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Length and Depth Ratios of Hydraulic Jump in Adverse Sloping Rectangular channels

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نسب الطول و العمق للقفزة المائية
فى القنوات المستطيلة ذات الميل العكسى

خلاصة

يوثر ميل القاع على كل من أبعاد و شكل القفزة الهيدروليكية التى تتكون فى القنوات المكشوفة إضافة إلى الفقد فى الطاقة نتيجة لتكون هذه القفزة و لما كانت الدراسات السابقة على القفزة الهيدروليكية المتكونة على ميل عكسى (عكس إتجاه السريان) قليلة إلى حد ما ،لذا فقد أجرى هذا البحث على قناة معملية مستطيلة القطاع قابلة للميل العكسى ، استخدم فيه خمسة ميول عكسية تتراوح قيمتها ما بين ٠,٠٠٢ إلى ٠,٠٠٥.
أخذت القياسات المختلفة للتصرفات و الأعماق و الأطوال و كذلك سرعة السريان عند بداية و نهاية القفزة فى حالة ثبوتها و من ثم تم عمل جداول و إعداد منحنيات تمثل العلاقات المختلفة التى يمكن استخدامها فى تعيين شكل القفزة المائية و الطاقة المفقودة نتجة لتكونها .
عمل تحقيق لمعادلات القفزة المتكونة على ميول موجبة (فى اتجاه السريان) لتحديد ما يمكن تطبيقه على القفزة المتكونة على ميول عكسية (سالبة) و تاثير هدة الميول على تصميم أحواض التهدئة .

Abstract

The adverse hydraulic jump has been investigated and analyzed in order to derive the limiting design parameters for this type of jump. The limiting design condition refers to the adverse jump when it occurs entirely on the adverse slope. A review of the literature shows that very little research has been carried out on this phenomenon. This study is carried out in the laboratory to find nondimensional relationships between the design parameters

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for a hydraulic jump on adverse sloping rectangular prismatic channels. The upstream Froude (F_1) number is considered to be sufficiently high to avoid undular jumps.

The obtained results show good correlation between the different parameters within the limitation $2.5 < F_1 < 9$, where the slope S_0 changes from -1/500, to - 1/200. It is noted that the length of a jump formed on an adverse slope is less than the corresponding one formed in a horizontal channel, with the same approaching conditions. This length is inversely proportional to the value of the adverse slope. However, the sequent depth ratios in the adverse sloping jump have lower values than those jumps in horizontal channels. The sequent depth ratio decreases as the adverse slope becomes steeper. In addition, the adverse sloping jump has lower energy dissipation than the equivalent horizontal one. Finally, the present results show that the length and depth of the stilling basin are less than those for the horizontal one and it becomes more economic.

Introduction

Hydraulic jump is one of the classical problems in the field of applied hydraulics. In the design of open channels, the possible occurrence of the jump,formed by the presence of some obstacles, such as stilling basins, slope changes, or bridge piers, changes the nature of the mathematical problem formulation and solution.

Hydraulic jump may be seen as an abrupt change in the depth and in the average velocity, with a sudden loss of specific energy, while mass and specific force of stream are preserved (neglecting friction).

A lot of hydraulic research engineers have performed experimental and theoretical analyses, based on the fundamental studies by Backmeteff and Matzke (1936) , Rouse, Siao and Rajaratnam (1959), Valiani (1997), and many others.

A few studies were made on hydraulic jump on adverse slope. This study is carried out in the laboratory and attempts are made to find the nondimensional relationships between the design parameters for a hydraulic jump on adverse sloping rectangular prismatic channels.

In the present work, the upstream Froude number is considered to be sufficiently high to avoid undular jumps, according to the limit (Marchi and Rubatta, 1981) : $F_1^2 \geq 3$.

Knowledge of the surface profile of a hydraulic jump is very important in designing the stilling basin where the jump takes place. It is also important to determine the pressures on the floor and vertical walls of the basin which are functions of the surface profile.

Theoretical model of hydraulic jumps

The prediction model for computing the sequent depth ratio and length-depth ratio of the hydraulic jump formed in an adverse sloping rectangular channel has been developed. It is based on the application of the one dimensional momentum and continuity equations. Hydraulic jumps on adverse sloping floor are used to dissipate the excess energy downstream of hydraulic structures. The Kindsvater (1942) equation for positively sloping hydraulic jump is modified here for the adverse sloping jump.

For jumps in adverse sloping channels, only few studies are available. According to Fig. (1), to simplify the derivation of the present study model, the following basic assumptions are considered: (i) continuity equation is valid; (ii) the channel is prismatic and rectangular with an adverse sloping bed; (iii) the flow is steady and the fluid is incompressible; (iv) uniform velocity distribution ($\beta_1 = \beta_2 = 1$) and hydrostatic pressure prevails at both the beginning and at the end of the jump; and (v) the frictional resistance of the flume, air entrainment and turbulence effects are neglected.

Referring to the sloping bed as the x-axis Figs. (1,2) the momentum equation in an adverse sloping prismatic rectangular channel may be written as:

$$\beta_2 \rho U_2 Q - \beta_1 \rho U_1 Q = P_1 - P_2 - W \sin\theta - F_f \qquad (1)$$

where P_1 and P_2 are the hydrostatic forces at the beginning and end of the hydraulic jump, respectively;

$$P_1 = \frac{\gamma Y_1 H_1}{2} = \frac{\gamma Y_1^2}{2} \cos\theta \qquad (2)$$

$$P_2 = \frac{\gamma Y_2 H_2}{2} = \frac{\gamma Y_2^2}{2} \cos\theta \qquad (3)$$

in which Y_1 and Y_2 are the conjugate depths of hydraulic jump, measured normal to the horizontal; H_1 and H_2 are the depths at the beginning and the end of the jump respectively, measured perpendicular to the floor; F_f = friction force; U_1 and U_2 are the mean velocities respectively; ρ is the density of water, Q is the discharge; θ is the channel slope; and β_1 and β_2 are the momentun coefficients.

The continuity equation is

$$Q = U_1 A_1 = U_2 A_2 \qquad (4)$$

Kindsvater (1942) and Chow (1959) represented the weight of water (W) between sections (1) and (2) in sloping channel, Fig. (1), by :

$$W = k \gamma L (H_1 + H_2) / 2 \qquad (5)$$

in which γ = specific weight of water; L = length of the jump ; and the other variables are defined in Figure.

Assuming $\beta_1 = \beta_2 = 1$, solution for equations (1) and (4) in the form of Belanger [4] equation may be written as :

$$H_2 / H_1 = \frac{1}{2} \left(\sqrt{1 + 8 G_1^2} - 1 \right) \qquad (6)$$

in which G_1 is defined as a modified Froude number. The function G_1 is given by

$$G_1^2 = \frac{\beta_1 F_1^2}{\cos\theta - \dfrac{kL \tan\theta}{(H_2 - H_1)} - \dfrac{2F_f}{\gamma (H_1^2 - H_2^2) \cos\theta}} \qquad (7)$$

if the friction force (F_f) is ignored then

$$G_1^2 = \frac{\beta_1 F_1^2}{\cos\theta - \frac{kL\tan\theta}{(H_2 - H_1)}} \qquad (8)$$

or

$$G_1^2 = \frac{F_1^2 \cos^3\theta}{1 - 2k\tan\theta} \qquad (9)$$

in which F_1 is the approaching Froude number $= \frac{U_1}{\sqrt{gH_1}}$; U_1 is the supercritical velocity; g is the acceleration of gravity; and k being a shape factor.
Rajaratnam [6,8] provided the following empirical expression for G_1 :

$$G_1 = F_1\, 10^{0.027\theta} \qquad (10)$$

Ohtsu and Yasuda [8] analyzed the jumps on sloping floors for wide range of slopes. They developed a momentum based equation for computing the sequent depth ratio of D-jump. Then they recommended the following equation for the design purposes:

$$\frac{Y_2}{H_1} = (0.077\theta^{1.27} + 1.41)(F_1 - 1) + 1 \quad ; \quad 4 \le F_1 \le 14, 0 \le \theta \le 19^0 \qquad (11)$$

where Y_2 and H_1 have the same definitions as before.
The data of Okada and Aki (1959) were fitted to an empirical relationship of the form

$$L/H_1 = (C_o + C_1 \sin\theta + C_2 \sin^2\theta) (F_1 - 2) + D_1 (1 + D_2 \sin^4\theta) \qquad (12)$$

where the empirical constants are:
a) for the L of the limiting jump of J. A. McCorquodale's study (1994) with $F_1 < 9$:

$C_o = 7.25$; $C_1 = 20.8$; $C_2 = 5$; $D_1 = 5$; $D_2 = 50$;

b) for the L of the stabilized jump of Okada et al. with $9 < F_1 < 13$:

$C_o = 7.2$; $C_1 = 21$; $C_2 = 19$; $D_1 = 7.7$; $D_2 = 175$.

In this study, attempts were made to extend the work of Husain et al. (1994) to verify the coefficients of nondimensional equations for the length and depth of an adverse sloping jump. They developed nondimensional equations for the profile coefficient k, sequent depth and the length of jump using multiple linear regression analysis, for a positive sloping jump, in the form of :

$$k = 1.152 + 0.025\theta + 0.031F_1 \qquad (13)$$

$$H_2/H_1 = -74.85 + 63.305k - 0.695\theta - 0.369F_1 \quad (14)$$

$$L/H_1 = -393.261 + 337.487k - 7.011\theta - 2.019F_1 \quad (15)$$

$$L/H_2 = -11.492 + 14.65k - 0.652\theta - 0.397F_1 \quad (16)$$

$$L/(H_2 - H_1) = -6.273 + 11.324k - 0.66\theta - 0.382F_1 \quad (17)$$

Energy loss

The equation of energy loss in the limiting adverse jump can be written as :

$$\Delta E = \left(\frac{\alpha_1 U_1^2}{2g} + H_1 \cos\theta\right) - \left(Z_2 + \frac{\alpha_2 U_2^2}{2g} + H_2 \cos\theta\right) \quad (18)$$

where Z_2 = L sinθ = bed elevation at section (2) relative to section (1) ; α = kinetic energy correction factor; the value of $\alpha_1 = 1.05$, $\alpha_2 = 1$; and E_1 and E_2 are the energies at the beginning and the end of the jump respectively.

Experimental arrangement and procedure

The experiments were carried out in the Hydraulics Laboratory of Faculty of Engineering , Cairo University, Experiments were conducted in a glass - sided re-circulating tilting flume with 10 m length working section. The flume bed is 30 cm wide and 45 cm deep and the maximum negative bed slope is 1 in 200. The water depths are measured by precise point gauges mounted in instrument carriages. The discharge is measured using a pre-calibrated orifice meter which is incorporated in the pump discharge pipework in conjunction with a 1 meter pressurized water manometer which indicates the flow rate in the system provided by the hand operated control valve. The bed slopes and experimental conditions are selected depending upon the limitations of the available facilities. The channel has a manually operated screw type jacking system for slope variations, and an electrical driven pumpset designed to deliver 30 l/s against a total head of 6 m. The test procedure consists of the following steps : (I) the flume is adjusted to the desired slope. (ii) the control valve is adjusted to a certain opening allowing a specific discharge to pass. (iii) the position of the jump is adjusted by means of the control gate and the tail gate without affecting the jump characteristics. (iv) after stable flow conditions are attained the conjugate water depths, length of jump, and manometer readings of orifice meter are measured. (v) the procedure is repeated for each bed slope with different valve openings (different discharges). The readings are taken when the initial Froude number is sufficiently high to avoid undular jumps, according to the classical limit (Marchi and Rubatta, 1981) : $F_1^2 \geq 3$.

The simulation of the hydraulic jump has been a challenging problem in hydraulics because of its complex nature. Moreover, in the design of hydraulic structures, it is necessary to know the location, length, conjugate depths, and the height. also the intensity of mixing process, and the amount of energy dissipated by the jump.

The main purpose of this study is :

(1) modeling and experimental verification of hydraulic jumps on adverse sloping channel,
(2) study of some important characteristics of the hydraulic jump on adverse sloping channel,
(3) prediction of a generalized relations for G_1 which is applicable to adverse sloping jump, and
(4) to find a practical method for the solution of sequent depth in adverse sloping jump.

In this study, several experiments of a stabilized adverse jump on slopes of -1/500, -1/400, -1/300, -1/250, -1/200 were carried out with approach Froude numbers in the range of normal design for conventional stilling basins to derive the design parameters for a hydraulic jump on adverse sloping rectangular prismatic channels.

Experimental results

Jump sequent depth ratio, length-depth ratio and energy loss ratio

The experimental sequent depth ratio H_2/H_1 for different values of F_1 on the five adverse slopes is shown in Fig (3). These data indicate that, for a given value of F_1, the ratio H_2/H_1 decreases as the magnitude of the adverse slope increases and for a given slope sequent depth ratio is directly proportional to F_1. The data of Okada et al. (1959) and the data of McCorquodale et al. (1994) show the same trend. This is good from the point of view of a hydraulic structure designer, because the height (and consequently pressure is small) and the length of hydraulic jump become small as the magnitude of adverse slope increases.
Also, the experimental data of L/H_1 against F_1, Fig.(4) shows that the values of L/H_1 are directly proportional to F_1 for a given adverse slope, but for a given F_1 the values of L/H_1 are inversely proportional to slope i.e., when F_1 increases the ratio L/H_1 decreases. Also, L/H_1 has a lower value for the adverse sloping jump than the horizontal jump as shown in Fig. (4).
Fig.(5) shows the relationship between F_1 and the nondimensional ratio $\Delta E/E_1$ (the relative energy loss). It indicates that the energy loss ratio (the relative energy loss) due to sloping adverse jump is inversely proportional to slope for a given Froude number, i.e., the relative energy loss decreases as the adverse slope increases but the energy loss ratio (the relative energy loss) decreases as the F_1 decreases for a given slope because of the part of energy is converted to overcome the position energy at section (2).
Equations (8), (9) or (10) give suitable values of the modified Froude number G_1 as compared by the corresponding values given by the experimental data, with correlation eoefficient of 0.99. Fig. (6) gives the values of G_1 versus F_1 for different slopes. It is elear from the figure that the modified Froude number G_1 has approximately the same values for different slopes at a given Froude number F_1. This means that G_1 can be considered independent on the slope of the channel and significantly dependent on F_1 on condition that the negative sign for tan θ must be considered. Also equation (12) gives suitable values for L/H_1 but equations (13) to (17) do not give reasonably results for adverse sloping jumps as shown in Table (1).

Conclusions

Based on the momentum and continuity principles, more practical and easy applicable prediction relationships (curves) are developed for computation of the sequent depth and length ratios of hydraulic jumps. Also the dissipated energy can be determined for hydraulic jumps in adverse sloping as well as in horizontal channels.

The obtained results show good agreement between the experimental data within the limitation $2.5 < F_1 < 9$, where the slope S_0 changes from -1/500, to - 1/200. It is noted that the length of an adverse jump is less than that of the equivalent horizontal jump and it is also inversely proportional to the adverse slope, with the same approaching conditions. But the sequent depth ratios in the adverse jump have lower values than those in a horizontal jump. The sequent depth ratio decreases as the adverse slope becomes steeper.

According to the above results, if the bed of a stilling basin has a limiting adverse slope without affecting the characteristics of the flow and the hydraulic jump, then the length and depth of the stilling basin are less than those for the horizontal one and it becomes more economic.

References

1- Alhamid, A. A, Husain, D. and Negm, A.M.," Prediction of the sequent depth ratio of hydraulic jump on rough sloping channel floors", J. of Irrig. and Power, Vol. 51, No. 2 , 1994.

2- Alhamid, A. A., and Negm, A.M., "Depth ratio of hydraulic jump in rectangular stilling basins", J. of Hydraulic Research, Vol. 34, No. 5, 1996.

3- Bradley, J.N., and Peterka, A.J., "Hydraulic design of stilling basins : stilling basin with sloping apron (Basin V)", J. of Hydraulic Division, Proc. ASCE, Vol. 83, No. 5, 1957.

4- Chow, V.T., "Open channel hydraulies", McGraw Hill, New York, 1959.

5- Hager, W.H., and Bretz, N.V., "Hydraulic jumps at positive and negative steps", J. of Hydraulic Research, Vol. 24, No. 4,1986.

6- Husain, D., Alhamid, A. A., and Negm, A.M., "Length and depth of hydraulic jumps on sloping floors", J. of Hydraulic Research, Vol. 32, No. 6, 1994.

7- Quraishi, A.A., and Al-Brahim, A. M., "Hydraulic jump in sloping channel with positive or negative step", J. of Hydraulic Research, Vol. 30, No. 6, 1992.

8- Negm, A. M., "Hydraulic jumps at positive and negative steps on sloping floor", J. of Hydraulic Research, Vol. 34, No. 3, 1996.

9- McCorquodale, J. A., and Mohamed, M. S., "Hydraulic jumps on adverse slopes", J. of Hydraulic Research, Vol. 32, No. 1, 1994.

Table (1) Comparison between various values which calculated using equations 12,14,15,16,17 and velues of this study

L/H1								L/H2				H2/H1				L/(H2 - H1)			
slope -1/400				slope -1/250				slope –1/400		slope – 1/250		slope –1/400		slope --1/250		slope —1/400		slope —1/250	
F1	study	eq. 12	eq. 15	F1	study	eq. 12	eq.15	study	eq. 16	study	eq. 16	study	eq. 14	study	eq. 14	study	eq. 17	study	eq. 17
3.21	12	13.82	40	4.894	21.12	26.22	37.2	3.704	5.609	5.193	5.759	3.24	4.496	4.067	5.8	5.357	6.727	6.886	6.73
3.66	12.22	17.14	39.08	3.916	18.82	19.05	39.17	3.395	5.635	4.482	6.148	3.6	5.134	4.2	6.161	4.701	6.713	5.882	7.103
2.69	12.24	10.04	41.04	4.401	20.82	22.61	38.19	4.054	5.579	4.306	5.955	3.02	3.771	4.836	5.982	6.061	6.743	5.429	6.918
4.49	22	23.15	37.42	4.904	23.81	26.3	37.18	4.681	5.682	4.451	5.755	4.7	6.288	5.349	5.796	5.946	6.687	5.474	6.726
3.2	15.81	13.78	40.01	5.481	27.45	30 53	36.01	4.172	5.609	4.403	5.526	3.791	4.488	6.235	5.583	5.667	6.727	5.243	6.506
5.75	28	32.37	34.87	3.374	16.67	15.08	40.27	4.667	5.754	4.984	6.363	6	8.057	3.344	6.361	5.6	6.648	7.111	7.31
3.98	18.72	19.49	38.43	2.815	14.58	10.98	41.39	4.534	5.653	4.875	6.585	4.128	5.585	2.991	6.567	5.984	6.703	7.324	7.524
4.47	24.62	23.05	37.45	3.557	17.47	16.42	39.9	5.161	5.681	4.606	6.29	4.769	6.269	3.793	6.293	6.531	6.688	6.255	7.241
4.42	25.25	22.66	37.55	3.767	14.21	17.96	39.47	5.179	5.678	3.776	6.207	4.875	6.194	3.763	6.216	6.516	6.689	5.143	7.16
4.84	28.42	25.75	36.7	4.72	18.93	24.95	37.55	5.538	5.702	4.492	5.828	5.132	6.786	4.214	5.864	6.879	6.676	5.889	6.796
mean	19.93	20.13	38.26		19.39	21.01	38.64	4.509	5.658	4.557	6.042	4.326	5.707	4.279	6.062	5.924	6.7	6.064	7.001
% of error		10	92			8	99		25		33		32		42		13		16

Note that the other slopes (-1/500, -1/300, -1/200) give the same results

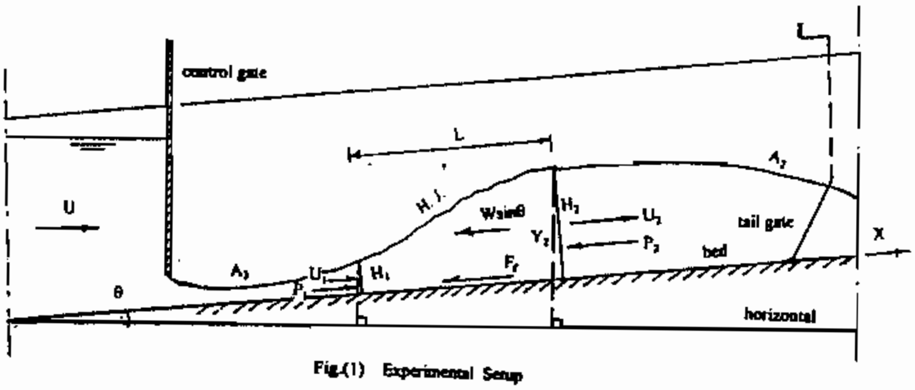

Fig.(1) Experimental Setup

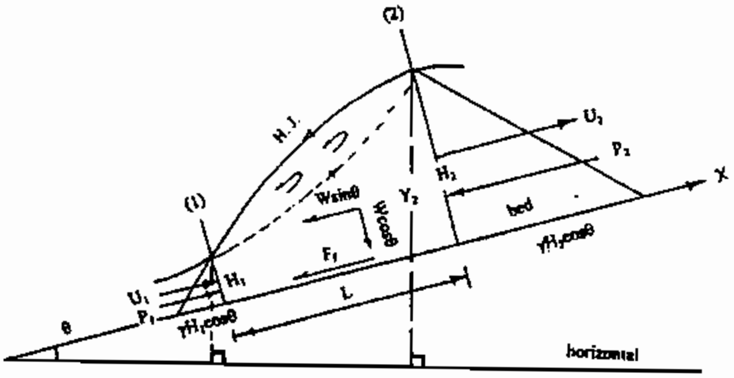

Fig.(2) Definition sketch and control volume of adverse hydraulic jump

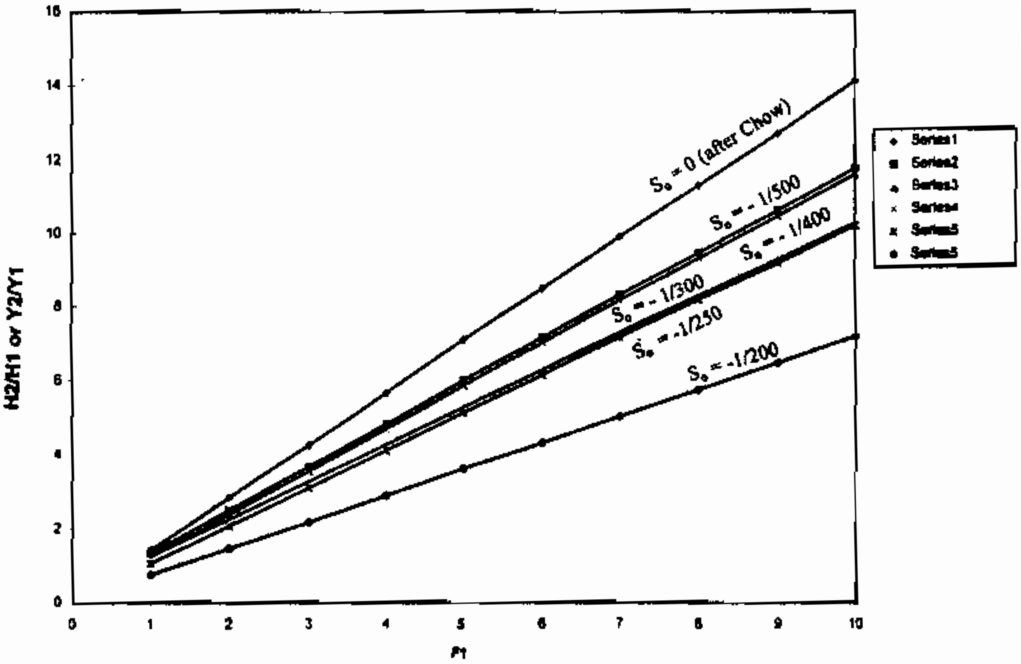

Fig.(3) Sequent depth ratio, H_2/H_1 ,as a function of Froude number, F_1, in adverse sloping channels.

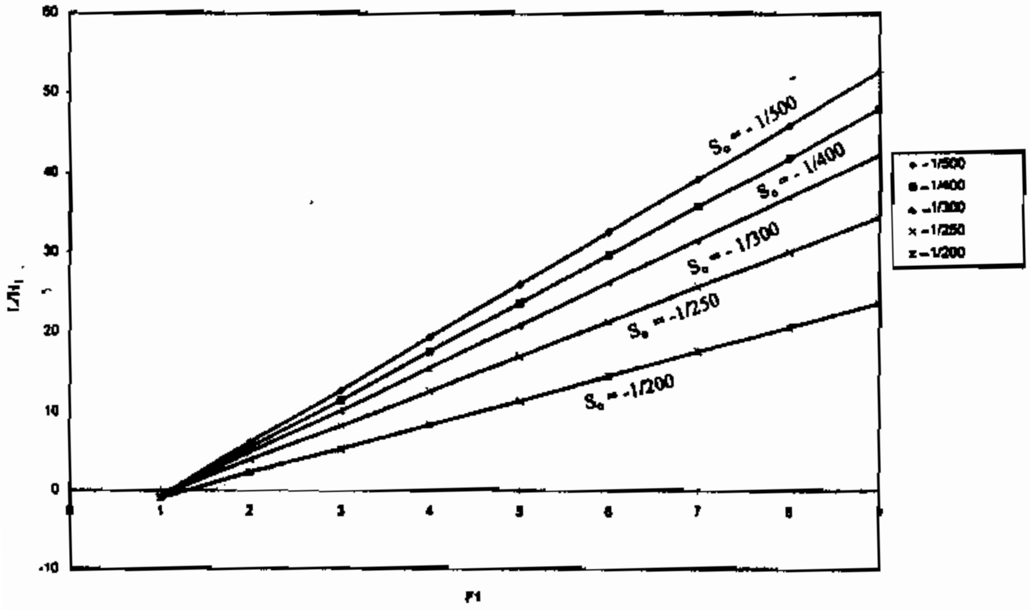

Fig.(4) Jump length ratio, L/H_1 , as a function of Froude number, F_1, in adverse sloping channels.

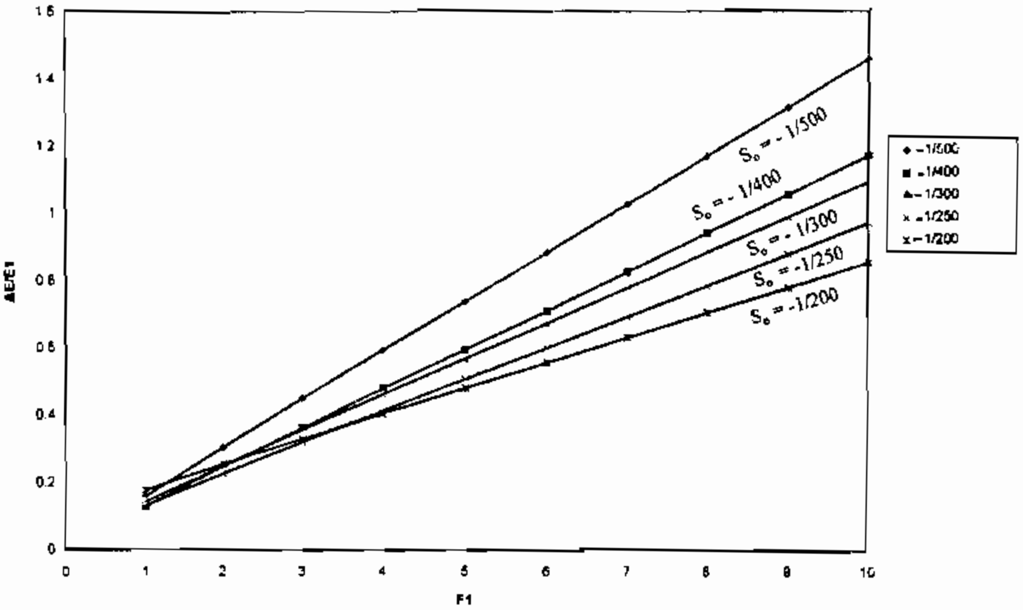

Fig.(5) The energy loss ratio (Relative energy loss), $\Delta E/E_1$, as a function of Froude, F_1, number in adverse sloping channels.

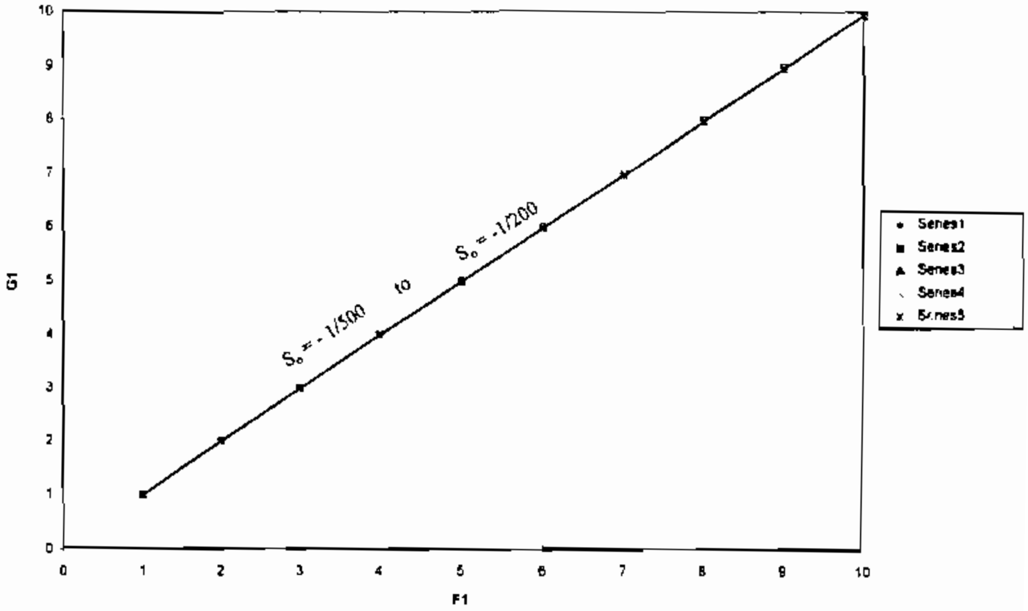

Fig.(6) The modified Froude number, G_1, as a function of Froude number, F_1, in adverse sloping channels