MANSOURA JOURNAL OF Mathematics
Official Journal of Faculty of Science, Mansoura University, Egypt
E-mail: scimag@mans.edu.eg ISSN: 2974-4946

Estimation of Some Lifetime Parameters of Inverse Power IshitaDistribution under Progressive Type-II Censored Data)

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Received:15/6/2022
Accepted:24/6/2022

Abstract: The topic of the article is inference about parameters of the inverse power Ishita distribution (IPID) using progressively type-II censored (Prog -II- C) samples. For the IPID parameters, maximum likelihood and Bayesian estimates are obtained. In addition, Bayesian estimates for symmetric and asymmetric loss functions like squared error loss and LINEX loss functions are provided. The Gibbs within Metropolis–Hasting samplers process is used to provide Using the Markov Chain Monte Carlo technique, Bayes estimates of unknown parameters and its credible intervals (CRIs) are obtained. At last, an application of the proposed approaches is considered a real-life data set, data set represents uncensored strengths of glass fibers data to assess the accuracy of the proposed estimators.

keywords: Inverse power Ishita distribution; progressive Type-II censoring; maximum likelihood estimation; Bayesian estimation; Gibbs and Metropolis–Hasting samplers; loss function**.**

1.Introduction

In the field of industrial and mechanical engineering, statisticians have spent a lot of effort investigating the failure of components and units, which are the fundamental components of operational systems. Their research includes observing functioning units until they fail, recording their lifetimes. There are several sorts of censoring schemes, including right, left, interval censoring, single or multiple censoring, and Type-I or type-II censoring, however traditional Type-I and type-II censoring methods do not allow units to be removed at any point other than the experiment's end. The hybrid censoring system, which was first presented by Epstein [1] Since it a combination of Type-I and Type-II techniques, we consider a more general censoring scheme called progressive type-II censoring scheme here. The progressively hybrid censoring scheme, which was introduced by Kundu and Joarder [2], has a favorable position in the reliability and life-testing over the last few years. The progressive type-II censoring system can also be proven as follows: on the life test, the tester assigns independent and identical units. The lifetime test is ended at the failure time of the mth $(m < n)$ unit, assuming there are n units to be tested at the start of the experiment. After the first failure, time t_1 is recorded, and R_1 units are selected at random from the remaining $n - 1$ survival units. As a result, when the second failure occurs, time t_2 is recorded, and R_2 units are randomly selected from the remaining $n - R_2 - 2$ survival units. This experiment ends when the mth failure occurs, which is known ahead of time, at time t_m, and $R_m = n - m - \sum_{i=1}^{m-1} R_i$ see Figure 1. As special cases. (if $R_i = 0$ and $i = 1, 2, \dots, m$ with $n = m$) and the conventional type-II right censoring

scheme if ($R_i = 0$ and $i = 1, 2, \dots, m - 1$ with $R_m = n - m$). Progressive type-II censoring scheme has been espoused by alot of authors with different breakdowns of failure time. For example, Balakrishnan and Aggarwala [3], Alshenawy [4], Balakrishnan [5], Ahmed [6] and Almetwally et al. [7]. Mann et al. [8], Lawless [9] and Meeker and Escobar [10] studied into the properties of progressively censored order statistics and gave a review with different advances in inductive processes depend on progressive Type-I and Type-II right censored samples, as well as identifying some interesting possible research topics. Balakrishnan and Cramer [11] provide a thorough survey of the literature on progressive censoring, as well as specifics on this progressive censoring technique and its various uses. The number of patients who leave a clinical test at every step, for example, is random and cannot be predicted, according to Tse et al. [12]. The removal pattern becomes increasingly random with each failure. See Balakrishnan and Sandhu [13] for further details on the gradually censored samples. Weibull, lognormal, or exponential lifetime distributions are used. Aggarwala and Balakrishnan [14] has studied inference for progressive type-II censored cases. Balakrishnan and Sandhu [13] and Aggarwala and Balakrishnan [14] a method for replicating generic progressive type-II censored samples taken from uniform or other continuous distributions Montanari et al. [15], as well as Eryilmaz and Bairamov [9], have investigated the parameters estimator of various lifetime distributions using progressive type-II censoring. Also, Balakrishnan and Kannan [11], Mousa and Jaheen [16]- [17], Mousa and Al-sagheer [18]. In right censored order statistics of progressive type-II, Salemi et al. [19] recently investigated A-optimal and D-optimal censoring strategies, Qin et al. [20] offer a novel spacing-based test statistic for determining whether When Maiti and Kayal [21] had access to a controlled step-wise sample of the second type, they used general progressive type-II censored data from an exponential distribution and parameters from a log-logistic distribution under the progressive type-II censored sample. when a controlled step-wise sample of the second type was accessible. The IPID is a generalized of the Ishita distribution, which was presented by Elnagar et al. 2022 [30] is thought to be a good model for the covid's-19 and glass fiber data failure times. Using complete data, they also examined the maximum likelihood estimators of the unknown parameters, and also their asymptotic confidence intervals.

Shanker and Shukla [22] introduced one parameter lifetime distribution depend on a two- component mixture of an exponential distribution having scale parameter θ and a gamma distribution having shape parameter

3 and scale parameter θ with their mixing proportion, the probability density function (pdf) of Ishita distribution with scale parameter θ is given by

$$f(y)=\frac{\theta^3}{\theta^3+2}(\theta+y^2)e^{-\theta y},$$

$y>0,\theta>0.$ (1)

and the cumulative distribution function (cdf) is given by

$$F(y)=1-\left[1+\frac{\theta y(\theta y+2)}{\theta^3+2}\right]e^{-\theta y},$$

$y>0,\theta>0.$ (2)

We applied the inverse power transformation to Ishit distribution getting the Inverse Power Ishita distribution, the random variable (rv) X is follow IPID, if its probability density function (PDF) is

$$f(y)=\frac{\alpha}{\theta^3+2}(\theta+y^{-2\alpha})y^{-\alpha-1}e^{-\theta y^{-\alpha\alpha}},$$

$y>0,\theta,\alpha>0,$ (3)

and its cumulative distribution function (CDF) is

$$F(y)=\left[1+\frac{\theta y^{-\alpha}(\theta y^{-\alpha}+2)}{\theta^3+2}\right]e^{-\theta y^{-\alpha\alpha}},$$

$y>0,\theta,\alpha>0$ (4)

Let $X_{1:m:n},X_{2:m:n},\ldots,X_{m:m:n}$; $1\le m\le n$ be a result of a progressively type-II censored sample lifetime test that included n units taken from a IPI (α,θ) distribution and the censoring scheme is $R_1,R_2,\ldots,R_m$.

Balakrishnan and Aggarwala [3] gives the joint PDF of a progressive type-II

$$f(x_{1:m:n},x_{2:m:n},\ldots,x_{m:m:n}) = C\prod_{i=1}^{m} f(x_{i:m:n;\alpha,\theta}).\left[1-F(x_{i:m:n;\alpha,\theta})\right]^{R_i}, m<n,$$

(5)

Where $C=n(n-R_1-1)\ldots(n-\sum_{i=1}^{m-1}(R_i-1))$.

The great review essay by Balakrishnan [5] provides a recent report on progressive censoring schemes. The remainder of the paper is organized as follows: MLEs of and are obtained in Section 2. In Section 3 investigate the confidence intervals for the unknown parameters. For various loss functions, such as the SEL and LINEX loss functions, Bayes estimates for unknown parameters were introduced in Section 4. In Section 5, a real data set was studied. Finally, in section 6, the research was conducted.

2. Maximum-Likelihood Estimation

We look at the problem of estimating IPID parameters in progressive type-II censored data using the maximum likelihood estimation method (MLEs).Let $x_{1:m:n}, x_{2:m:n}, \dots, x_{m:m:n}$be a progressive type-II censored sample from the IPI distribution with PDF Eq. (1) and parameters α and θ. The likelihood function has the following

$$L(\alpha,\theta) = c\frac{\alpha^m \theta^{3m}}{(\theta^3+2)^m} e^{-\theta\sum_{i=1}^m y_i^{-\alpha}} \prod_{i=1}^m [(\theta + y_i^{-2\alpha}) y_i^{-\alpha-1}] \prod_{i=1}^m \left[1 - \left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right] e^{-\theta y_i^{-\alpha}}\right]^{R_i}. \quad (6)$$

On progressively type-II censored samples, the log likelihood function of the parameters α and θ is obtained as

$$l = \ln L(\alpha,\theta) = \ln c + m\ln\alpha + 3m\ln\theta - m\ln(\theta^3+2) - \theta\sum_{i=1}^m y_i^{-\alpha} + \sum_{i=1}^m \ln(\theta + y_i^{-2\alpha}) - (\alpha+1)\sum_{i=1}^m \ln y_i + \sum_{i=1}^m R_i \ln\left[1 - \left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right] e^{-\theta y_i^{-\alpha}}\right]. \quad (7)$$

The first partial derivatives of Eq. (7) with respect to α and θ are calculated and solving this nonlinear system of equations $\frac{\partial}{\partial\alpha}\ln L(\alpha,\theta) = 0$ and $\frac{\partial}{\partial\theta}\ln L(\alpha,\theta) = 0$, the maximum likelihood estimates ($\hat{\alpha}$ and $\hat{\theta}$) are obtained.

Accordingly, results in

$$\frac{m}{\alpha} + \sum_{i=1}^m (\theta y_i^{-\alpha} - 1)\ln y_i + \sum_{i=1}^m R_i \left[\frac{\theta y_i^{-\alpha} \ln y_i \, e^{-\theta y_i^{-\alpha}}}{1 - \left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right] e^{-\theta y_i^{-\alpha}}}\right] = 0 \quad (8)$$

and

$$\frac{3m}{\theta} - \frac{3m\theta^2}{\theta^3+2} - \sum_{i=1}^m y_i^{-\alpha} + \sum_{i=1}^m \frac{1}{\theta + y_i^{-2\alpha}} + \sum_{i=1}^m R_i\, y_i^{-\alpha} e^{-\theta y_i^{-\alpha}} \left[\frac{\left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right] - \left[\frac{2(\theta^3+2)(\theta y_i^{-\alpha}+1) - 3\theta^2(\theta y_i^{-\alpha}+2)}{(\theta^3+2)^2}\right]}{1 - \left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right] e^{-\theta y_i^{-\alpha}}}\right] = 0, \quad (9)$$

since Eqs. (8-9) cannot be solved analytically, so to obtain the estimations, a numerical method like Newton-Raphson method should be employed. In Ahmed [6], the algorithm is given in detail.

3. Asymptotic confidence intervals

The entries of the Fisher information matrix's inverse give us the approximate variances and covariance's of the MLEs, $\hat{\alpha}$ and $\hat{\theta}$; $I_{ij} = E[-[\partial^2 l(\Psi)/\partial\psi_i\partial\psi_j]]$,

where $i,j = 1,2$ and $\Psi = (\psi_1,\psi_2) = (\alpha,\theta)$. Unfortunately, precise asymptotic forms for these equations are difficult to get. The Fisher information matrix $\hat{I}_{ij} = -[\partial^2 l(\Psi)/\partial\psi_i\partial\psi_j\,]_{\Psi=\hat{\Psi}}$ is then used, we will utilize the expectation to determine the parameters' confidence intervals (CIs), which is obtained through inference. Consequently, the observed information matrix is

$$\hat{I}_{(\hat{\alpha},\hat{\theta})} = \begin{bmatrix} \frac{-\partial^2 l}{\partial\alpha^2} & \frac{-\partial^2 l}{\partial\alpha\,\partial\theta} \\ \frac{-\partial^2 l}{\partial\theta\,\partial\alpha} & \frac{-\partial^2 l}{\partial\theta^2} \end{bmatrix}_{(\alpha=\hat{\alpha},\theta=\hat{\theta})}. \quad (10)$$

As a result, the asymptotic variance-covariance matrix $\hat{V}$ for MLEs is derived by converting the observed information matrix $\hat{I}(\alpha,\theta)$ or similar to the asymptotic variance-covariance matrix

$$\hat{V} = \hat{I}^{-1} = \begin{bmatrix} \widehat{var(\alpha)} & cov(\alpha,\theta) \\ cov(\alpha,\theta) & \widehat{var(\theta)} \end{bmatrix}_{(\hat{\alpha},\hat{\theta})}. \quad (11)$$

It is evident that $(\hat{\alpha}, \hat{\theta})$ is approximately distributed as multivariate normal with mean (α,θ) and covariance matrix $I^{-1}(\alpha,\theta)$ given specific symmetry requirements, see Bebbington et al. [23]. As a result, the $(1 -$

$\gamma)100\%$ convergent confidence intervals (ACIs) for α and θ may be calculated as follows:

$$\hat{\alpha} \pm z_{\frac{\gamma}{2}}\sqrt{\widehat{var(\alpha)}}, \hat{\theta} \pm z_{\frac{\gamma}{2}}\sqrt{\widehat{var(\theta)}}, \quad (12)$$

where $z_{\frac{\gamma}{2}}$ is the right-tail probability percentile standard normal distribution for $\frac{\gamma}{2}$ see Lawless [24].

4. Bayes Estimation

The Bayesian process treats the parameters as random variables, and the distributions for these random variables are illustrated by a joint prior distribution, which is established before control the failure data, according to Zellner [25] discussed Bayesian estimation. The Bayesian procedure regards the parameters as random variables and the distributions for these random variables are illustrated by a joint prior distribution, which is created before control the failure data. Because of the reduction of given data in reliability analysis in the most practical conditions, the Bayesian approach will supply successfully in these conditions, let the prior

knowledge in the analysis of the failure data. The SEL and LINEX loss functions, as well as Bayesian estimations of the unknown parameters α and θ. The independent parameters α and θ have the following gamma prior distributions

$$\pi_1(\alpha) \propto \alpha^{a_1-1}\exp(-b_1\alpha), \alpha > 0,$$

$$\pi_2(\theta) \propto \alpha\theta^{a_2-1}\exp(-b_2\theta), \theta > 0, \quad (13)$$

wherever all the hyper parameters a_i and $b_i, i = 1, 2$ are assumed to be non-negative and known. The posterior distribution is derived from the likelihood function Eq. (4) and the prior distribution Eq. (13), and with the posterior distribution of α and θ denoted by $\pi^*(\alpha, \theta|x)$

$$\pi^*(\alpha, \theta|x) = \frac{L(\alpha, \theta; x)\pi_1(\alpha)\pi_2(\theta)}{\int_0^\infty\int_0^\infty L(\alpha, \theta; x)\,\pi_1(\alpha)\pi_2(\theta)d\alpha d\theta}. (14)$$

4.1. Loss Function

We need to design an asymmetric loss function to make statistical Bayesian inference more practical and relevant. The loss function, as defined by Press and James [26], is a real-valued function that satisfies all feasible estimates and parameters. We must adopt an asymmetric loss function for the purpose of obtain statistical Bayesian inference extra sensible and relevant. According to Press and James [29], the loss function is a real valued function that satisfy all possible estimations and parameters.

4.1.1. Squared Error Loss Function

The square error loss function is defined in estimation problems as:

$$L(\psi, \hat{\psi}) = (\hat{\psi} - \psi)^2. \quad (15)$$

Then, for any function of α and θ, the Bayes estimate is $g(\alpha, \theta)$ under the SEL function given as follows

$$\hat{g}_{BS}(\alpha, \theta \mid x) = E_{\alpha,\theta|x}(g(\alpha, \theta)), \quad (16)$$

where

$$E_{\alpha,\theta|x}(g(\alpha,\theta)) = \frac{\int_0^\infty\int_0^\infty g(\alpha,\theta)\,\pi_1(\alpha)\pi_2(\theta)L(\alpha,\theta|x)d\alpha d\theta}{\int_0^\infty\int_0^\infty \pi_1(\alpha)\pi_2(\theta)L(\alpha,\theta|x)d\alpha d\theta}. (15)$$

4.1.2. Linear Exponential (LINEX) Loss Function

The LINEX loss function $L(\Delta)$ for a parameter ψ is proposed via Varian and Hal [27] as:

$$L(\Delta) = (e^{c\Delta} - c\Delta - 1), c \neq 0, \Delta = \hat{\psi} - \psi, \quad (18)$$

hence the bayes estimate of a function $g(\alpha, \theta)$ under LINEX loss function given by

$$\hat{g}_{BL}(\alpha, \theta \mid x) = -\frac{1}{c}\log[E(e^{-g(\alpha,\theta)} \mid x)], c \neq 0, (19)$$

$$E(e^{-g(\alpha,\theta)}) = \frac{\int_0^\infty\int_0^\infty e^{-g(\alpha,\theta)}\,\pi_1(\alpha)\pi_2(\theta)L(\alpha,\theta|x)d\alpha d\theta}{\int_0^\infty\int_0^\infty \pi_1(\alpha)\pi_2(\theta)L(\alpha,\theta|x)d\alpha d\theta}. \quad (16)$$

It's important noting that the ratio of multiple integrals in Eqs. (15) and (16) cannot be expressed explicitly. To produce samples from the joint posterior density function in Eq. (14), the MCMC approach is used. We use the Gibbs within Metropolis-Hasting samplers process to implement the MCMC technique. The joint

posterior distribution may could be expressed according to

$$\pi^*(\alpha,\theta|x) \propto m\frac{\alpha^{m-1}\theta^{3m-1}}{(\theta^3+2)^m}e^{-\theta\sum_{i=1}^m y_i^{-\alpha}}\prod_{i=1}^m[(\theta+y_i^{-2\alpha})y_i^{-\alpha-1}]\prod_{i=1}^m\left[1-\left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right]e^{-\theta y_i^{-\alpha}}\right]^{R_i}. (17)$$

The conditional posterior densities of α and θ may could be represented according to

$$\pi^*{}_1(\alpha|\theta,x) \propto \alpha^{m-1}e^{-\theta\sum_{i=1}^m y_i^{-\alpha}}\prod_{i=1}^m[(\theta+y_i^{-2\alpha})y_i^{-\alpha-1}]\prod_{i=1}^m\left[1-\left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right]e^{-\theta y_i^{-\alpha}}\right]^{R_i}, (18)$$

and

$$\pi^*{}_2(\theta|\alpha,x) \propto \frac{\theta^{3m-1}}{(\theta^3+2)^m}e^{-\theta\sum_{i=1}^m y_i^{-\alpha}}\prod_{i=1}^m[(\theta+y_i^{-2\alpha})y_i^{-\alpha-1}]\prod_{i=1}^m\left[1-\left[1+\frac{\theta y_i^{-\alpha}(\theta y_i^{-\alpha}+2)}{\theta^3+2}\right]e^{-\theta y_i^{-\alpha}}\right]^{R_i}. (18)$$

Because the conditional posteriors of α and θ in the previous equations do not qualify for a specified distribution, the Metropolis–Hasting sampler must be used to apply the MCMC approach. Tierney and Luke [28] proposed the Metropolis–Hastings algorithm within Gibbs sampling, which generates the posterior samples as:

1.Begin with an educated guess of $(\alpha^{(0)},\theta^{(0)})$.

Set = 1 .

2.Calculate $\alpha^{(j)}$ and $\theta^{(j)}$ from Eqs. (18) and (23) by using M-H technique described below, with the normal indicated distribution $N(\alpha^{j-1},var(\alpha))$ and $N(\theta^{j-1},var(\theta))$, where The main diagonal of the inverse Fisher information matrix can be used to calculate $var(\alpha)$ and $var(\theta)$.

a)Create α^* proposal from $N(\alpha^{j-1},var(\alpha))$ and θ^* from $N(\theta^{j-1},var(\theta))$.

b)Calculate the probability of acceptance $\rho_\alpha = \min\left[1,\frac{\pi^*{}_1(\alpha^*|\theta^{j-1},x)}{\pi^*{}_1(\alpha^{j-1}|\theta^{j-1},x)}\right]$ and $\rho_\theta = \min\left[1,\frac{\pi^*{}_2(\theta^*|\alpha^j,x)}{\pi^*{}_2(\theta^{j-1}|\alpha^j,x)}\right]$

c) Generate a u_1 and u_2 from a Uniform (0, 1) distribution.

d)If $u_1 = \rho_\alpha$, accept the proposition and set $\alpha^j = \alpha^*$ else set $\alpha^j = \alpha^{j-1}$.

e)If $u_2 = \rho_\theta$, accept the proposition and set $\theta^j = \theta^*$ else set $\theta^j = \theta^{j-1}$.

Calculate α^j and θ^j.

Set $j = j + 1$.

Repeat Steps 3-6 N times.

To evaluate the CRIs of α and θ of $\psi_k^i, i = 1,2,\dots,N, k = 1,2$ and $(\psi_1,\psi_2) = (\alpha,\theta)$ as $\psi_k^1 < \psi_k^2 < \cdots < \psi_k^N$, then the $(1-\gamma)100\%$ CRIs of ψ_k is $\hat{\alpha}_{BS} = \frac{1}{N-M}\sum_{j=M+1}^N \alpha^j$,

And $\hat{\theta}_{BS} = \frac{1}{N-M}\sum_{j=M+1}^N \theta^j$.

And the estimates for the mentioned parameters under LINEX loss function are:

$$\alpha_{BL} = \frac{-1}{c}\log\left[\frac{1}{N-M}\sum_{i=M+1}^N e^{-c\alpha^i}\right],$$

and

$$\theta_{BL} = \frac{-1}{c}\log\left[\frac{1}{N-M}\sum_{i=M+1}^N e^{-c\theta^i}\right].$$

5. Application of Real-life data

In this section real data set is analyzed. The data appeared in the work introduced by Mahmoud and Mandouh [29] uncensored strengths of glass fibers data. The considered glass fibers data uncensored strengths of glass fibers data. A progressively type II censored sample of size $m = 10$ simulated randomly from the sample of size $n = 20$ with censoring scheme $(5,5,0,0,0,0,0,0,0,0)$.

We use data appeared in the work introduced by Mahmoud and Mandouh [29] strengths of glass fibers data: 1.014, 1.081, 1.082, 1.185, 1.223, 1.248, 1.267, 1.271, 1.272, 1.275, 1.276, 1.278, 1.286, 1.288, 1.292, 1.304, 1.306, 1.355, 1.361, 1.364, 1.379, 1.409, 1.426, 1.459, 1.46, 1.476, 1.481, 1.484, 1.501, 1.506, 1.524, 1.526, 1.535, 1.541, 1.568, 1.579, 1.581, 1.591, 1.593, 1.602, 1.666, 1.67, 1.684, 1.691, 1.704, 1.731, 1.735, 1.747, 1.748, 1.757, 1.800, 1.806, 1.867, 1.876, 1.878, 1.91, 1.916, 1.972, 2.012, 2.456, 2.592, 3.197, and 4.121.

The inverse power Ishita (IPID) is fitted the data set by using the method of maximum likelihood and the outcomes are compared with the other competitive models namely Inverse Power Sujatha (IPS), Ishita (I), Sujatha (S), Lindely (L) and Exponential distributions respectively. Next, some criteria like the

Akaike information criterion (AIC), Bayesian information criterion (BIC), and Consistent Akaike information criterion (CAIC), −2 ln(L) Kolmogorov-Samirnov Statistics (K-S), the Cramer-von Mises (W^*), Anderson-Darling (A^*) and P-value statistics are used to verify which of the aforementioned distributions fits the research data better. The formula for computing the vales of AIC, BIC and CAIC are respectively provided by

$$AIC = 2k - 2l,$$

$BIC = kln(n) - 2l,$ and

$$CAIC = \frac{2nk}{n-k-1} - 2l.$$

where l denotes the log-likelihood function evaluated at the maximum likelihood estimates, k is the number of model parameters and n is the sample size.

Distribution	K-S	P-Value	-2ln(L)	AIC	BIC	W^*	A^*	CAIC
IPI	0.07683	0.85105	20.0346	44.0693	48.3556	0.06972	0.53051	44.2693
IPS	0.07726	0.84625	20.0856	44.1713	48.4576	0.07064	0.5383	44.3713
Ishita	0.39886	3.93895×10^{-9}	87.6274	177.255	179.398	1.00794	14.6483	177.32
Sujatha	0.40959	1.32028×10^{-9}	81.4149	166.973	164.895	1.00794	14.4846	164.895
Lindely	0.43474	9.09346×10^{-11}	85.4759	172.952	175.095	1.00794	15.8556	173.017
Exponential	0.47214	1.26787×10^{-9}	93.2229	188.446	190.589	1.00794	18.2276	188.511

Table 1: Estimates of model parameter with standard errors and corresponding values of model selection criteria for the distributions fitted to glass fiber data.

Tables 1, show that the present results by (IPID) have the least values for (AIC), (BIC), (CAIC), $-2\ ln(L)$ Kolmogorov-Samirnov Statistics (K-S), (W^*), (A^*) and the highest value for P-value statistics indicating the best fit for the data.

Progressively Type-II failure data presented in Table 2 have the MLEs parameters $\hat{\alpha}$ and $\hat{\theta}$ giving in Table 3. For different values of the shape parameter c of the LINEX loss function for the parameters α and θ, Bayes estimates respect to both SEL and LINEX functions are produced which indicated in Table 4 and showed that Bayesian methods have good performance, The 95% ACIs and CRIs for the parameters α and θ are computed. It is well known that LINEX loss function becomes symmetric for c tending to zero. From Table (3), the results of SEL and LINEX loss function, at $c = 0.0001$, are the same. This indicates that the recommended approaches are accurate. Figures 1, 2 display diagrams for simulation numbers of the parameters α and θ for glass fibers data, and they confirm a good mixing performance.

Table 2: The Progressively Type-II failure glass fibers data.

x_i	0.92	0.928	0.997	0.9971
R_i	5	5	0	0
x_i	1.061	1.117	1.162	1.183
R_i	0	0	0	0
x_i	1.187	1.192	1.196	1.213
R_i	0	0	0	0
x_i	1.215	1.2199	1.22	1.224
R_i	0	0	0	0
x_i	1.225	1.228	1.237	1.24
R_i	0	0	0	0

Table 3: Point estimates for the parameters α, θ, glass fibers data.

Parameter	MLE	SEL	LINEX		
			$C_1 = -2$	$C_2 = 2$	$C_3 = 0.000$
α	11.8184	11.491	11.5167	11.472	11.4931
θ	1.91185	1.8323	1.83367	1.8379	1.8323

Table 4: 95% ACIs and CRI α and θ glass fibers data.

Parameter	MLE	MCMC
α	(6.81276,16.824)	(11.2878,11.891)
θ	(1.27567,2.54804)	(1.79865,1.8838)

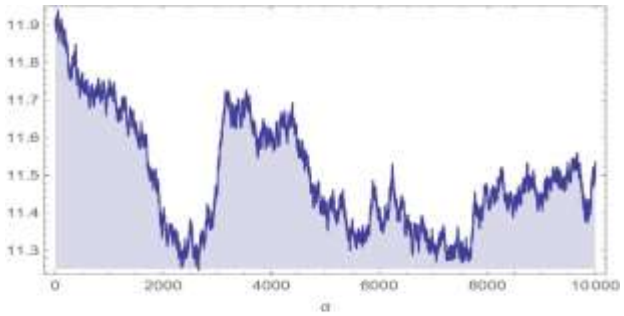

Fig1: Simulation numbers of the parameter α for glass fibers data.

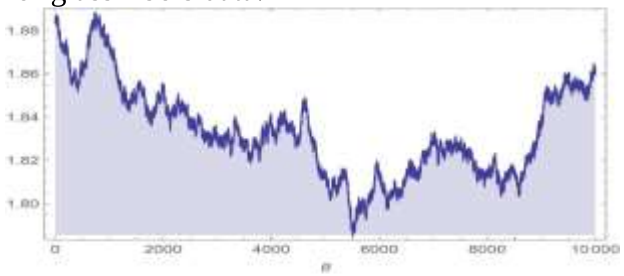

Fig 2: Simulation numbers of the parameter θ for glass fibers data.

6. conclusion

In this study, we discuss an iterative technique for deriving MLEs from an inverse power Ishita lifetime distribution using progressively Type-II censored samples. Another estimation approach, based on Bayes

estimates, is also considered. The Bayes estimates were produced using loss functions; however, they are not available in explicit form. We propose that the Bayes estimators and associated IPID credible intervals be computed using the MCMC technique. We derive the posterior summaries of interest, such as credible intervals for the parameters, in a simple way using typical MCMC simulation methods for a Bayesian analysis of the model. The findings of this study can be used to the estimate problem of the IPI distribution using complete data, which are usually type-II censoring samples. The proposed methods are demonstrated with numerical example.

7.References

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