Menofia University **Faculty of Engineering** **Shebien El-kom** **Academic Year : 2016-2017** **Department : Basic Eng. Sci.**		**Subject : P.D.E.** **Max. Marks: 100** **Grade 600 master** **Time Allowed : 3 hours** **Date : 7/ 6 / 2017**

Question 1 [30 marks]

For a circular disk the Laplace equation in polar coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \theta^2}\right)=0$$

With boundary conditions $u(a,\theta)=f(\theta)$,

Roundedness at origin $|u(0,\theta)|<\infty$,

Periodicity $u(r,-\pi)=u(r,\pi)$ and $\frac{\partial u}{\partial \theta}(r,-\pi)=\frac{\partial u}{\partial \theta}(r,\pi)$

Solve this equation?

Question 2 [10 marks]

Prove that if $f'(x)$ is piecewise smooth, then the Fourier sine series of a continuous function can only be differentiated term by term $f(0)=0$ *and* $f(L)=0$

Question 3 [20 marks]

Solve heat equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$, $-\infty<x<\infty$,

With B.Cs $u(-\infty,t)=0$ *and* $u(\infty,t)=0$ And I.C. $u(x,0)=f(x)$

Question 4 [40marks]

A) Solve heat equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+xe^{-t}$, $0<x<L$, $t>0$

With B.Cs $u(0,t)+u'(0,t)=1$ *and* $u(0,t)=2$

And I.C. $u(x,0)=f(x)$

B) Solve the wave equation Using Fourier Transform

$$\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2},\qquad -\infty<x<\infty$$

With conditions: $u(x,0)=f(x)$, $\frac{\partial u(x,0)}{\partial t}=0$

With my best wishes

Dr. Islam M. Eldesoky