REAL-TIME MODEL-REFERENCE CONTROL OF NON-LINEAR PROCESSES

التحكم في المنظومات اللاخطيه باستخدام النموذج المرجعي في الزمن الحقيقي

BY

K. M. SOLIMAN

Control & Computers Dept., Faculty of Engineering, Mansoura University

يقدم البحث استراتيجيه بسيطة لتصميم وتنفيذ أجهزة التحكم التوافقيه ذات النموذج المرجعي. ويتم تنفيذ التصميم باستخدام متجه الخطأ بين الحالات الفعليه المقاسة للمنظومه وبين الحالات المناظره في النموذج المرجعي وبعد نجعل معدل هذا الانحراف سالبا وذلك عن طريق اختيار داله ليابونوف مناسبة ويمكن تطبيق الطريقه المقترحة على العمليات الصناعيه اللاخطيه دونما حاجة لتقريب المنظومة للصوره الخطيه .

تم تطبيق الطريقة المقترحة على منظومة ذراع الي ذات درجتين من درجات الحريه وثبت فاعليتها من خلال نتائج المحاكاة الرقميه ويحتوي البحث على بعض الاعتبارات العملية التي يجب اخذها في الاعتبار عند تطبيق الطريقة المقترحة على منظومات التحكم في الزمن الحقيقي، كما ان البحث يبين اهمية الطريقة المقترحة من زاوية التحكم التوافقي في الزمن الحقيقي .

■■ Abstract :

This paper demonstrates a strategy for the design and implementation of model reference controllers. The design technique is based on constructing an error vector between the plant measurable states and the states of the proposed controller, then forcing the gradient of this error vector to be negative via the use of a suitable Liapunov function. The suggested design procedure is applicable for non-linear processes without any need to carry out linearization approximations. A simulated two degrees of freedom robot arm is used to exemplify the suggested technique. A few practical considerations are then addressed to investigate the causality of the proposed controller and its applicability to real-time process control systems. Conclusion is submitted with a few comments regarding both the adaptivity and real-time compatibility of the proposed controller.

■■ Introduction :

Control systems are designed to perform specific tasks. The designed control system is supposed to fulfill some requirements that are usually stated as performance specifications. For simple linear systems, these performance specifications may have precise numerical values in terms of time-domain measures, i.g. steady state accuracy, rise time, maximum overshoot, ...etc., or in terms of frequency domain measures, i.g. gain margin, phase margin, band width, ...etc. For such simple linear systems, it is always possible to construct effective mathematical models for them, then using the well-known techniques of the classical control theory to build the required controller (compensator) [4,8]. It is also possible to combine educated trial and error methods to ease the design procedure as the range in which the system parameters change is usually narrow. The use of time-domain approaches, i.g. state space representations could also be used - especially for regulation problems - but after performing a linearization approximation round the required operating point.

Since all physical plants are usually non-linear to some degree, it is implied that the foregoing argument will yield satisfactory performance only over a limited range of operation. If the assumption of linearity in the process is no longer valid, the design techniques of the classical control theory don't hold. In such a case, the model-reference approach may be of a great help [9].

The desired output of the process for a given input could be obtained by constituting a model. This model, called the reference model, need not be an actual hardware, but simply a simulated mathematical one on a microcomputer system. The output of the reference model is compared with that of the actual process to perform an error vector which is used directly in generating the control signal. The choice of the reference model offers the designer a great flexibility, as it could be chosen a linear one regardless of whether the process is linear or not. It is also possible to use RLS estimation techniques [10] to generate an equivalent model of the process which is continuously identified, hence assuring the adaptivity of the designed controller.

■■ The Mathematical Approach :

Let the system under consideration be expressed in a state space form as follows :

$$\dot{X} = f(x , u , t) \tag{1}$$

where $f(x,u,t)$ is a general expression for the system either (non) linear and/or stationary (time-variant). For stationary linear systems, this general expression will be reduced to :

$$\dot{X} = A X + B U \tag{2}$$

where X is the state vector;
U is the forcing input vector; and
A , B are nxn & nx1 matrices respectively (n being no. of states)

Further let the reference model be a stationary linear one that could be expressed as follows :

$$\dot{X}_d = A_d X_d + B_d V \tag{3}$$

where X_d is the desired states vector;
V is the desired output (Reference Input) vector; and
A_d , B_d are nxn & nx1 matrices respectively

Constructing an error vector between the desired states and the actual states of the system yields :

$$e = x_d - x \tag{4}$$

From which the error gradient could be given by :

$$\begin{aligned} \nabla e = \dot{e} &= \dot{x}_d - \dot{x} \\ &= \{ A_d X_d + B_d V \} - f(x , u , t) \\ &= A_d \{ e + X \} + B_d V - f(x , u , t) \\ &= A_d e + A_d X + B_d V - f(x , u , t) \end{aligned} \tag{5}$$

Since stability is of major concern, the model-reference controller will be designed such that equation (5) is stabilized with (e = 0) as an equilibrium state. Using the second method of Liapunov, we can test for the error vector stability by defining an N(e) to be a Liapunov function as follows [4,8] :

$$N(e) = e^T P e \quad , \quad P \text{ is a +ve definite symmetric matrix}$$

Then

$$\dot{N}(e) = \dot{e}^T P e + e^T P \dot{e}$$
$$= \{ e^T A_d^T + X^T A_d^T - f^T(x,u,t) + V^T B_d^T \} P e$$
$$+ e^T P \{ A_d e + A_d X - f(x,u,t) + B_d V \}$$
$$= e^T (A_d^T P + P A_d) e + 2 S \quad (6)$$

where $S = e^T P \{ A_d X + B_d V - f(x,u,t) \}$

Now if the following equation is satisfied

$$A_d^T P + P A_d = - Q \quad , Q \text{ is +ve definite} \quad (7)$$

a sufficient condition for equation (6) to be negative definite is that :

$$S = e^T P \{ A_d X + B_d V - f(x,u,t) \} \le 0 \quad (8)$$

It is evident that the mathematical complexity could be very much eased by letting S = 0 , for which the solution for the desired control vector is independent on P , or :

$$A_d X + B_d V - f(x,u,t) = 0$$

Thus :

$$f(x,u,t) = A_d X + B_d V \quad (9)$$

Now, equation (9) represents the design criteria for the proposed model-reference controller. This equation could be solved directly to find u (the control vector) as a function of both the system and the reference model states. This strategy will be exemplified in the next section.

■■ Application for a Robot Arm Model :

Figure (1) shows a schematic diagram of a planner two degrees of freedom robot arm for which [1-3] :

m_1 = 1.5 Kg. ,

m_2 = 1 Kg. ,

m_L = 0.5 Kg. ,

L_1 = 30 cm. ,

L_2 = 20 cm. and

g = 9.81 m/sec^2

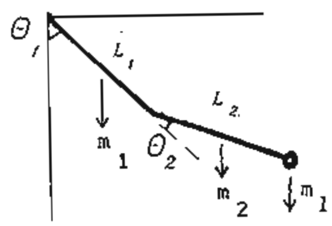


Fig. (1) - The Robot System.

Now, defining I_i to be the inertial moment of link i, or :

$$I_1 = \frac{1}{3} m_1 L_1^2 \quad , I_2 = \frac{1}{3} m_2 L_2^2 + m_L L_2^2 \quad (10)$$

Also, defining L_{ci} to be the center of gravity of link i, or :

$$L_{c1} = \frac{L_1}{2} \quad , L_{c2} = \frac{m_2 + 2 m_L}{m_2 + m_L} \cdot \frac{L_2}{2} \quad (11)$$

Introducing the following notations :

$$\left.\begin{aligned} D_{11} &= I_1 + I_2 + (m_2 + m_L)\,(L_1^2 + 2\,L_1\,L_{c2}\cos\theta_2) \\ D_{22} &= I_2 \\ D_{12} &= D_{21} = D_{22} + (m_2 + m_L)\,(L_1\,L_{c2}\cos\theta_2) \end{aligned}\right\} \quad (12)$$

$$\left.\begin{aligned} H &= (m_2 + m_L)\,L_1\,L_{c2}\sin\theta_2 \\ G_2 &= (m_2 + m_L)\,g\,L_{c2}\sin(\theta_1 + \theta_2) \\ G_1 &= G_2 + m_1\,g\,L_{c1}\sin\theta_1 + (m_2 + m_L)\,g\,L_1\sin\theta_1 \end{aligned}\right\} \quad (13)$$

Now, the dynamic equations of the robot system could take the form :

$$\left.\begin{aligned} u_1 &= D_{11}\,\ddot{\theta}_1 + D_{12}\,\ddot{\theta}_2 - H\,(\,2\,\dot{\theta}_1\,\dot{\theta}_2 + \dot{\theta}_2^2\,) + G_1 \\ u_2 &= D_{21}\,\ddot{\theta}_1 + D_{22}\,\ddot{\theta}_2 + H\,(\,2\,\dot{\theta}_1^2\,) + G_2 \end{aligned}\right\} \quad (14)$$

where u_i is the torque applied at link i .

It is very obvious that equation (14) is time-invariant, non-linear and dynamically coupled. These discrepancies must be taken into account when choosing the reference model in order to minimize their effects. Since each link is characterized by a second order model, the reference model could be chosen to be composed of two decoupled linear second order systems as follows :

$$\dot{X}_d = A_d\,X_d + B_d\,V$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_n^2 & -2\xi\omega_n^{\;2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_n^2 & -2\xi\omega_n^2 \end{bmatrix} \begin{bmatrix} x_{1d} \\ x_{2d} \\ x_{3d} \\ x_{4d} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \omega_n^2 & 0 \\ 0 & 0 \\ 0 & \omega_n^2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (15)$$

This choice has the effect of decoupling the robot dynamic equations, thus the robot system could be treated as a series of mechanically independent arms. Also a careful choice of both ξ and ω_n can guarantee a satisfactory response.

The next step will be to express the robot model in a state space form such that :

$$\dot{X} = f(x,u)$$

Defining the following two intermediate variables :

$$\left.\begin{aligned} F_1 &= u_1 + H\,(\,2\,\dot{\theta}_1\,\dot{\theta}_2 + \dot{\theta}_2^2\,) - G_1 = D_{11}\,\ddot{\theta}_1 + D_{12}\,\ddot{\theta}_2 = u_1 + P_1 \\ F_2 &= u_2 - H\,(\,2\,\dot{\theta}_1^2\,) + G_2 \qquad\quad = D_{21}\,\ddot{\theta}_1 + D_{22}\,\ddot{\theta}_2 = u_2 + P_2 \end{aligned}\right\} \quad (16)$$

where :

$$\left.\begin{aligned} P_1 &= H\,(\,2\,\dot{\theta}_1\,\dot{\theta}_2 + \dot{\theta}_2^2\,) - G_1 \\ P_2 &= -H\,(\,2\,\dot{\theta}_1^2\,) + G_2 \end{aligned}\right\} \quad (17)$$

the following two equations could be attained :

$$\left.\begin{aligned} \ddot{\theta}_1 &= \frac{1}{\Delta}\,(D_{22}F_1 - D_{12}F_2) \\ \ddot{\theta}_2 &= \frac{1}{\Delta}\,(-D_{21}F_1 + D_{11}F_2) \end{aligned}\right\} \qquad (18)$$

where $\Delta = D_{11}D_{22} - D_{12}D_{21}$

which have the following state space form :

$$\left.\begin{aligned} x_1 &= \theta_1 & \dot{x}_1 &= \dot{\theta}_1 = x_2 \\ x_2 &= \dot{\theta}_1 & \dot{x}_2 &= \ddot{\theta}_1 = \frac{1}{\Delta}\,(D_{22}F_1 - D_{12}F_2) \\ x_3 &= \theta_2 & \dot{x}_3 &= \dot{\theta}_2 = x_4 \\ x_4 &= \dot{\theta}_2 & \dot{x}_4 &= \ddot{\theta}_2 = \frac{1}{\Delta}\,(-D_{21}F_1 + D_{11}F_2) \end{aligned}\right\} \qquad (19)$$

Thus,

$$\dot{X} = f(x,u) = \begin{bmatrix} x_2 \\ \frac{1}{\Delta}\,(D_{22}F_1 - D_{12}F_2) \\ x_4 \\ \frac{1}{\Delta}\,(-D_{21}F_1 + D_{11}F_2) \end{bmatrix} \qquad (20)$$

And from equation (9), the following form could be attained :

$$\dot{X} = A_d X + B_d V = \begin{bmatrix} x_2 \\ -\omega_n^2 x_1 - 2\xi\omega_n x_2 + \omega_n^2 V_1 \\ x_4 \\ -\omega_n^2 x_3 - 2\xi\omega_n x_4 + \omega_n^2 V_2 \end{bmatrix} \qquad (21)$$

Now with reference to equations (9) and (16), equating both equations (20) and (21) can yield the desired control signal u.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} -\omega_n^2 x_1 - 2\xi\omega_n x_2 + \omega_n^2 V_1 \\ -\omega_n^2 x_3 - 2\xi\omega_n x_4 + \omega_n^2 V_2 \end{bmatrix} \qquad (22)$$

Thus :

$$\left.\begin{aligned} u_1 &= F_1 - P_1 \\ u_2 &= F_2 - P_2 \end{aligned}\right\} \qquad (23)$$

where P_i , F_i are substituted from equations (17) and (22) respectively.

Simulation Results :

In order for the design analysis to be complete, the response of the robot system will be evaluated at two different situations :

a- *Regulation :* or point to point movements, where the robot is required to reach a desired point in its working space in terms of its degrees of freedom; (θ_1 , θ_2) [5,6].

b- *Servomechanism :* in which the robot is supposed to follow a given trajectory or a predefined path [6,7].

o *Point-TO-Point Movements :*

Assuming the robot links to have zero initial conditions ($\theta_{1i} = \theta_{2i} = 0$) and that the desired positions are :

$$\theta_{1f} = 0.3 \text{ rad.} \quad , \quad \theta_{2f} = 0.4 \text{ rad.}$$

Since the chosen reference model -for each link- is a standard second order system having the following transfer function :

$$\frac{\omega_d(s)}{V(s)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \qquad (24)$$

where : $v(t) = [\theta_{1f} \quad \theta_{2f}]^T$, or $V(s) = [\frac{\theta_{1f}}{S} \quad \frac{\theta_{2f}}{S}]^T$

the robot system could be thought of as being required to follow a step input. To avoid overshooting and to have the fastest possible response, we can choose the poles of equation (24) to be of a critically damped system (ξ = 1). For the robot system to have a band width of approximately 1 hertz, ω_n should be $\geq$ 8 rad/sec.

Thus, S $\leq$ -8 , -8 as depicted in figure (2). Corresponding results for this case are shown in figure (3).

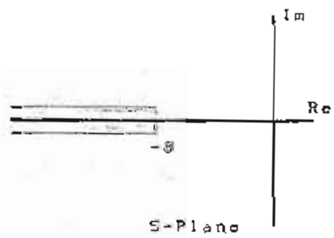


Fig. (2)

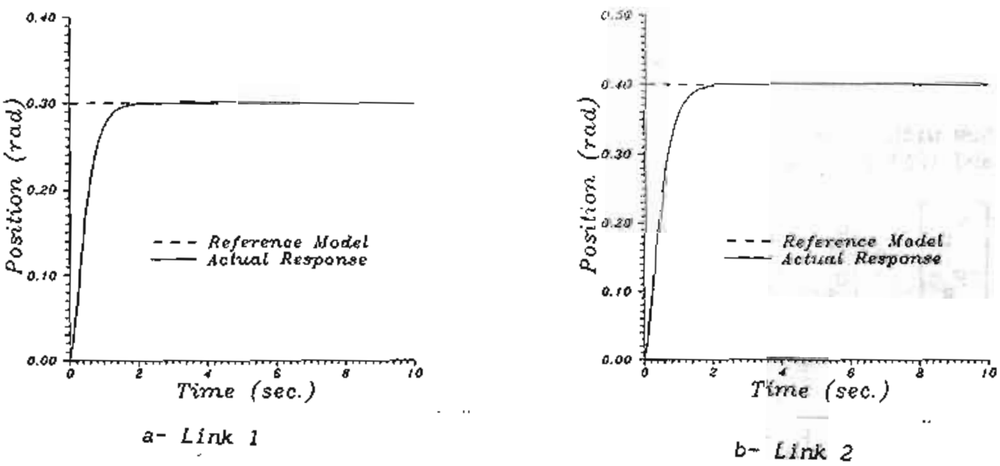


Fig. (3)
Response for Point-To-Point Movements.

o *Trajectory Following :*

In this case, $V(t) = \theta(t)$, which indicates that for rapidly-changing trajectories, the chosen poles of the reference model must have a small time constant in order to cope with the fast changes in the forcing input signal. This could be attained by either increasing ω_n or decreasing ξ such that :

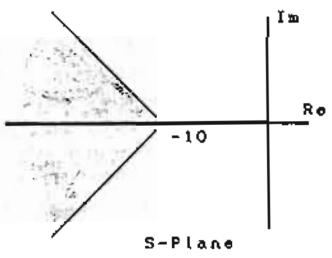


Fig. (4)

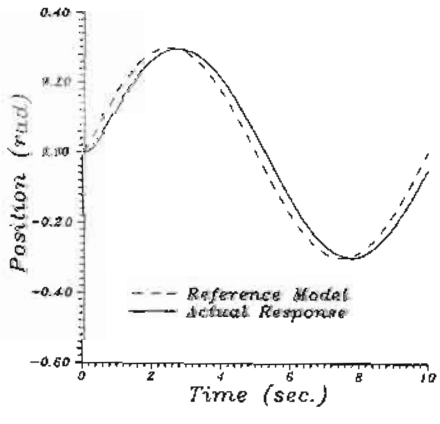
$$0.7 \le \xi \le 1.0$$

and $$\omega_n \ge 10$$

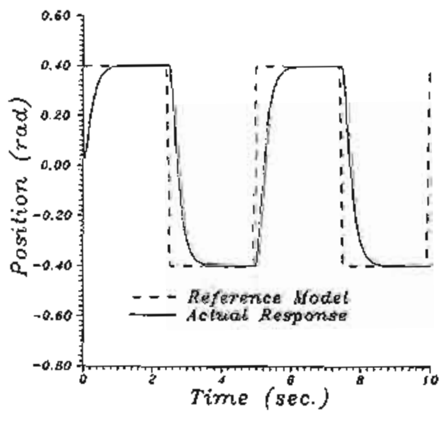
Figure (4) shows the region in the complex S domain that corresponds to the chosen poles of the reference model.
Figure (5) shows the response of the closed loop system assuming the required trajectories to be :

$\theta_{1d}(t) = 0.3 \sin(\pi t/5)$

$\theta_{2d}(t) = 0.4$ square (t) , period = 2 sec.



a- *Link 1*



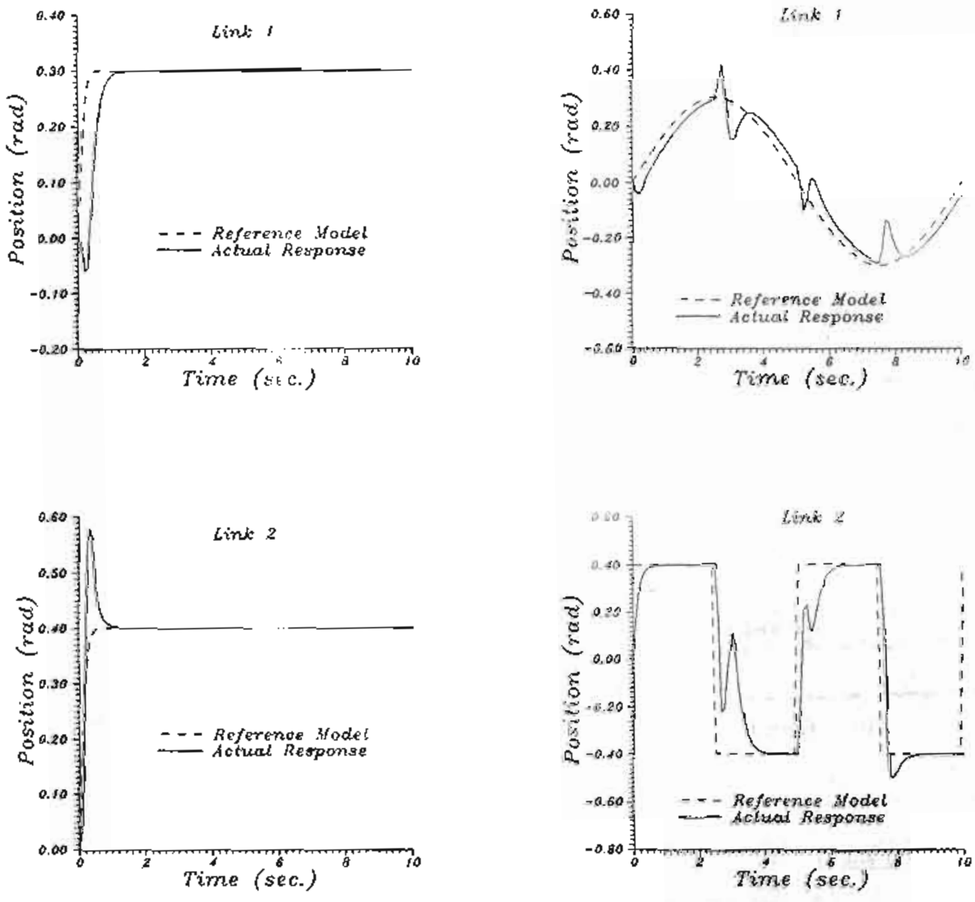
b- *Link 2*

Fig. (5)
Response for Trajectory Following Case.

Design Practicalities :

From the previous analysis, it was found that the design procedure is perfectly sound and that the robot system could follow any desired trajectory with almost zero deviation. If the desired trajectory is smooth enough, i. e. the rate in which θ changes with time is small enough, there will be no large changes in the required control signal supplied by the actuators. Hence, the causality of the proposed controller is assured. However, for large changes in θ , i. g. step variations, there will be large changes in the control signal that could force the actuator into a saturation level resulting in the actuator not being able to supply the required control signal. This means that a considerable deviation is expected between the desired path and the actual one. The following curves show the actual response of the robot system subject to the following constraints :-

a- The max supplied torque = ± 5.00 N.m.
b- The max torque variation = ± 50.0 N.m/sec. (25)
c- The max BW of the closed loop system = 4 rad/sec.



a- Regulation Case b- Servomechanism Case

Fig. (6)
Actual Response of The Closed Loop System.

It is very obvious that the designed controller exhibits the well known characteristics of reset windup actions associated with unproperly designed PID controllers due to the fact that the saturation level of the physical controller (actuator) was not taken into consideration. To overcome this difficulty, a relaxation must be made on the choice of the closed loop poles of the proposed reference model, i. e. the resultant time constant must not be too small, to avoid over-excitation of the overall system, thus assuring that the control signal required will be within the reasonable range dictated by the physical limitations of the actuators' hardware.

For the case of the two degrees of freedom robot system, and for $\xi = 1$, it was found that the proper range of ω_n that could satisfy the constraints of equation (25) is :

$$3 \text{ rad/sec.} < \omega_n < 7 \text{ rad/sec.}$$

For other dynamic systems, the acceptable range of the closed loop poles of the reference model could be obtained by carefully calculating the maximum expected variations in the system states, then back-substituting in the dynamic model of the system to infer the maximum control signal required. This is not an easy task, especially for non-linear systems, however a series of repeated trials could yield the proper choice of the reference model.

** Adaptive Implementation of The Proposed Controller :

If the process to be controlled is required to accommodate environmental changes surrounding it and/or internal variations of its parameters, it would be necessary to continuously identify the process parameters so as to achieve satisfactory performance over a wide range of operation conditions. This could be easily fulfilled by invoking a real-time identification scheme. Thus if a linear model is proposed for the actual process, equation (9) could take the form :

$$A X + B U = A_d X + B_d V \qquad (26)$$

or,

$$B U = (A_d - A) X + B_d V \qquad (27)$$

:s,

$$B^T B U = B^T (A_d - A) X + B^T B_d V \qquad (28)$$

or,

$$U = (B^T B)^{-1} B^T (A_d - A) X + (B^T B)^{-1} B^T B_d V \qquad (29)$$

Thus, equation (29) is the design equation of the model-reference controller where A , B are linear matrices to be identified.

One final comment concerning the flexibility offered in choosing the reference model should be clarified, is that the choice of the reference model must incapsulate all the constraints imposed on the physical limitations of the process and the relations between the internal parameters of the process itself and its sensitivity to environmental changes. Taking the robot system as an example, it was found that the response is sluggish when the hardware limitations of the actuators were taken into account. To overcome this problem, it is necessary either to change the reference model in equation (15) to account for the coupling effects, or to completely change the design equation deduced from equation ?) to find a faster error gradient. These two suggested solutions are te complex, and it is up the controller designer to decide which tion is the most appropriate for the underlying process.

■■ Conclusion :

This paper introduces a strategy for designing real-time model-reference controllers for non-linear processes that could be directly applied without any need to carry out linearization approximations.

A non-linear two degrees of freedom planner robot was considered to exemplify the suggested technique. The controller performance was evaluated at two different situations. In point-to-point movements, it was seen that the robot arm acts exactly as a conventional linear second-order model which assures the predominance of the chosen reference model on the behavior of the closed loop system regardless of the inherent non-linearities in the robot model. Also, in trajectory following case, it was seen that robot arm has fast and smooth response which assures the effectiveness of the proposed scheme and its ability to follow up continuous changes in the surrounding environment. Then real-time realization of the proposed controller was considered, where all the physical constraints imposed by the hardware level of the robot system (actuators' time delays, maximum supplied torque, ...etc) were taken into account. Then a few improvements in the chosen reference model were suggested in order to reach the best attainable response. Finally a generalization is made so that the proposed controller could be applied directly to any physical process via the use of adaptive identification algorithms in which the process is continuously identified to yield an equivalent linear model for it, then this linear model is used directly to calculate the required control signal which assures the adaptivity of the proposed controller and its ability to accommodate environmental changes.

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