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RECURRENT ALGORITHM FOR THE INITIAL VALUE PROBLEM OF SPACE DYNAMICS

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Abstract

In this paper, an efficient and simple algorithm of the initial value problem of space dynamics will be developed using recurrent computations of Lagrangian coefficients of the transition matrix. Numerical applications of the algorithm are given for planetary and artificial satellites motion. The results are optimistic in the way that, any desired accuracy could be achieved in solving initial value problem of space dynamics whatever the type of the orbit may be.

1. Introduction

We will develop solutions of the two-body initial value problem expressed in inertial cartesian coordinates. In principle, the reduction of the two-body problem to six independent integrals of motion is sufficient for a solution [Danby,1988]. However, in many practical applications it is desirable to have the solution in inertial cartesian coordinates. Of these applications, are for examples, the patched conic [Vallado, 1997] which find much use in all space mission analysis, and the closest approach [Sharaf and Sharaf, 1998] which is of great importance for visual observations of planetary surfaces and also for randezvous considerations of space probes.

The algorithm is of recurrent nature, which facilitate the computations of large number of coefficients needed for accurate determination of the position and velocity vectors. Also, the solution is that there is no need to solve Kepler's equation and its variants for parabolic and hyperbolic orbits. Finally, the algorithm is universal in the sense that, it could be applied for any type of the conic orbits. Numerical applications are also given for some problems of planetary and artificial satellites motions.

2. Basic Formulations

2.1. TWO – BODY FORMULATIONS

The equation describing the relative motion of the two bodies in rectangular coordinates is

$$\frac{d}{dt}\mathbf{v} = \frac{d^2}{dt^2}\mathbf{r} = -\frac{\mu}{r^3}\mathbf{r}, \qquad (2.1)$$

where $\mathbf{v}$ is the velocity , μ is the gravitational parameter and $\mathbf{r}$ is the position vector given as

$$\mathbf{r} = x\underline{i}_x + y\underline{i}_y + z\underline{i}_z, \qquad (2.2)$$

$\underline{i}_x$, $\underline{i}_y$ and $\underline{i}_z$ are the unit vectors along the coordinate axes x ,y and z respectively, and

$$r = (x^2 + y^2 + z^2)^{1/2}. \qquad (2.3)$$

Equation (2.1) is unchanged if we replace $\mathbf{r}$ with $-\mathbf{r}$. Thus Equation (2.1) gives the motion of the body of mass m_2 relative to the body of the mass m_1, or the motion of m_1 relative to m_2. Also observe that if we replace t with – t, Equation (2.1) remains unchanged.

Among the integrals of the two-body problem are the conservation of angular momentum;

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} = \text{constant}, \qquad (2.4)$$

and the conservation of the total energy

$$v^2 - \frac{2\mu}{r} = \text{constant} \qquad (2.5)$$

where $\mathbf{v} = \dot{x}\underline{i}_x + \dot{y}\underline{i}_y + \dot{z}\underline{i}_z$, (2.6)

$$v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2.$$

2.2. LAGRANGIAN COEFFICIENTS

The two body initial value problem may be formulated as: Given position and velocity vectors $\mathbf{r}_n$ and $\mathbf{v}_n$ at the initial time t_n, and given a second time t_l find $\mathbf{r}_l$ and $\mathbf{v}_l$. The transition matrix $\Phi_{l,n}$, needed to extrapolate $\mathbf{r}_l$, $\mathbf{v}_l$. from $\mathbf{r}_n$, $\mathbf{v}_n$ is given as

$$\Phi_{l,n} = \begin{vmatrix} F_{l,n} G_{l,n} \\ \\ \dot{F}_{l,n} \dot{G}_{l,n} \end{vmatrix} \qquad (2.7)$$

that is $\mathbf{r}_l = F_{l,n}\,\mathbf{r}_n + G_{l,n}\,\mathbf{v}_n$, (2.8)

$$\mathbf{v}_l = \dot{F}_{l,n}\,\mathbf{r}_n + \dot{G}_{l,n}\,\mathbf{v}_n, \qquad (2.9)$$

where the elements $F_{l,n}$, $G_{l,n}$ and their time derivatives $\dot{F}_{l,n}$, $\dot{G}_{l,n}$ are known as *Lagrangian Coefficients.* For any type of the two-body conic motion, these coefficients satisfy the same two properties:

1- For any two points on an orbit $\mathbf{r}_n$ and $\mathbf{r}_l$ corresponding to the epochs t_n and t_l we get from the conservation of angular momentum [Equation (2.4)] and Equations (2.8) and (2.9) that

$$F_{l,n} \dot{G}_{l,n} - G_{l,n} \dot{F}_{l,n} = 1. \quad (2.10)$$

This Equation implies that, given any three of the four functions F,G, $\dot{F}$ and $\dot{G}$, we can solve for the remaining one, also this equation means that the determinant of the transition matrix is unity.

2- For any three points on an orbit $\mathbf{r}_n$, $\mathbf{r}_l$ and $\mathbf{r}_N$ corresponding to three epochs t_n, t_l and t_N, then [Bond, 1996]

$$\Phi_{N,n} = \Phi_{N,l} * \Phi_{l,n} \quad (2.11)$$

This property is very useful when the time difference $t_N - t_n$ is sufficiently large, we may have to repeat the process by decrementing the time interval several times. The transition matrices thus sequentially generated are, of course, multiplied together to produce the final desired transition matrix.

Equation (2.11) implies that

$$F_{N,n} = F_{N,l} * F_{l,n} + G_{N,l} * \dot{F}_{l,n} , \qquad (2.12.1)$$

$$G_{N,n} = F_{N,l} * G_{l,n} + G_{N,l} * \dot{G}_{l,n} , \qquad (2.12.2)$$

$$\dot{F}_{N,n} = \dot{F}_{N,l} * F_{l,n} + \dot{G}_{N,l} * \dot{F}_{l,n} , \qquad (2.12.3)$$

$$\dot{G}_{N,n} = \dot{F}_{N,l} * G_{l,n} + \dot{G}_{N,l} * G_{l,n} , \qquad (2.12.4)$$

1.1. RECURRENT COMPUTATIONS OF LAGRANGIAN COEFFICIENTS AS FUNCTIONS OF TIME

The Lagrangian Coefficients $F_{l,n}$ and $G_{l,n}$ satisfy the differential equation [Battine, 1999]

$$\ddot{Q} + uQ = 0 , \qquad (2.13)$$

and, for Q ≡ $F_{l,n}$,the initial conditions are

$$Q(t_n) = F_{l,n} = 1 \text{ and } \dot{Q}(t_n) = \dot{F}_{l,n} = 0 , \qquad (2.14)$$

while, for Q ≡ $G_{l,n}$, we have

$$Q(t_n) = G_{l,n} = 0 \text{ and } \dot{Q}(t_n) = F\dot{G}_{l,n} = 1 . \qquad (2.15)$$

Coupled with Equation (2.13), the first order differential equations

$$\frac{dv}{dt} = -3uz ; \frac{dz}{dt} = E - u - 2z^2 ; \frac{dE}{dt} = -2z(u + E), \qquad (2.16)$$

where the *Lagrange's fundamental invariants* u, z and E are defined as

$$u = \mu / r^{3} \; ; \; z = \frac{1}{r^{2}} \langle \mathbf{r}, \mathbf{v} \rangle \; ; \; E = \frac{1}{r^{2}} \langle \mathbf{v}, \mathbf{v} \rangle , \quad (2.17)$$

where $\langle \mathbf{A}, \mathbf{B} \rangle$ is used to denote the scalar product of the two vectors **A, B**.

Utilize the standard technique of a power series solution of Equation (2.13), together with Equations (2.16) in the form

$$Q = \sum_{m=1}^{\infty} Q_m (t - t_n)^{m-1}, \quad u = \sum_{m=1}^{\infty} u_m (t - t_n)^{m-1};$$

$$z = \sum_{m=1}^{\infty} z_m (t - t_n)^{m-1}; \qquad E = \sum_{m=1}^{\infty} E_m (t - t_n)^{m-1},$$

we get for the coefficients the recursion formulae $\forall \; m \geq 1$.

$$Q_{m+2} = - \frac{1}{m(m+1)} \sum_{m=1}^{m} u_l Q_{m-l+1}, \tag{2.18}$$

$$u_{m+2} = - \frac{3}{m} \sum_{l=1}^{m} u_l z_{m-l+1}, \tag{2.19}$$

$$z_{m+1} = \frac{1}{m} \{ E_m - u_m - 2 \sum_{l=1}^{m} z_l z_{m-l+1} \}, \tag{2.20}$$

$$E_{m+1} = - \frac{2}{m} \sum_{l=1}^{m} z_l \{ u_{m-l+1} + E_{m-l+1} \}. \tag{2.21}$$

The initial conditions for these formulae could be computed from the initial values $\mathbf{r}_n$ and $\mathbf{v}_n$ as

$$Q_1 = \begin{cases} \rightarrow 1 \text{ For } Q \equiv F_{l,n} \\ \rightarrow 0 \text{ For } Q \equiv G_{l,n} \end{cases} \quad ; \quad Q_2 = \begin{cases} \rightarrow 0 \text{ For } Q \equiv F_{l,n} \\ \rightarrow 1 \text{ For } Q \equiv G_{l,n} \end{cases} \quad (2.22)$$

where $u_1 = \mu / r^3 \quad ; z_1 = \{x_n \dot{x}_n + y_n \dot{y}_n + z_n \dot{z}_n\} / r_n^2 \quad ; \; E_1 = v_n^2 / r_n^2$ (2.23)

$$r_n = (x_n^2 + y_n^2 + z_n^2)^{1/2} \quad ; v_n^2 = \dot{x}_n^2 + \dot{y}_n^2 + \dot{z}_n^2 . \qquad (2.24)$$

Equations (2.18) to (2.24) determine recursively the $F_{l,n}$ and $G_{l,n}$ functions at any time t_l , the functions $\dot{F}_{l,n}$ and $\dot{G}_{l,n}$ are simply the respective time derivatives of $F_{l,n}$ and $G_{l,n}$.

3.□. Computational Developments

3.1 CANONICAL UNITS

We can work faster by eliminating useless conversions between routines and by not changing validated routines to incorporate new constants. The overall effect is programs that are faster, more accurate, and easier to maintain. Values for the commonly used astrodynamic constants and their relationship to canonical units are listed in Table A.

3.2. BASIC ROUTINES

* Routine to compute Lagrangian coefficients F and G from the recurrent formulations of Subsection 2.3.
* Routine to evaluate a polynomial for a given value of the argument, and is done by means of nested multiplication.
* Routine to find derivative of a polynomial, for which, the dimension of the resulting polynomial is set at dimension of the given polynomial less one. Derivative is then calculated by multiplying coefficients by their respective exponents.
* Routine to find the correct quadrant of the angle $\beta = \tan^{-1}(y/x)$, and is done by adding and subtracting π or 2π to $\gamma = \tan^{-1}(|y/x|)$, to bring β into the correct quadrant according to the sign of x and y.
* Routine to compute the geocentric distance (r), geocentric right ascension (α) and declination (δ) and their time rates for a body of known values of **r** and **v** at a given epoch t. Symbolically: t , **r**, **v** $\rightarrow r, \alpha, \delta, \dot{r}, \dot{\alpha}, \dot{\delta}$. The formulations of this routine are

$$r = |\ \mathbf{r}|\ , \qquad (3.1)$$

$$\sin\delta = z/r \qquad ; \cos\delta = \sqrt{x^2 + y^2}\,/r\ , \qquad (3.2)$$

$$\mathrm{P} = \sqrt{x^2 + y^2} \neq 0, \qquad (3.3)$$

$$\sin\alpha = \frac{y}{\mathrm{P}}\ ; \qquad (3.4)$$

$$\text{Else} \quad W = \sqrt{\dot{x}^2 + \dot{y}^2}\ , \qquad (3.5)$$

$$\sin\alpha = \frac{\dot{y}}{W} \qquad ; \cos\alpha = \frac{\dot{x}}{W}, \qquad (3.6)$$

$$\dot{r} = \frac{1}{r} < \mathbf{r}\ , \mathbf{v} >, \qquad (3.7)$$

$$\alpha = \frac{\dot{y}x - \dot{x}y}{\mathrm{P}^2}\ , \qquad (3.8)$$

$$\dot{\delta} = \frac{r\dot{z} - \dot{r}z}{r\mathrm{P}}\ . \qquad (3.9)$$

3.3. ACCURACY CHECKES

The accuracy of the basic computations ($F_{l,n}, G_{l,n}$ and $\mathbf{r}_l$ and $\mathbf{v}_l$) could be checked at any time t_l by the following conditions

$$CH1 = \left| F_{l,n} * \dot{G}_{l,n} - G_{l,n} * \dot{F}_{l,n} - 1 \right| = 0\ , \qquad (3.10)$$

$$CH2 = \left| \frac{2}{r_l r_n}(r_l - r_n) - (v_l^2 - v_n^2) \right| = 0\ . \qquad (3.11)$$

$CH1$ is the condition given by Equation (2.10) for Lagrangian coefficients and $CH2$ is the condition for the constancy of energy of Equation (2.5) [

with $\mu = 1$ according to the adopted canonical units of Table A],

$$r_k = (x_k^2 + y_k^2 + z_k^2)^{1/2}; \quad v_k^2 = (\dot{x}_k^2 + \dot{y}_k^2 + \dot{z}_k^2). \tag{3.12}$$

3.4. NUMERICAL APPLICATIONS

3.4.1. APPLICATION 1

Compute the heliocentric motion of Mercury for 100 days using an interval of two days between positions. Assume the following data for arbitrary epoch time 2001 January 11:

- Equinox of coordinates : J2000.0
- k=0.01720209895=$1/TU_*$
- $\mu = 1$
- Mass of Mercury : 0.000000166 M_*
- $t_n = 2451920.5$ (this is Julian day number JD corresponding to 2001 January11)

* $\mathbf{r}_n$ ={ 0.3297222,-0.1854921,-0.1332786 }

* $\mathbf{v}_n$ ={0.01023801,0.02214297,0.01076614 }

These data, are taken from *Astronomical Almanac 2001*.

According to the adopted canonical units, the components of $\mathbf{v}_n$ are to be multiplied by TU_*.

The results of this application are listed in Table I in which, the first five columns are respectively, the data, rectangular coordinates (X,Y,Z) and the heliocentric distance R. While the last two columns of the table are respectively the values of the accuracy checks of Equations. Table I-1, Compares the values of R with the corresponding values of Mercury's radius vector given in *Astronomical Almanac 2001*. The column on the right is the residual found by subtracting the computed and the reference values. It can be seen from the residuals that, for Mercury, the two-body motion agrees fairly well with the actual perturbed orbit.

3.4.2. Application 2

Compute the heliocentric motion of Venus for 300 days using an interval of two days between positions. Assume the following data for the arbitrary epoch of time 2001 January 11:

* Equinox of coordinates : J2000.0

* $k = 0.01720209895 = 1/TU_*$

* $\mu = 1$

* Mass of Venus : 0.000002448M_*

* t_n=2451920.5(this is the Julian day number JD corresponding to 2001 January11)

* $\mathbf{r}_n$ = { 0.3288277, 0.5932406, 0.2460807 } ,

* $\mathbf{v}_n$ ={- 0.01806820, 0.00790963, 0.00470191 }

These data, are taken from *Astronomical Almanac 2001*. Again, according to the adopted canonical units, the components of $\mathbf{v}_n$ are to be multiplied by TU_*.

The results of this application are listed in Table II and are arranged as in Table I . Table II-1, Compares the values of the computed heliocentric distance of Venus with the corresponding values given in *Astronomical Almanac 2001*. The column on the right is the residual found by subtracting the computed and the reference values .

It can be seen from the residuals that, for Venus , the two-body motion agrees to some extent with the actual perturbed orbit

3.4.3. Application 3

For the present application we used three artificial satellites with initial position and velocity vectors $\mathbf{r}_n$ and $\mathbf{v}_n$ at t_n = 13 January 2001 (say) listed in Table B together with the eccentricities and the types of their orbits. To adopte canonical units and other practical units the following transformations rules were used.

*** Constants**

$F_1 = \pi / 180 = 0.017453292$; $F_2 = 1 \frac{DU_{\oplus}}{TU_{\oplus}} = 7.90536828 Km/s$ (see Table A)

$F_3 = 1^{\circ} / solars = 1.0 / 0.0710151110315201 rad / TU_{\oplus}$

$r_{\oplus} = 6378.145 Km$ (see Table A)

*** Input**

$\mathbf{r}_n / r_{\oplus} \rightarrow \mathbf{r}_n$ ($DU_{\oplus}$)

$\mathbf{v}_n / F_2 \rightarrow \mathbf{v}_n$ ($DU_{\oplus} / TU_{\oplus}$)

TABLE A

ASTRODYNAMIC CONSTANTS

Geocentric	Canonical Units	Metric Units
Mean Equatorial Radius, $r_\oplus$	$1\ DU_\oplus$	6378.145Km
Time Unit	$1\ TU_\oplus$	806.8118744 sec
Speed Unit	$1\frac{DU_\oplus}{TU_\oplus}$	$7.90536828\ \frac{Km}{sec}$
Gravitational Parameter, $\mu_\oplus$	$1\frac{DU_\oplus^3}{TU_\oplus^2}$	$3.986012 \times 10^5 \frac{Km^3}{sec^2}$
Heliocentric		
Mean Distance, Earth to Sun	$1\ AU_*$	$1.4959965 \times 10^8 Km$
Time Unit	$1TU_*$	5.0226757×10^6 sec
Speed Unit	$1\frac{AU_*}{TU_*}$	$29.784852\ \frac{Km}{sec}$
Gravitational Parameter, μ_*	$1\frac{AU_*^3}{TU_*^2}$	$1.3271544\ 10^{11} \frac{Km^3}{sec^2}$

TABLE B

Initial position and Velocity Vectors of the Test Orbits

Orbit 1	**Orbit 2**	**Orbit 3**
Elliptic	Hyperbolic	Parabolic
$x = 5096.530625 km$	$x = -10316.00709$	$x = 5.015496663$
$y = 3997.328251$	$y = -6389.956846$	$y = -673.5048965$
$z = -1767.35171$	$z = -4005.124124$	$z = -7154.564650$
$\dot{x}$ =4.6783016085km/s	$\dot{x} = 4.452701327$	$\dot{x} = 7.68083023$
$\dot{y} = 0.602386847$	$\dot{y} = 1.566664537$	$\dot{y} = 3.902921787$
$\dot{z} = 4.217758697$	$\dot{z} = -10.87305394$	$\dot{z} = -6.058643631$
$e = 0.599895261$	$e = 3.493577836$	$e = 1$

TABLE I

Heliocentric Motion Of Mercury For 100 Days Starting from 11 January 2001

Julian Date 245	X	Y	Z	R	CH1	CH2
1920.500	.32972220	-.18549210	-.13327860	.40110751	1.000	.000
1922.500	.34702938	-.13960112	-.11056032	.39005314	1.000	.000
1924.500	.35741481	-.09094096	-.08564500	.37861678	1.000	.000
1926.500	.36000948	-.04031561	-.05887196	.36701237	1.000	.000
1928.500	.35399418	.01125462	-.03070125	.35550121	1.000	.000
1930.500	.33867097	.06250713	-.00173492	.34439538	1.000	.000
1932.500	.31356285	.11192917	.02726843	.33405594	1.000	.000
1934.500	.27853902	.15778451	.05539496	.32488235	1.000	.000
1936.500	.23395120	.19819274	.08160375	.31728961	1.000	.000
1938.500	.18074941	.23127335	.10479176	.31167134	1.000	.000
1940.500	.12053143	.25534814	.12389685	.30835196	1.000	.000
1942.500	.05548175	.26916630	.13802434	.30753771	1.000	.000
1944.500	-.01181544	.27209095	.14656603	.30928093	1.000	.000
1946.500	-.07865527	.26418546	.14927523	.31347042	1.000	.000
1948.500	-.14250791	.24617113	.14627488	.31985164	1.000	.000
1950.500	-.20125125	.21927772	.13800176	.32806900	1.000	.000
1952.500	-.25329477	.18504270	.12511219	.33771601	1.000	.000
1954.500	-.29759546	.14511766	.10838022	.34837977	1.000	.000
1956.500	-.33359792	.10111883	.08861156	.35967291	1.000	.000
1958.500	-.36113793	.05453257	.06658315	.37125156	1.000	.000
1960.500	-.38034036	.00666892	.04300771	.38282232	1.000	.000
1962.500	-.39152895	-.04135052	.01851793	.39414172	1.000	.000
1964.500	-.39515472	-.08858404	-.00633639	.40501178	1.000	.000
1966.500	-.39174329	-.13425083	-.03108366	.41527375	1.000	.000
1968.500	-.38185819	-.17770835	-.05532221	.42480170	1.000	.000
1970.500	-.36607681	-.21842987	-.07871079	.43349651	1.000	.000
1972.500	-.34497550	-.25598405	-.10095924	.44128074	1.000	.000
1974.500	-.31912115	-.29001739	-.12181994	.44809429	1.000	.000
1976.500	-.28906733	-.32023960	-.14108040	.45389095	1.000	.000
1978.500	-.25535336	-.34641164	-.15855701	.45863568	1.000	.000
1980.500	-.21850549	-.36833631	-.17408986	.46230246	1.000	.000
1982.500	-.17903940	-.38585083	-.18753851	.46487274	1.000	.000
1984.500	-.13746360	-.39882129	-.19877870	.46633426	1.000	.000
1986.500	-.09428349	-.40713872	-.20769980	.46668032	1.000	.000
1988.500	-.05000591	-.41071649	-.21420298	.46590937	1.000	.000
1990.500	-.00514405	-.40948918	-.21820006	.46402490	1.000	.000
1992.500	.03977733	-.40341263	-.21961303	.46103565	1.000	.000
1994.500	.08421635	-.39246553	-.21837431	.45695615	1.000	.000
1996.500	.12760827	-.37665239	-.21442775	.45180765	1.000	.000
1998.500	.16935957	-.35600840	-.20773058	.44561939	1.000	.000
2000.500	.20884214	-.33060627	-.19825650	.43843037	1.000	.000
2002.500	.24538779	-.30056570	-.18600015	.43029172	1.000	.000
2004.500	.27828372	-.26606586	-.17098331	.42126970	1.000	.000
2006.500	.30676986	-.22736155	-.15326326	.41144945	1.000	.000
2008.500	.33003956	-.18480371	-.13294380	.40093962	1.000	.000
2010.500	.34724592	-.13886453	-.11018933	.38987790	1.000	.000
2012.500	.35751691	-.09016718	-.08524226	.37843718	1.000	.000
2014.500	.35998387	-.03951876	-.05844365	.36683209	1.000	.000
2016.500	.35382888	.01205685	-.03025559	.35532471	1.000	.000
2018.500	.33835660	.06329313	-.00128247	.34422791	1.000	.000
2020.500	.31309436	.11267354	.02771464	.33390344	1.000	.000

TABLE I1

Comparison Between The Components Values Of Mercury vector with The Corresponding values given in Astronomical Almanac 2001

T	Reference	Computed	Residual
1920.500	.40110750	.40110751	.00000001
1922.500	.39005310	.39005314	.00000004
1924.500	.37861680	.37861678	.00000002
1926.500	.36701230	.36701237	.00000007
1928.500	.35550110	.35550121	.00000011
1930.500	.34439530	.34439538	.00000008
1932.500	.33405580	.33405594	.00000014
1934.500	.32488220	.32488235	.00000015
1936.500	.31728941	.31728961	.00000020
1938.500	.31167110	.31167134	.00000024
1940.500	.30835170	.30835196	.00000026
1942.500	.30753750	.30753771	.00000021
1944.500	.30928080	.30928093	.00000013
1946.500	.31347040	.31347042	.00000002
1948.500	.31985180	.31985164	.00000016
1950.500	.32806930	.32806900	.00000030
1952.500	.33771650	.33771601	.00000049
1954.500	.34838040	.34837977	.00000063
1956.500	.35967370	.35967291	.00000079
1958.500	.37125250	.37125156	.00000094
1960.500	.38282330	.38282232	.00000098
1962.500	.39414280	.39414172	.00000108
1964.500	.40501290	.40501178	.00000112
1966.500	.41527490	.41527375	.00000115
1968.500	.42480280	.42480170	.00000110
1970.500	.43349760	.43349651	.00000109
1972.500	.44128180	.44128074	.00000106
1974.500	.44809530	.44809429	.00000101
1976.500	.45389190	.45389095	.00000095
1978.500	.45863650	.45863568	.00000082
1980.500	.46230320	.46230246	.00000074
1982.500	.46487340	.46487274	.00000066
1984.500	.46633480	.46633426	.00000054
1986.500	.46668080	.46668032	.00000048
1988.500	.46590980	.46590937	.00000043
1990.500	.46402520	.46402490	.00000030
1992.500	.46103590	.46103565	.00000025
1994.500	.45695630	.45695615	.00000015
1996.500	.45180770	.45180765	.00000005
1998.500	.44561940	.44561939	.00000001
2000.500	.43843030	.43843037	.00000007
2002.500	.43029150	.43029172	.00000022
2004.500	.42126940	.42126970	.00000030
2006.500	.41144910	.41144945	.00000035
2008.500	.40093920	.40093962	.00000042
2010.500	.38987740	.38987790	.00000050
2012.500	.37843660	.37843718	.00000058
2014.500	.36683150	.36683209	.00000059
2016.500	.35532410	.35532471	.00000061
2018.500	.34422720	.34422791	.00000071
2020.500	.33390280	.33390344	.00000064

TABLE II

Heliocentric Motion Of Venus For 300 Days Starting From 11 January 2001

Julian Date 245	X	Y	Z	R	CH1	CH2
1920.500	.32882770	.59324060	.24608070	.72153848	1.000	.000
1922.500	.29219218	.60811682	.25509185	.72128648	1.000	.000
1924.500	.25463526	.62107538	.26329858	.72104084	1.000	.000
1926.500	.21627457	.63207341	.27067416	.72080232	1.000	.000
1928.500	.17723051	.64107421	.27719446	.72057169	1.000	.000
1930.500	.13762593	.64804740	.28283804	.72034969	1.000	.000
1932.500	.09758567	.65296903	.28758622	.72013703	1.000	.000
1934.500	.05723623	.65582168	.29142316	.71993439	1.000	.000
1936.500	.01670530	.65659458	.29433594	.71974242	1.000	.000
1938.500	-.02387864	.65528360	.29631456	.71956174	1.000	.000
1940.500	-.06438672	.65189133	.29735204	.71939294	1.000	.000
1942.500	-.10469014	.64642711	.29744442	.71923655	1.000	.000
1944.500	-.14466058	.63890694	.29659079	.71909308	1.000	.000
1946.500	-.18417064	.62935354	.29479329	.71896300	1.000	.000
1948.500	-.22309428	.61779622	.29205715	.71884672	1.000	.000
1950.500	-.26130721	.60427083	.28839062	.71874463	1.000	.000
1952.500	-.29868735	.58881964	.28380499	.71865706	1.000	.000
1954.500	-.33511522	.57149120	.27831454	.71858429	1.000	.000
1956.500	-.37047436	.55234020	.27193652	.71852656	1.000	.000
1958.500	-.40465172	.53142729	.26469105	.71848405	1.000	.000
1960.500	-.43753805	.50881889	.25660109	.71845690	1.000	.000
1962.500	-.46902827	.48458693	.24769236	.71844521	1.000	.000
1964.500	-.49902183	.45880867	.23799325	.71844901	1.000	.000
1966.500	-.52742305	.43156639	.22753472	.71846828	1.000	.000
1968.500	-.55414144	.40294711	.21635020	.71850297	1.000	.000
1970.500	-.57909200	.37304235	.20447548	.71855296	1.000	.000
1972.500	-.60219553	.34194774	.19194857	.71861809	1.000	.000
1974.500	-.62337885	.30976277	.17880960	.71869815	1.000	.000
1976.500	-.64257508	.27659039	.16510065	.71879288	1.000	.000
1978.500	-.65972385	.24253669	.15086563	.71890196	1.000	.000
1980.500	-.67477146	.20771052	.13615011	.71902506	1.000	.000
1982.500	-.68767109	.17222314	.12100118	.71916175	1.000	.000
1984.500	-.69838294	.13618783	.10546729	.71931162	1.000	.000
1986.500	-.70687430	.09971951	.08959806	.71947416	1.000	.000
1988.500	-.71311970	.06293432	.07344415	.71964886	1.000	.000
1990.500	-.71710094	.02594931	.05705704	.71983514	1.000	.000
1992.500	-.71880715	-.01111804	.04048892	.72003242	1.000	.000
1994.500	-.71823479	-.04815017	.02379243	.72024006	1.000	.000
1996.500	-.71538764	-.08502981	.00702059	.72045738	1.000	.000
1998.500	-.71027677	-.12164041	-.00977348	.72068370	1.000	.000
2000.500	-.70292045	-.15786648	-.02653666	.72091829	1.000	.000
2002.500	-.69334412	-.19359398	-.04321604	.72116040	1.000	.000
2004.500	-.68158021	-.22871069	-.05975908	.72140925	1.000	.000
2006.500	-.66766807	-.26310657	-.07611378	.72166407	1.000	.000
2008.500	-.65165378	-.29667407	-.09222884	.72192404	1.000	.000
2010.500	-.63358996	-.32930853	-.10805384	.72218833	1.000	.000
2012.500	-.61353562	-.36090843	-.12353939	.72245611	1.000	.000
2014.500	-.59155589	-.39137577	-.13863726	.72272654	1.000	.000
2016.500	-.56772184	-.42061630	-.15330056	.72299877	1.000	.000
2018.500	-.54211018	-.44853985	-.16748386	.72327193	1.000	.000
2020.500	-.51480302	-.47506059	-.18114337	.72354519	1.000	.000
2022.500	-.48588760	-.50009725	-.19423698	.72381767	1.000	.000

TABLE II
(CONTINUED)
Heliocentric Motion Of Venus For 300 Days Starting From 11 January 2001

Julian Date 245	X	Y	Z	R	CH1	CH2
2024.500	-.45545595	-.52357338	-.20672449	.72408854	1.000	.000
2026.500	-.42360463	-.54541755	-.21856765	.72435696	1.000	.000
2028.500	-.39043439	-.56556355	-.22973029	.72462207	1.000	.000
2030.500	-.35604983	-.58395060	-.24017846	.72488308	1.000	.000
2032.500	-.32055910	-.60052347	-.24988046	.72513917	1.000	.000
2034.500	-.28407350	-.61523263	-.25880696	.72538954	1.000	.000
2036.500	-.24670718	-.62803443	-.26693110	.72563344	1.000	.000
2038.500	-.20857675	-.63889113	-.27422852	.72587011	1.000	.000
2040.500	-.16980095	-.64777103	-.28067746	.72609882	1.000	.000
2042.500	-.13050024	-.65464856	-.28625877	.72631889	1.000	.000
2044.500	-.09079648	-.65950428	-.29095601	.72652962	1.000	.000
2046.500	-.05081253	-.66232498	-.29475545	.72673040	1.000	.000
2048.500	-.01067190	-.66310364	-.29764611	.72692059	1.000	.000
2050.500	.02950162	-.66183947	-.29961982	.72709963	1.000	.000
2052.500	.06958433	-.65853786	-.30067116	.72726696	1.000	.000
2054.500	.10945300	-.65321037	-.30079754	.72742210	1.000	.000
2056.500	.14898518	-.64587468	-.29999917	.72756456	1.000	.000
2058.500	.18805960	-.63655451	-.29827903	.72769391	1.000	.000
2060.500	.22655653	-.62527952	-.29564290	.72780977	1.000	.000
2062.500	.26435806	-.61208528	-.29209931	.72791179	1.000	.000
2064.500	.30134850	-.59701305	-.28765951	.72799966	1.000	.000
2066.500	.33741470	-.58010976	-.28233745	.72807311	1.000	.000
2068.500	.37244635	-.56142778	-.27614970	.72813192	1.000	.000
2070.500	.40633631	-.54102481	-.26911545	.72817592	1.000	.000
2072.500	.43898093	-.51896369	-.26125641	.72820497	1.000	.000
2074.500	.47028032	-.49531223	-.25259676	.72821900	1.000	.000
2076.500	.50013863	-.47014300	-.24316309	.72821794	1.000	.000
2078.500	.52846438	-.44353311	-.23298429	.72820182	1.000	.000
2080.500	.55517064	-.41556401	-.22209151	.72817067	1.000	.000
2082.500	.58017537	-.38632127	-.21051805	.72812460	1.000	.000
2084.500	.60340159	-.35589426	-.19829925	.72806373	1.000	.000
2086.500	.62477763	-.32437601	-.18547243	.72798826	1.000	.000
2088.500	.64423735	-.29186284	-.17207672	.72789840	1.000	.000
2090.500	.66172034	-.25845415	-.15815303	.72779443	1.000	.000
2092.500	.67717208	-.22425210	-.14374387	.72767667	1.000	.000
2094.500	.69054413	-.18936135	-.12889326	.72754546	1.000	.000
2096.500	.70179428	-.15388875	-.11364659	.72740120	1.000	.000
2098.500	.71088668	-.11794301	-.09805050	.72724434	1.000	.000
2100.500	.71779195	-.08163441	-.08215276	.72707533	1.000	.000
2102.500	.72248732	-.04507450	-.06600210	.72689471	1.000	.000
2104.500	.72495667	-.00837573	-.04964808	.72670300	1.000	.000
2106.500	.72519062	.02834886	-.03314099	.72650080	1.000	.000
2108.500	.72318659	.06498594	-.01653162	.72628873	1.000	.000
2110.500	.71894881	.10142228	.00012880	.72606741	1.000	.000
2112.500	.71248834	.13754505	.01678881	.72583755	1.000	.000
2114.500	.70382307	.17324216	.03339686	.72559983	1.000	.000
2116.500	.69297766	.20840263	.04990145	.72535498	1.000	.000
2118.500	.67998354	.24291688	.06625131	.72510376	1.000	.000
2120.500	.66487881	.27667712	.08239552	.72484694	1.000	.000
2122.500	.64770816	.30957765	.09828369	.72458531	1.000	.000
2124.500	.62852278	.34151519	.11386614	.72431968	1.000	.000

TABLE II
(CONTINUED)
Heliocentric Motion Of Venus For 300 Days Starting From 11 January 2001

Julian Date 245	X	Y	Z	R	CH1	CH2
2126.500	.60738018	.37238922	.12909400	.72405088	1.000	.000
2128.500	.58434409	.40210233	.14391940	.72377973	1.000	.000
2130.500	.55948430	.43056047	.15829563	.72350709	1.000	.000
2132.500	.53287641	.45767330	.17217725	.72323379	1.000	.000
2134.500	.50460167	.48335449	.18552028	.72296070	1.000	.000
2136.500	.47474674	.50752199	.19828234	.72268868	1.000	.000
2138.500	.44340344	.53009834	.21042274	.72241857	1.000	.000
2140.500	.41066847	.55101088	.22190270	.72215122	1.000	.000
2142.500	.37664315	.57019206	.23268540	.72188748	1.000	.000
2144.500	.34143308	.58757965	.24273616	.72162819	1.000	.000
2146.500	.30514788	.60311696	.25202256	.72137415	1.000	.000
2148.500	.26790078	.61675307	.26051450	.72112619	1.000	.000
2150.500	.22980833	.62844298	.26818437	.72088509	1.000	.000
2152.500	.19099000	.63814782	.27500712	.72065161	1.000	.000
2154.500	.15156783	.64583497	.28096036	.72042650	1.000	.000
2156.500	.11166600	.65147822	.28602443	.72021049	1.000	.000
2158.500	.07141048	.65505786	.29018249	.72000426	1.000	.000
2160.500	.03092858	.65656076	.29342058	.71980848	1.000	.000
2162.500	-.00965143	.65598046	.29572768	.71962377	1.000	.000
2164.500	-.05020078	.65331720	.29709574	.71945074	1.000	.000
2166.500	-.09059060	.64857795	.29751974	.71928993	1.000	.000
2168.500	-.13069234	.64177638	.29699771	.71914188	1.000	.000
2170.500	-.17037825	.63293285	.29553070	.71900705	1.000	.000
2172.500	-.20952175	.62207439	.29312288	.71888589	1.000	.000
2174.500	-.24799788	.60923454	.28978142	.71877879	1.000	.000
2176.500	-.28568375	.59445335	.28551657	.71868610	1.000	.000
2178.500	-.32245892	.57777718	.28034155	.71860811	1.000	.000
2180.500	-.35820582	.55925859	.27427258	.71854508	1.000	.000
2182.500	-.39281017	.53895616	.26732877	.71849721	1.000	.000
2184.500	-.42616133	.51693431	.25953211	.71846467	1.000	.000
2186.500	-.45815273	.49326306	.25090736	.71844755	1.000	.000
2188.500	-.48868219	.46801784	.24148197	.71844591	1.000	.000
2190.500	-.51765228	.44127919	.23128602	.71845977	1.000	.000
2192.500	-.54497066	.41313251	.22035209	.71848906	1.000	.000
2194.500	-.57055036	.38366777	.20871515	.71853370	1.000	.000
2196.500	-.59431012	.35297921	.19641244	.71859355	1.000	.000
2198.500	-.61617463	.32116498	.18348337	.71866840	1.000	.000
2200.500	-.63607477	.28832687	.16996936	.71875802	1.000	.000
2202.500	-.65394786	.25456992	.15591367	.71886210	1.000	.000
2204.500	-.66973785	.22000203	.14136134	.71898033	1.000	.000
2206.500	-.68339549	.18473368	.12635892	.71911230	1.000	.000
2208.500	-.69487849	.14887748	.11095441	.71925760	1.000	.000
2210.500	-.70415165	.11254781	.09519704	.71941576	1.000	.000
2212.500	-.71118696	.07586043	.07913712	.71958626	1.000	.000
2214.500	-.71596366	.03893212	.06282586	.71976855	1.000	.000
2216.500	-.71846830	.00188025	.04631523	.71996204	1.000	.000
2218.500	-.71869478	-.03517760	.02965773	.72016612	1.000	.000
2220.500	-.71664432	-.07212402	.01290626	.72038012	1.000	.000

TABLE III

Comparison Between The Components Values Of Venus radius vector with The Corresponding Values given in Astronomical Almanac 2001

T	Reference	Computed	Residual
1920.500	.72153850	.72153848	.00000002
1922.500	.72128650	.72128648	.00000002
1924.500	.72104080	.72104084	.00000004
1926.500	.72080230	.72080232	.00000002
1928.500	.72057170	.72057169	.00000001
1930.500	.72034980	.72034969	.00000011
1932.500	.72013720	.72013703	.00000017
1934.500	.71993460	.71993439	.00000021
1936.500	.71974270	.71974242	.00000028
1938.500	.71956210	.71956174	.00000036
1940.500	.71939330	.71939294	.00000036
1942.500	.71923700	.71923655	.00000045
1944.500	.71909360	.71909308	.00000052
1946.500	.71896360	.71896300	.00000060
1948.500	.71884740	.71884672	.00000068
1950.500	.71874540	.71874463	.00000077
1952.500	.71865800	.71865706	.00000094
1954.500	.71858530	.71858429	.00000101
1956.500	.71852760	.71852656	.00000104
1958.500	.71848520	.71848405	.00000115
1960.500	.71845810	.71845690	.00000120
1962.500	.71844640	.71844521	.00000119
1964.500	.71845030	.71844901	.00000129
1966.500	.71846960	.71846828	.00000132
1968.500	.71850440	.71850297	.00000143
1970.500	.71855440	.71855296	.00000144
1972.500	.71861960	.71861809	.00000151
1974.500	.71869970	.71869815	.00000155
1976.500	.71879450	.71879288	.00000162
1978.500	.71890370	.71890196	.00000174
1980.500	.71902690	.71902506	.00000184
1982.500	.71916370	.71916175	.00000195
1984.500	.71931380	.71931162	.00000218
1986.500	.71947650	.71947416	.00000234
1988.500	.71965140	.71964886	.00000254
1990.500	.71983800	.71983514	.00000286
1992.500	.72003550	.72003242	.00000308
1994.500	.72024350	.72024006	.00000344
1996.500	.72046130	.72045738	.00000392
1998.500	.72068000	.72068370	.00000370
2000.500	.72092310	.72091829	.00000481
2002.500	.72116580	.72116040	.00000540
2004.500	.72141530	.72140925	.00000605
2006.500	.72167080	.72166407	.00000673
2008.500	.72193140	.72192404	.00000736
2010.500	.72219650	.72218833	.00000817
2012.500	.72246500	.72245611	.00000889
2014.500	.72273630	.72272654	.00000976
2016.500	.72300940	.72299877	.00001063

TABLE III
(continued)

T	Reference	Computed	Residual
2018.500	.72328340	.72327193	.00001147
2020.500	.72355750	.72354519	.00001231
2022.500	.72383090	.72381767	.00001323
2024.500	.72410260	.72408854	.00001406
2026.500	.72437190	.72435696	.00001494
2028.500	.72463790	.72462207	.00001583
2030.500	.72489980	.72488308	.00001672
2032.500	.72515670	.72513917	.00001753
2034.500	.72540790	.72538954	.00001836
2036.500	.72565260	.72563344	.00001916
2038.500	.72589010	.72587011	.00001999
2040.500	.72611950	.72609882	.00002068
2042.500	.72634030	.72631889	.00002141
2044.500	.72655160	.72652962	.00002198
2046.500	.72675300	.72673040	.00002260
2048.500	.72694370	.72692059	.00002311
2050.500	.72712330	.72709963	.00002367
2052.500	.72729100	.72726696	.00002404
2054.500	.72744650	.72742210	.00002440
2056.500	.72758930	.72756456	.00002474
2058.500	.72771880	.72769391	.00002489
2060.500	.72783490	.72780977	.00002513
2062.500	.72793690	.72791179	.00002511
2064.500	.72802480	.72799966	.00002514
2066.500	.72809820	.72807311	.00002509
2068.500	.72815680	.72813192	.00002488
2070.500	.72820060	.72817592	.00002468
2072.500	.72822930	.72820497	.00002433
2074.500	.72824290	.72821900	.00002390
2076.500	.72824130	.72821794	.00002336
2078.500	.72822460	.72820182	.00002278
2080.500	.72819280	.72817067	.00002213
2082.500	.72814590	.72812460	.00002130
2084.500	.72886842	.72806373	.00080469
2086.500	.72800780	.72798826	.00001954
2088.500	.72791690	.72789840	.00001850
2090.500	.72781180	.72779443	.00001737
2092.500	.72769280	.72767667	.00001613
2094.500	.72756030	.72754546	.00001484
2096.500	.72741470	.72740120	.00001350
2098.500	.72725640	.72724434	.00001206
2100.500	.72708580	.72707533	.00001047
2102.500	.72690360	.72689471	.00000889
2104.500	.72671020	.72670300	.00000720
2106.500	.72650630	.72650080	.00000550
2108.500	.72629250	.72628873	.00000377
2110.500	.72606930	.72606741	.00000189
2112.500	.72583760	.72583755	.00000005
2114.500	.72559790	.72559983	.00000193
2116.500	.72535110	.72535498	.00000388
2118.500	.72509800	.72510376	.00000576

TABLE III1
(continued)

T	Reference	Computed	Residual
2120.500	.72483920	.72484694	.00000774
2122.500	.72457550	.72458531	.00000981
2124.500	.72430790	.72431968	.00001178
2126.500	.72403710	.72405088	.00001378
2128.500	.72376400	.72377973	.00001573
2130.500	.72348940	.72350709	.00001769
2132.500	.72321420	.72323379	.00001959
2134.500	.72293930	.72296070	.00002140
2136.500	.72266540	.72268868	.00002328
2138.500	.72239360	.72241857	.00002497
2140.500	.72212450	.72215122	.00002672
2142.500	.72185920	.72188748	.00002828
2144.500	.72159840	.72162819	.00002979
2146.500	.72134290	.72137415	.00003125
2148.500	.72109360	.72112619	.00003259
2150.500	.72085130	.72088509	.00003379
2152.500	.72061670	.72065161	.00003491
2154.500	.72039060	.72042650	.00003590
2156.500	.72017370	.72021049	.00003679
2158.500	.71996670	.72000426	.00003756
2160.500	.71977040	.71980848	.00003808
2162.500	.71958520	.71962377	.00003857
2164.500	.71941180	.71945074	.00003894
2166.500	.71925080	.71928993	.00003913
2168.500	.71910270	.71914188	.00003918
2170.500	.71896790	.71900705	.00003915
2172.500	.71884690	.71888589	.00003899
2174.500	.71874020	.71877879	.00003859
2176.500	.71864800	.71868610	.00003810
2178.500	.71857060	.71860811	.00003751
2180.500	.71850830	.71854508	.00003678
2182.500	.71846120	.71849721	.00003601
2184.500	.71842970	.71846467	.00003497
2186.500	.71841360	.71844755	.00003395
2188.500	.71841320	.71844591	.00003271
2190.500	.71842830	.71845977	.00003147
2192.500	.71845900	.71848906	.00003006
2194.500	.71850510	.71853370	.00002860
2196.500	.71856650	.71859355	.00002705
2198.500	.71864300	.71866840	.00002540
2200.500	.71873430	.71875802	.00002372
2202.500	.71884010	.71886210	.00002200
2204.500	.71896010	.71898033	.00002023
2206.500	.71909400	.71911230	.00001830
2208.500	.71924110	.71925760	.00001650
2210.500	.71940120	.71941576	.00001456
2212.500	.71957360	.71958626	.00001266
2214.500	.71975790	.71976855	.00001065
2216.500	.71995330	.71996204	.00000874
2218.500	.72015940	.72016612	.00000672
2220.500	.72037540	.72038012	.00000472

- **Out put**

$$r * \; r_{\oplus} \rightarrow r\,(\text{km})$$

$$\delta / F_1 \rightarrow \delta^{\circ}$$

$$\alpha / F_1 \rightarrow \alpha^{\circ}$$

$$\dot{r} * F_2 \rightarrow \dot{r}\ (\text{km/s})$$

$$\dot{\delta} / F_3 \rightarrow \dot{\delta}(^{\circ}/s)$$

$$\dot{\alpha} / F_3 \rightarrow \dot{\alpha}\ (^{\circ}/s)$$

The results of these applications are listed in Tables III, IV and V.

In concluding the present paper, we stress that, efficient and simple algorithm for the two-body initial value problem of space dynamics has been established. Its efficiency is due to some factors, of them are:

- Its recurrent nature which facilitates the computations of large number of the coefficients needed for accurate predictions of **r** and **v** .
- The solution of **r** and **v** is that there is no need to solve Kepler's equation and its variants for parabolic and hyperbolic orbits.
- The algorithm is universal in the sense it could be applied for any conic orbits.
- The algorithm could also be used whatever the time difference Δt between the initial and final points. By repeating the process of decrementing the time interval several times as mentioned after Equation (2.11)

TABLE III

Geocentric Distance, Right Ascension, Declination and Their Time Rates For Satellite No.1 During 100 days Starting From 13 January 2001

Julian Date 245	$R(Km)$	α°	δ°	$\dot{r}(Km/s)$	$\dot{\alpha}(\circ/s)$	$\dot{\delta}(\circ/s)$
1922.5	7021.589	35.7103	-10.2276	2.3276	-.0188	.0403
1924.5	7273.225	33.5643	-5.5644	1.8689	-.0171	.0376
1926.5	7470.689	31.5787	-1.1741	1.4242	-.0161	.0357
1928.5	7615.444	29.6883	3.0206	.9899	-.0155	.0343
1930.5	7708.556	27.8423	7.0828	.5629	-.0153	.0335
1932.5	7750.708	25.9967	11.0670	.1400	-.0155	.0330
1934.5	7742.206	24.1085	15.0226	-.2818	-.0160	.0330
1936.5	7682.989	22.1315	18.9965	-.7057	-.0170	.0333
1938.5	7572.625	20.0102	23.0365	-1.1348	-.0185	.0341
1940.5	7410.304	17.6723	27.1932	-1.5722	-.0206	.0353
1942.5	7194.826	15.0162	31.5226	-2.0213	-.0238	.0370
1944.5	6924.597	11.8880	36.0881	-2.4853	-.0286	.0392
1946.5	6597.632	8.0377	40.9614	-2.9673	-.0361	.0421
1948.5	6211.627	3.0249	46.2162	-3.4695	-.0485	.0456
1950.5	5764.166	356.0003	51.8985	-3.9910	-.0709	.0491
1952.5	5253.317	345.1596	57.9048	-4.5234	-.1151	.0502
1954.5	4679.227	326.5197	63.4900	-5.0370	-.2055	.0391
1956.5	4048.514	294.4012	65.6118	-5.4436	-.3209	-.0141
1958.5	3386.793	256.7341	57.8145	-5.4855	-.2755	-.1213
1960.5	2773.444	230.1051	35.9418	-4.4655	-.1778	-.2447
1962.5	2404.467	210.9581	-.2038	-1.2824	-.1551	-.3444
1964.5	2510.684	188.5972	-40.3014	2.9029	-.2446	-.2919
1966.5	3014.147	143.5008	-64.0222	5.0935	-.5144	-.0837
1968.5	3663.362	89.4715	-62.1394	5.5426	-.3059	.0777
1970.5	4317.505	65.7023	-51.6212	5.2975	-.1248	.0879
1972.5	4926.175	54.8091	-41.8184	4.8287	-.0665	.0751
1974.5	5474.165	48.4163	-33.5371	4.3012	-.0431	.0634
1976.5	5958.417	44.0131	-26.4799	3.7714	-.0315	.0547
1978.5	6379.962	40.6527	-20.3266	3.2576	-.0251	.0482
1980.5	6741.047	37.9009	-14.8367	2.7639	-.0211	.0435
1982.5	7044.083	35.5281	-9.8366	2.2897	-.0186	.0400
1984.5	7291.256	33.3981	-5.1989	1.8324	-.0170	.0374
1986.5	7484.392	31.4224	-.8271	1.3886	-.0160	.0355
1988.5	7624.919	29.5373	3.3546	.9550	-.0155	.0342
1990.5	7713.873	27.6930	7.4085	.5285	-.0153	.0334
1992.5	7751.905	25.8455	11.3886	.1058	-.0155	.0330
1994.5	7739.293	23.9520	15.3439	-.3161	-.0161	.0330
1996.5	7675.945	21.9655	19.3214	-.7403	-.0171	.0334
1998.5	7561.397	19.8296	23.3689	-1.1700	-.0186	.0342
2000.5	7394.810	17.4702	27.5375	-1.6082	-.0209	.0354
2002.5	7174.951	14.7824	31.8836	-2.0584	-.0242	.0371
2004.5	6900.194	11.6067	36.4713	-2.5237	-.0291	.0394
2006.5	6568.525	7.6820	41.3729	-3.0073	-.0369	.0424
2008.5	6177.618	2.5455	46.6615	-3.5112	-.0499	.0458
2010.5	5725.069	355.2974	52.3777	-4.0341	-.0734	.0493
2012.5	5209.033	344.0131	58.3931	-4.5665	-.1203	.0500
2014.5	4629.959	324.4687	63.8606	-5.0759	-.2155	.0369
2016.5	3995.363	291.2528	65.4403	-5.4661	-.3251	-.0212
2018.5	3333.481	254.0955	56.5858	-5.4564	-.2661	-.1310
2020.5	2730.732	228.3965	33.5080	-4.2992	-.1730	-.2549

TABLE IV

Geocentric Distance, Right Ascension, Declination and Their Time Rates For Satellite No.2 During 100 days Starting From 13 January 2001

Julian Date 245	R(Km)	α°	δ°	$\dot{r}(Km/s)$	$\dot{\alpha}(\circ/s)$	$\dot{\delta}(\circ/s)$
1922.5	12723.693	212.3768	-24.6352	.0575	.0053	-.0532
1924.5	12792.321	213.0429	-30.9952	1.0833	.0059	-.0526
1926.5	12982.338	213.7880	-37.2243	2.0756	.0066	-.0511
1928.5	13288.047	214.6318	-43.2151	3.0076	.0075	-.0487
1930.5	13700.969	215.6000	-48.8839	3.8600	.0087	-.0457
1932.5	14210.850	216.7273	-54.1746	4.6225	.0102	-.0424
1934.5	14806.667	218.0615	-59.0573	5.2924	.0121	-.0390
1936.5	15477.453	219.6699	-63.5240	5.8729	.0148	-.0355
1938.5	16212.873	221.6505	-67.5815	6.3710	.0184	-.0321
1940.5	17003.555	224.1507	-71.2452	6.7955	.0236	-.0289
1942.5	17841.232	227.4007	-74.5331	7.1558	.0311	-.0259
1944.5	18718.751	231.7738	-77.4585	7.4610	.0426	-.0229
1946.5	19630.001	237.8973	-80.0224	7.7193	.0609	-.0198
1948.5	20569.809	246.8312	-82.1988	7.9381	.0903	-.0164
1950.5	21533.814	260.2005	-83.9106	8.1236	.1350	-.0120
1952.5	22518.356	279.4297	-85.0060	8.2812	.1833	-.0060
1954.5	23520.359	302.5437	-85.3262	8.4153	.1917	.0006
1956.5	24537.241	323.3112	-84.9257	8.5297	.1495	.0056
1958.5	25566.833	338.2460	-84.0804	8.6276	.1013	.0081
1960.5	26607.304	348.2349	-83.0391	8.7115	.0677	.0091
1962.5	27657.108	355.0073	-81.9386	8.7835	.0468	.0092
1964.5	28714.938	359.7807	-80.8432	8.8455	.0337	.0090
1966.5	29779.680	3.2832	-79.7815	8.8989	.0252	.0087
1968.5	30850.388	5.9455	-78.7659	8.9451	.0195	.0083
1970.5	31926.251	8.0300	-77.8006	8.9850	.0155	.0078
1972.5	33006.575	9.7028	-76.8863	9.0196	.0126	.0074
1974.5	34090.764	11.0732	-76.0214	9.0495	.0104	.0070
1976.5	35178.304	12.2155	-75.2037	9.0755	.0087	.0066
1978.5	36268.748	13.1818	-74.4305	9.0980	.0074	.0063
1980.5	37361.711	14.0096	-73.6990	9.1176	.0064	.0059
1982.5	38456.859	14.7266	-73.0064	9.1345	.0056	.0056
1984.5	39553.898	15.3535	-72.3501	9.1491	.0049	.0053
1986.5	40652.575	15.9063	-71.7275	9.1618	.0043	.0051
1988.5	41752.665	16.3975	-71.1363	9.1727	.0039	.0048
1990.5	42853.974	16.8367	-70.5744	9.1822	.0035	.0046
1992.5	43956.328	17.2318	-70.0397	9.1902	.0031	.0043
1994.5	45059.578	17.5892	-69.5304	9.1971	.0028	.0041
1996.5	46163.588	17.9140	-69.0448	9.2029	.0026	.0040
1998.5	47268.243	18.2105	-68.5814	9.2078	.0024	.0038
2000.5	48373.436	18.4823	-68.1387	9.2119	.0022	.0036
2002.5	49479.077	18.7324	-67.7153	9.2153	.0020	.0035
2004.5	50585.083	18.9632	-67.3101	9.2180	.0018	.0033
2006.5	51691.382	19.1769	-66.9220	9.2202	.0017	.0032
2008.5	52797.910	19.3754	-66.5498	9.2219	.0016	.0030
2010.5	53904.609	19.5603	-66.1927	9.2231	.0015	.0029
2012.5	55011.428	19.7328	-65.8498	9.2239	.0014	.0028
2014.5	56118.321	19.8943	-65.5202	9.2243	.0013	.0027
2016.5	57225.250	20.0457	-65.2033	9.2244	.0012	.0026
2018.5	58332.177	20.1880	-64.8982	9.2243	.0011	.0025
2020.5	59439.072	20.3219	-64.6043	9.2239	.0011	.0024

TABLE V

Geocentric Distance, Right Ascension, Declination and Their Time Rates For Satellite No.3 During 100 days Starting From 13 January 2001

Julian Date 245	R(Km)	α°	δ°	$\dot{r}(Km/s)$	$\dot{\alpha}(\circ/s)$	$\dot{\delta}(\circ/s)$
1922.5	7886.531	347.7083	-83.1085	5.9736	.3322	.0432
1924.5	8614.552	8.4740	-77.5714	6.1423	.0866	.0456
1926.5	9357.226	15.2929	-72.4157	6.2240	.0372	.0402
1928.5	10106.083	18.6152	-67.9157	6.2495	.0206	.0349
1930.5	10855.650	20.5884	-63.9999	6.2385	.0131	.0305
1932.5	11602.400	21.9037	-60.5725	6.2043	.0092	.0268
1934.5	12344.084	22.8488	-57.5497	6.1552	.0068	.0237
1936.5	13079.285	23.5646	-54.8632	6.0970	.0053	.0211
1938.5	13807.142	24.1281	-52.4582	6.0333	.0042	.0190
1940.5	14527.165	24.5852	-50.2909	5.9667	.0035	.0172
1942.5	15239.110	24.9647	-48.3258	5.8989	.0029	.0156
1944.5	15942.899	25.2859	-46.5341	5.8310	.0025	.0143
1946.5	16638.567	25.5620	-44.8925	5.7637	.0021	.0131
1948.5	17326.221	25.8025	-43.3813	5.6975	.0019	.0121
1950.5	18006.020	26.0143	-41.9845	5.6328	.0017	.0112
1952.5	18678.151	26.2026	-40.6884	5.5697	.0015	.0104
1954.5	19342.818	26.3714	-39.4816	5.5084	.0013	.0097
1956.5	20000.239	26.5239	-38.3544	5.4489	.0012	.0091
1958.5	20650.632	26.6624	-37.2984	5.3913	.0011	.0085
1960.5	21294.216	26.7891	-36.3064	5.3354	.0010	.0080
1962.5	21931.208	26.9054	-35.3722	5.2814	.0009	.0076
1964.5	22561.819	27.0127	-34.4904	5.2291	.0009	.0071
1966.5	23186.254	27.1122	-33.6563	5.1784	.0008	.0068
1968.5	23804.710	27.2047	-32.8656	5.1294	.0007	.0064
1970.5	24417.379	27.2910	-32.1147	5.0820	.0007	.0061
1972.5	25024.444	27.3719	-31.4004	5.0360	.0007	.0058
1974.5	25626.080	27.4477	-30.7197	4.9915	.0006	.0055
1976.5	26222.456	27.5191	-30.0701	4.9483	.0006	.0053
1978.5	26813.733	27.5865	-29.4492	4.9065	.0005	.0051
1980.5	27400.064	27.6502	-28.8549	4.8659	.0005	.0048
1982.5	27981.596	27.7106	-28.2855	4.8265	.0005	.0046
1984.5	28558.471	27.7679	-27.7391	4.7883	.0005	.0045
1986.5	29130.822	27.8224	-27.2142	4.7511	.0004	.0043
1988.5	29698.779	27.8744	-26.7095	4.7150	.0004	.0041
1990.5	30262.463	27.9240	-26.2236	4.6799	.0004	.0040
1992.5	30821.994	27.9713	-25.7554	4.6458	.0004	.0038
1994.5	31377.482	28.0167	-25.3039	4.6125	.0004	.0037
1996.5	31929.036	28.0601	-24.8679	4.5802	.0004	.0036
1998.5	32476.759	28.1018	-24.4468	4.5487	.0003	.0035
2000.5	33020.750	28.1418	-24.0395	4.5180	.0003	.0033
2002.5	33561.103	28.1803	-23.6454	4.4880	.0003	.0032
2004.5	34097.909	28.2174	-23.2637	4.4589	.0003	.0031
2006.5	34631.256	28.2531	-22.8938	4.4304	.0003	.0030
2008.5	35161.227	28.2876	-22.5351	4.4026	.0003	.0029
2010.5	35687.903	28.3209	-22.1870	4.3754	.0003	.0029
2012.5	36211.360	28.3530	-21.8490	4.3489	.0003	.0028
2014.5	36731.674	28.3841	-21.5206	4.3230	.0003	.0027
2016.5	37248.914	28.4142	-21.2013	4.2977	.0002	.0026
2018.5	37763.152	28.4433	-20.8908	4.2730	.0002	.0026
2020.5	38274.451	28.4716	-20.5886	4.2488	.0002	.0025

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طريقة حسابية تعاودية للمسألة الاستهلالية لديناميكا الفضاء

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الملخص العربى

تم في هذا البحث تكوين طريقة حسابية فعالة وبسيطة للمسألة الاستهلالية لديناميكا الفضاء وذلك باستخدام حسابات تعاودية لمعاملات لاجرانج لمصفوفة الانتقال . اشتمل البحث أيضاً على تطبيقات عددية للطريقة على الحركة الكوكبية وكذلك على حركة الأقمار الصناعية وقد أشارت النتائج فعالية الطريقة في الحصول على أية دقة مطلوبة لحل المسألة الاستهلالية لديناميكا الفضاء ولأي نوع من أنواع المدارات .