a first order transfer function since its effect results in a change in magnitude and a phase lag.

$$N(S) = K/(Ts+1)$$

$$N = K/(1+w^2T^2)\;, \quad \phi = -\tan^{-1} wT$$

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