

**EFFECT OF POROSITY ON THE NATURAL CONVECTION
HEAT TRANSFER IN AN INCLINED ANNULAR
POROUS MEDIUM**

تأثير المسامية على انتقال الحرارة بالحمل الطبيعي في وسط مسامي حلقي مائل

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جذب موضوع انتقال الحرارة في الاوساط المسامية انتباه الباحثين في الآونة الأخيرة نظرا لتزايد التطبيقات الهندسية والتكنولوجية في هذا المجال مثل عمليات استخلاص البترول والصناعات الكيماوية وعمليات العزل الحراري وزيادة الطاقة الحرارية في المجمعات الشمسية، التخلص من النفايات النووية وفي تبريد وعزل المفاعلات النووية وفي تبريد الكابلات الكهربائية.

يقدم هذا البحث دراسة تأثير المسامية على انتقال الحرارة بالحمل الحر خلال وسط مسامي حلقي مثلث ذو نسبة بعديّة 41 ونسبة القطر الخارجي الي القطر الداخلي 2,143 تكون الوسط من اسطوانتين مركزيتين يمثل الفراغ المحصور بينهما وسطا حلقيًا باقطار 45/21 مم وطول 492 مم. ملئ الوسط الحلقي بكرات من الصلب وعرض لفيض حراري ثابت عند السطح الداخلي بينما يحفظ السطح الخارجي عند درجة حرارة ثابتة. تغيير الفيض الحراري على السطح الداخلي من 4 إلى 324 وات حيث يتغير رقم رايلي من 10 حتى 550. الهدف الرئيسي هو ايجاد تأثير درجة مسامية الوسط على انتقال الحرارة ممثلا في درجات الحرارة ورقم نوسيلت، حيث تغير أقطار كرات الصلب المستخدمة لتكون 3,11 و 3,97 و 6,35 مم ذات نفاذية 0,42 و 0,43 و 0,45. أظهرت النتائج ان تغير درجة مسامية الوسط الحلقي له تأثيرا كبيرا على خواص انتقال الحرارة. فاستخدام اوساط مسامية ينقص من درجتى الحرارة المتوسطة والقصى لسطح الفيض الحراري مما يسمح بإمكانية زيادة الفيض الحراري. وبرغم ان زيادة درجة المسامية تنقص من معامل انتقال الحرارة بالتوصيل الا انها تزيد من معامل انتقال الحرارة بالحمل مما يزيد من معامل انتقال الحرارة الكلي كما تزيد من رقم نوسيلت المتوسط. عند رقم رايلي يساوي 40 يبدأ انتقال الحرارة بالحمل الطبيعي، وبزيادة رقم رايلي يزداد كل من رقم نوسيلت المتوسط ومعامل انتقال الحرارة بالحمل. كما تم الحصول على علاقة رياضية تربط رقم نوسيلت المتوسط \bar{Nu} مع كل من زاوية الميل θ ورقم رايلي Ra ورقم دارسي Da .

ABSTRACT

Experimental study was carried out for the steady laminar natural convection of air in an inclined annulus porous medium, with constant heat flux heated inner cylinder and an isothermally cooled outer cylinder. The primary objective of the study is to show the effect of porosity of the annulus on the local and maximum temperatures of the heated inner cylinder and the local and mean Nusselt number. The geometrical parameters of the annulus are: diameters of 21/45mm, gap width=12mm, length of 492 mm, and an aspect ratio (length/gap width) = 41. The operating parameters are: heat flux $4 \leq Q \leq 324$ W, Rayleigh number $10 \leq Ra \leq 550$ and inclination angle $\theta = 0, 30, 60$ and 90° . The spherical bead diameter are varied from $d = 3.11$ to 3.97 to 6.35 mm with porosity of 0.42, 0.43 and 0.45 respectively. The results show great influence of the porosity on the local and maximum surface temperatures as well as the local and mean Nusselt number. The average Nusselt number is correlated with the inclination angle and the Rayleigh and Darcy numbers.

INTRODUCTION

Natural convection heat transfer in porous enclosures commonly occurs in nature, and engineering and technological applications. This phenomenon plays an important role in such diverse applications including thermal insulators, storage of solar energy in underground containers, underground cable systems, heat exchangers, food industry, biomedical applications and heat transfer from nuclear fuel rod bundle in nuclear reactors and interior of its canister in either storage or disposal.

There are large number of publications in the literature dealing with natural convection in horizontal and vertical porous annulus. A complete review for the previous researches for the porosity and inclination angle effects on the heat transfer characteristics in both the horizontal and vertical porous annulus is presented in the work of El Kady et al. [1].

Natural convection in an inclined porous annulus were studied numerically by Fukuda et al. [2], experimentally by Takata et al [3] and analytically with experimental proof by Takata et al [4]. They investigated the 3-D flow configuration with temperature distribution and heat transfer analysis for porosity $\epsilon=0.39$, $Da = 2.51 \times 10^{-6}$, $A=7.2$, $R=2$, and $10 \leq RaDa \leq 300$. The effect of the inclination angle on the natural convection of an inclined annular porous medium was studied experimental by El Kady et al [1]. The annulus was filled with spherical beads of $d = 6.35$ mm with porosity of 0.45. The operating parameters are: $4 \leq Q \leq 324$ W, $10 \leq Ra \leq 550$, $Da = 4.583 \times 10^{-4}$ and inclination angle $0 \leq \theta \leq 90^\circ$.

Natural convective heat transfer in an inclined porous annulus has received very little attention. In the available literature, only one experimental study has been carried out by Takata et al [3] for limited parameters. The present study is a step to overcome the lack of the available experimental data in this field. An experimental investigation has been carried out for the free convection in an inclined porous annulus. The main goal is to show the effect of porosity on the heat transfer characteristics. Three diameters of stainless steel beads were used with $d = 3.11$, 3.97 and 6.35 mm, porosity $\epsilon = 0.42$, 0.43 and 0.45 , permeability $K = 1.2 \times 10^{-8}$, 2.1×10^{-8} , and 6.6×10^{-8} , and Darcy number $Da = 8.33 \times 10^{-5}$, 1.456×10^{-5} , and 4.583×10^{-4} .

EXPERIMENTAL APPARATUS

Schematic layout of the experimental test apparatus is shown in Fig. 1. An annular section (1) which is filled with stainless steel spherical beads (2) is portable by means of a pivot (9). Three cases of different bead diameter $d = 3.11$, 3.97 and 6.35 mm are studied. The change of inclination angle is achieved by a protractor (10). The inner cylinder (3) is made from copper with length of 540mm and diameters of 13/21 mm, and is heated by an electric heater (4). The outer cylinder (5) which is made of copper with length of 540 and diameters of 45/50 mm is kept approximately at constant temperature by circulating cooling water through a PVC cylinder (6) with diameters 75/70 mm. City water is forced from the bottom tap (7), and leaves at the upper tap (8). The water temperatures at the inlet and the outlet of the shell were measured by means of two copper-constantan thermocouples (11). The electric heater (4) has a diameter of

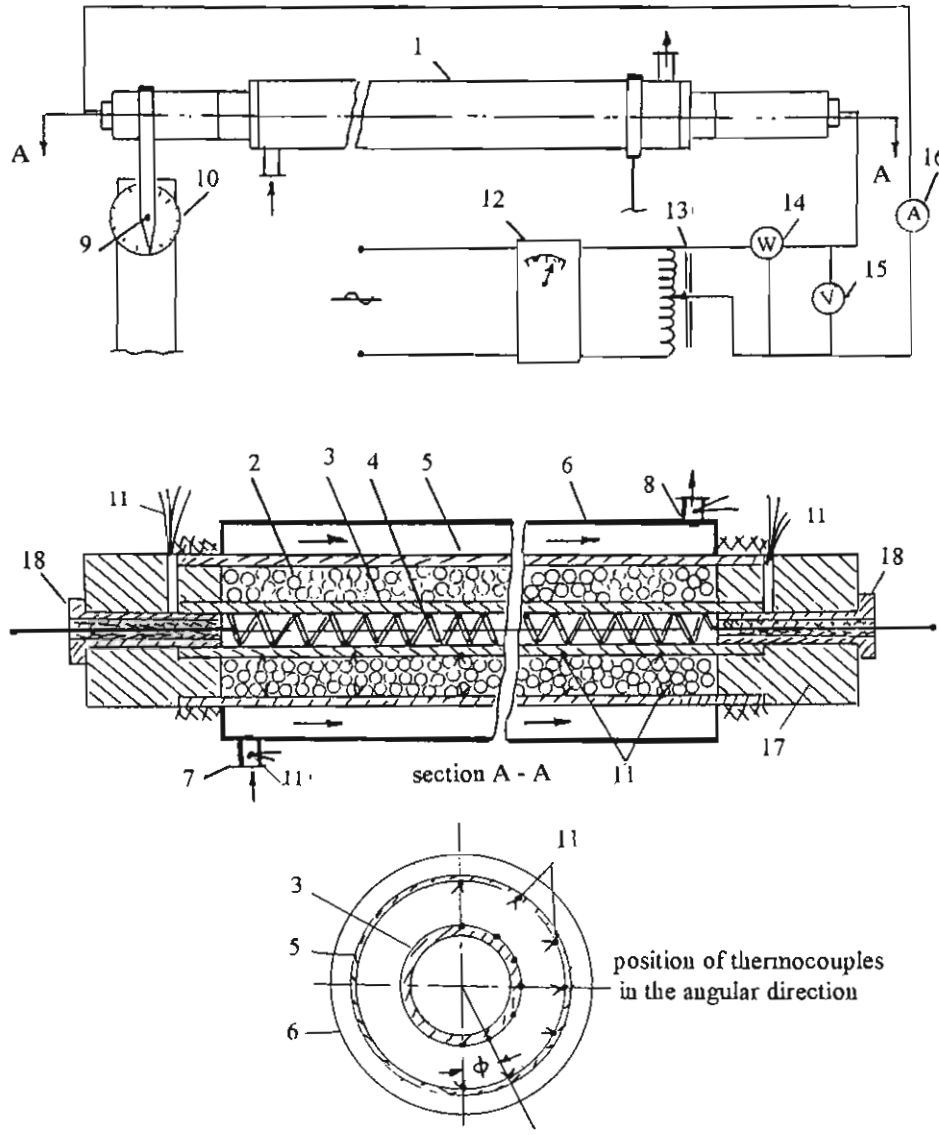


Fig. 1 Schematic layout of the experimental test apparatus
 (1) annular section, (2) stainless steel spherical beads, (3) inner cylinder, (4) electric heater. (5) outer cylinder, (6) PVC cylinder, (7) bottom tap, (8) upper tap, (9) pivot, (10) protractor, (11) copper-constantan thermocouples, (12) voltage regulator, (13) variable voltage auto-transformer, (14) Wattmeter, (15) voltmeter, (16) ammeter, (17) thermal insulation, (18) teflon pins.

0.5 mm and its effective length is 492 mm. The electric heater is covered by electric insulation from porcelain which gives constant heat flux along its effective length. The total electric resistance of the heater equals 100 ohm. The consumed power in the heater is adjusted by a variable voltage auto-transformer (13), voltage regulator (12), and measured by voltmeter (15), ammeter (16) and Wattmeter (14). In order to avoid the heat loss in the longitudinal direction of the annulus, thermal insulation (17) is used at the external two ends of the annulus. The inner cylinder is fixed concentrically inside the outer one by means of teflon pins (18).

The temperatures of the inner and outer surfaces of the annulus were measured by copper-constantan thermocouples (19) of 0.3 mm diameter.

The characteristic parameters in the present study are the local Nusselt number Nu , the mean Nusselt number \bar{Nu} , the Rayleigh number Ra and the Darcy number Da . These parameters are calculated as follows:

$$Nu = q \delta / [k_m (\bar{t}_i - \bar{t}_o)], \quad \bar{Nu} = q \delta / [k_m (\bar{t}_i - \bar{t}_o)]$$

$$Ra = g\beta\delta^3 (\bar{t}_i - \bar{t}_o) (\rho c)_f / \nu k_m, \quad \text{and} \quad Da = K / \delta^2$$

where \bar{t}_i and \bar{t}_o are the mean temperatures of the inner and outer cylinders respectively, δ is the characteristic length = $(r_o - r_i)$, q is the heat flux per unit area, Q is the input power, K is the permeability of the porous medium and equals to $\{d^2 \epsilon^3 / 180(1 - \epsilon)^2\}$ as mentioned by El Kady [5], d is the bead diameter, ϵ is the porosity, and h is the overall coefficient of heat transfer at the inner cylinder = $q / (\bar{t}_i - \bar{t}_o)$.

β , ρ , c , and ν are the coefficient of thermal expansion, density, specific heat, and the kinematic viscosity of the air at its mean temperature, respectively.

The effective thermal conductivity k_m is determined by applying a low electric power of about 1-4W to the electric heater, where the heat transfer process is contributed only by conduction.

RESULTS AND DISCUSSION

The effect of porosity on the heat transfer characteristics for the inclined porous annulus was examined. The boundary conditions are constant heat flux on the inner cylinder wall and constant temperature at the outer one. The annulus has a radius ratio $R=2.134$ and an aspect ratio $A=41$. Input heat flux at the inner cylinder wall was changed from 0.77 to 11.16 kW/m² with $10 \leq Ra \leq 550$. The porous media is saturated with air which has $Pr=0.7$. Different inclination angles $\theta = 0, 30, 60,$ and 90 were considered. Three diameters of stainless steel beads were used with $d = 3.11, 3.97$ and 6.35 mm, porosity $\epsilon = 0.42, 0.43$ and 0.45 , permeability $K = 1.2 \times 10^{-8}, 2.1 \times 10^{-8}$, and 6.6×10^{-8} , and Darcy number $Da = 8.33 \times 10^{-5}, 1.456 \times 10^{-5}$, and 4.583×10^{-4} .

Temperature distribution

The effect of porous medium existence is shown in Fig. 2 on the temperature distribution of the inner cylinder wall along the longitudinal direction for porous and non-porous vertical annulus and $Q=49W$. The existence of the porous media causes a

great reduction of the inner cylinder wall temperature. t_i decreases from 145 to 46 by a reduction of about 66% by using porous medium of $\epsilon = 0.45$ and $Q=49W$. The existence of the porous media gives the chance to increase the heat power generated from the inner cylinder without raising its temperatures. The temperature of the inner cylinder by using a porous medium of $\epsilon = 0.45$ with $Q= 225W$ still lower than the corresponding values by using non porous medium with $Q=49W$.

Figure 3 presents the temperature distribution on the inner cylinder wall along the longitudinal direction, while Fig. 4 shows the temperature distribution on the inner wall along the angular direction at $X/L=0.5$ for different inclination angles $\theta = 0, 30, 60,$ and 90° , an input power $Q=324W$ and $\epsilon=0.42, 0.43,$ and 0.45 . The inner cylinder wall temperature t_i decreases as the porosity increases. This is because, with the increase of the porosity ϵ , the resistance to the buoyancy force decreases, which enables the fluid to move faster carrying more heat from the inner wall to the outer one leading to lower inner wall temperature.

Figure 3 presents that for non-horizontal annulus, as the inclination angle increases, a drop in the temperature along the inner wall with negative temperature gradient in the midsection is observed and is followed by a positive temperature gradient which can be explained by the existence of two cells of flow inside the annulus as discussed by El Kady et al. [1].

Figure 4 shows that for non-vertical annulus and with the decrease of inclination angle θ , t_i in the upper half increases sharply and decreases again from $\phi=120^\circ$ to 180° making a peak at $\phi=150^\circ$ indicating that the separation may occur near $\phi=150^\circ$ as discussed by El Kady et al. [1]. The rate of the increase of the peak temperature increases with the increase of porosity. Finally at the horizontal position the maximum peak temperature occurs by the case of higher porosity $\epsilon = 0.45$.

Figure 5 shows the variation of inner cylinder wall mean temperature versus input power at the vertical position and different porosities. Two regions are recognized and aligned by $Q=230W$. In the lower region which characterise with low heat energy where $Q<230W$, with the increase of porosity t_i increases. In the upper region which characterise by higher heat energy where $Q>230W$, with the increase of porosity t_i decreases

The effect of the heat power Q on the maximum inner wall temperature t_{max} is presented in Fig. 6 for the three different porosities. At low input power $Q < 280W$, t_{max} is higher for higher porosity while t_{max} is lower for higher porosities at high input power $Q>280W$.

The behavior of both \bar{t}_i and t_{max} with the increase of porosity in Figs 5 and 6 is mainly due to the effect of the convective part of heat transfer. For the low heat energy region the air flow has relatively low velocity, this increases the conduction part of heat transfer and decreases the convection part and the flow is conduction dominated heat transfer. With the increase of porosity the conduction heat transfer decreases, therefore, \bar{t}_i and t_{max} increases. With further increase of the heat energy Q the air flow velocity increases, the convection part of heat transfer increases and the conduction part decreases. In the convection dominated heat transfer flow, the increase of porosity increases the convection part of heat transfer, Therefore, \bar{t}_i and t_{max} decreases accordingly.

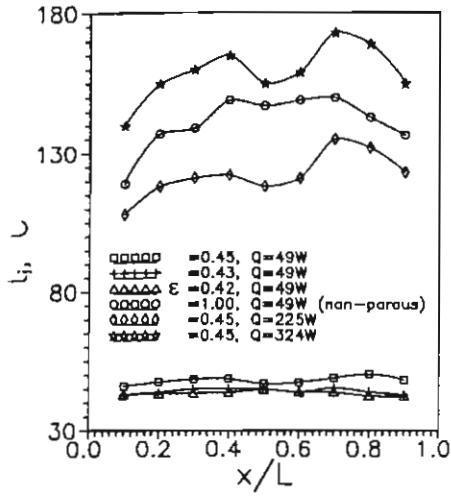


Fig. 2 temperature distribution on the inner cylinder wall of the annulus in the longitudinal direction for $\theta = 90^\circ$

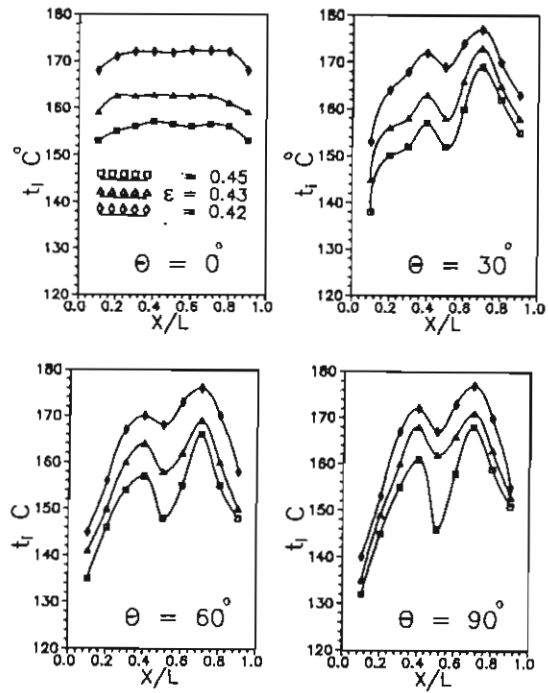


Fig. 3 temperature distribution on the inner cylinder wall of the annulus in the longitudinal direction for $Q=324W$ and $\epsilon = 0.42, 0.43$ and 0.45

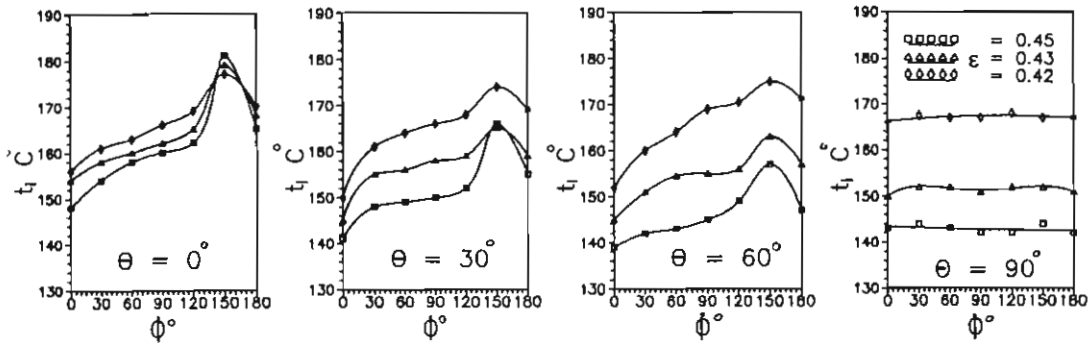


Fig. 4 temperature distribution on the inner cylinder wall along the angular direction for $Q=324W$ and $\epsilon = 0.42, 0.43$ and 0.45

Heat transfer

The effect of porosity on heat transfer coefficient, local and mean Nusselt number, are presented to express the heat transfer process. The variation of heat transfer coefficient with input power Q is performed in Fig. 7 at the vertical position for the three used porosities $\varepsilon = 0.42, 0.43, \text{ and } 0.45$ and the non porous case where $\varepsilon=1$. The heat transfer coefficient h is higher for porous annulus than that for non-porous annulus by nearly 15 times. At very low input power, $Q=4W$, heat transfer process takes place by conduction only. With the increase of porosity, the conduction heat transfer coefficient decreases. With further increase of the input power, h increases due to the increase of the convective part of heat transfer. As Q increases from 80 to 324W, h increases from 67 to 85 W/m^2K for $\varepsilon=0.45$, from 70 to 80 W/m^2K for $\varepsilon=0.43$ and from 72 to 79 W/m^2K for $\varepsilon=0.42$.

Figure 8 shows the effect of heat flux on the convective part of heat transfer coefficient h_c for the vertical annulus at different porosities. h_c increases as the input heat flux increases. The rate of increase of h_c is higher for higher porosity than the lower porosity. As heat flux increases from 3 to 10 kW/m^2 , h_c increases from 8 W/m^2K for all cases to 14 W/m^2K for $\varepsilon=0.42$, 18 W/m^2K for $\varepsilon=0.43$, and to 24 W/m^2K for $\varepsilon=0.45$. Due to the increasing rate of h_c for higher porosity than the lower porosity, two regions are recognized in Fig. 7 and aligned by $Q=230W$. In the lower region which characterise with low heat energy where $Q<230W$, h decreases with the increase of porosity while in the upper region which characterise by higher heat energy where $Q>230W$, h increases with the increase of porosity.

Local Nusselt number Nu distribution on the inner cylinder wall along the longitudinal direction is presented in Fig 9 for $Q=225W$, the three porosities $\varepsilon = 0.42, 0.43 \text{ and } 0.45$ and at different inclination angles $\theta = 0, 30, 60 \text{ and } 90^\circ$. With the increase of the porosity ε , the resistance to the buoyancy force decreases, which enables the fluid to move faster carrying more heat from the inner wall to the outer one and increases Nu . For non-horizontal annulus, as the inclination angle increases, an increase in the local Nusselt number along the inner wall with positive gradient in the midsection is observed and is followed by a negative gradient which can be explained by the existence of two cells of flow inside the annulus as discussed by El Kady et al. [1].

The distribution of Local Nusselt number distribution on the inner wall along the angular direction is presented in Fig. 10. Nu is nearly constant for vertical annulus. For the non-vertical annulus and with the decrease of inclination angle θ , Nu in the upper half decreases sharply and increases again from $\phi=120^\circ$ to 180° making a bottom value at $\phi=150^\circ$ indicating that the separation may occur near $\phi=150^\circ$ as discussed by El Kady et al. [1]. The rate of the decrease of the bottom value of Nu_j increases with the increase of porosity.

The effect of porosity on the heat transfer rates in nondimensional forms is shown in Fig.11 which presents the relation between Nu and Ra for different inclination angles $\theta = 0, 30, 60 \text{ and } 90^\circ$. \bar{Nu} takes nearly constant value for $Ra \leq 40$ that is the conduction regime where $\bar{Nu}_c = (R-1)/\ln R$ [6]. For $Ra > 40$, \bar{Nu} increases as Ra increases expressing the existence of convective heat transfer. Also, \bar{Nu} increases with the increase of the porosity due to the increase of the convective heat. The average Nusselt

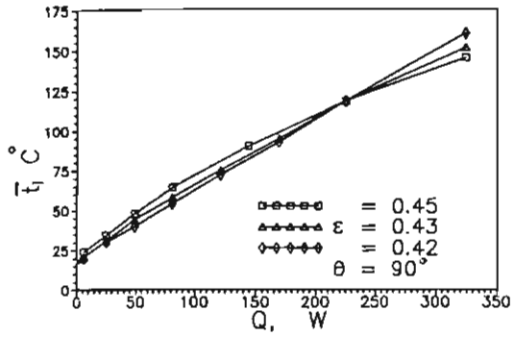


Fig. 5 variation of mean temperature of the inner cylinder wall of the annulus with input power for $\epsilon=0.42, 0.43$ and 0.45 and $\theta = 90^\circ$

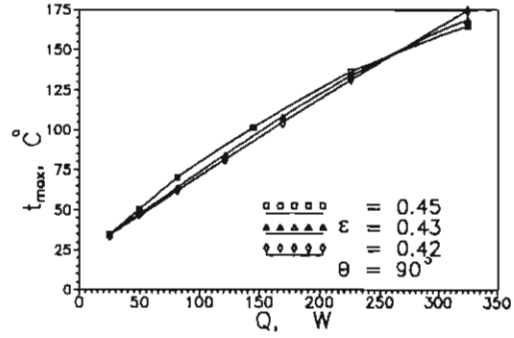


Fig. 6 variation of maximum temperature of the inner cylinder wall of the annulus with input power for $\epsilon=0.42, 0.43$ and 0.45 and $\theta = 90^\circ$

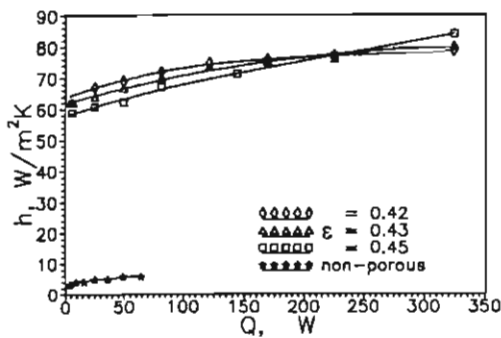


Fig. 7 variation of the heat transfer coefficient of the inner inner wall with input power for porous and non-porous annulus

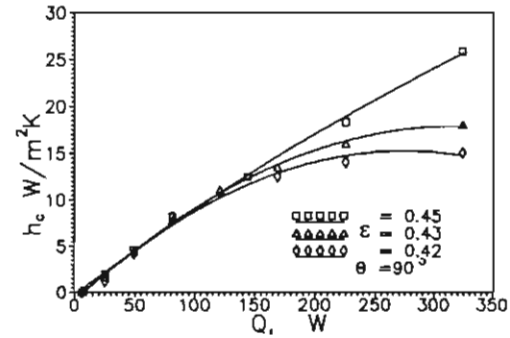


Fig. 8 variation of the convective heat transfer coefficient of the inner inner wall with input power for porous annulus at $\theta = 90^\circ$

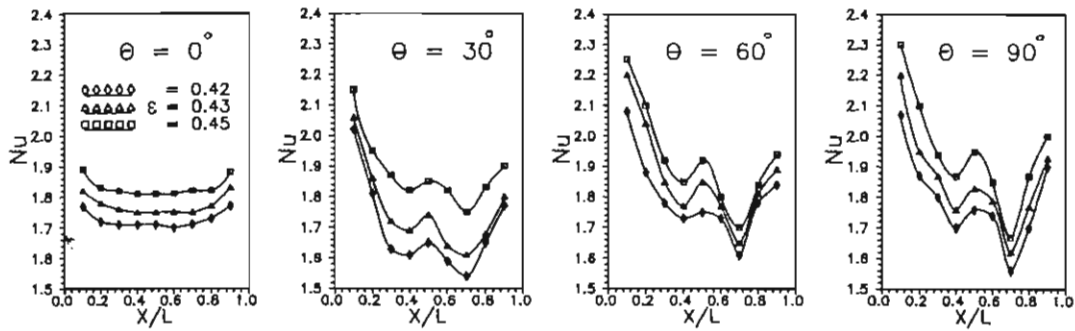


Fig. 9 local Nusselt number of the inner cylinder wall at the longitudinal direction for $Q=324W$ and $\epsilon = 0.42, 0.43$ and 0.45

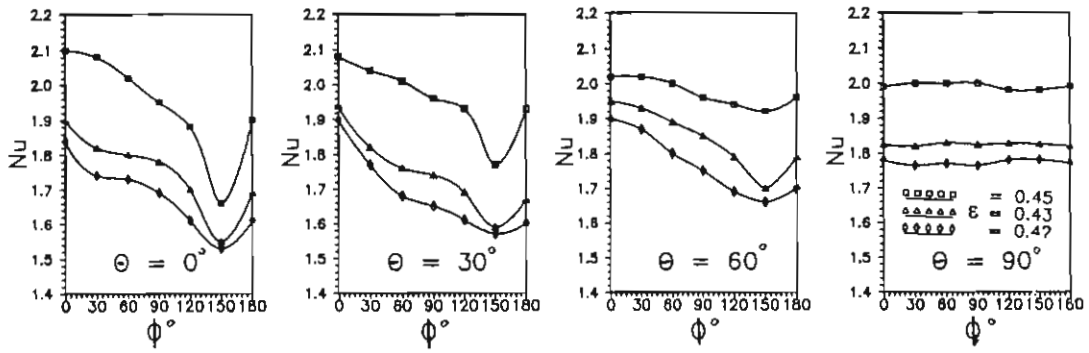


Fig.10 local Nusselt number of the inner cylinder wall at the angular direction for $Q=324W$ and $\epsilon = 0.42, 0.43$ and 0.45

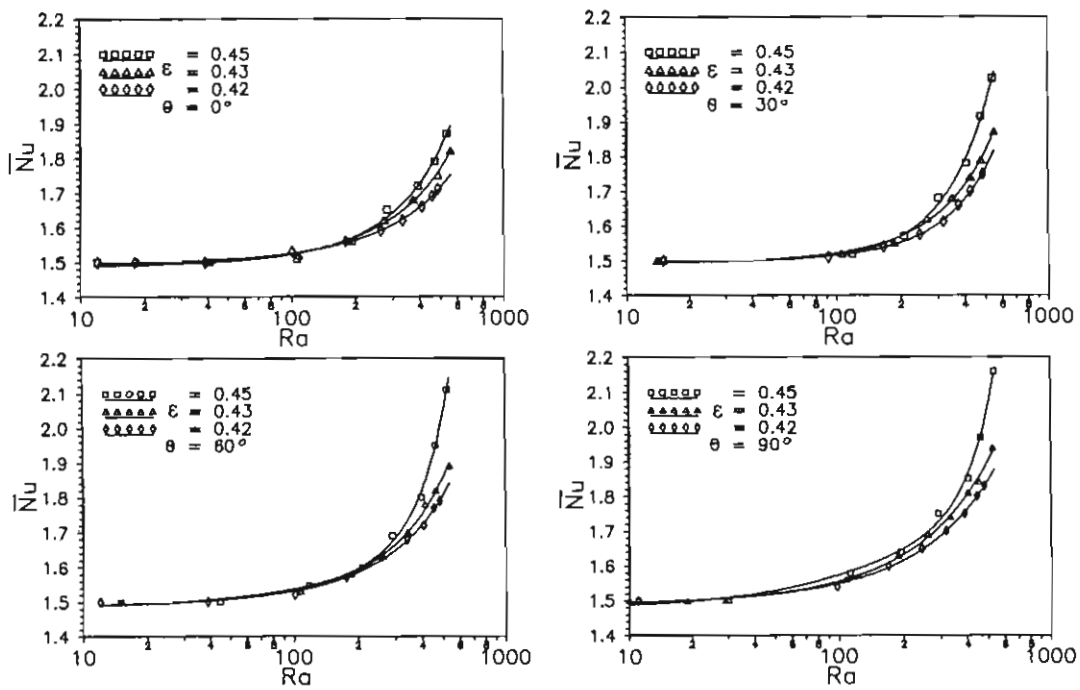


Fig. 11 distribution of mean Nusselt number on the inner wall with Rayleigh number for $\theta = 0, 30, 60$ and 90°

number is correlated with the inclination angle θ , the Rayleigh number and Darcy number as follows:

$$\bar{Nu} = 1.497 + 3.18 \times 10^{-4} (0.695 + \sin\theta) Ra + 5.35 \times 10^{-5} Da^{0.515} Ra^2$$

for $8.33 \times 10^{-5} \leq Da \leq 4.583 \times 10^{-4}$

CONCLUSIONS

Natural convective heat transfer had been experimentally investigated in an inclined annular porous medium with $A = 41$ and radius ratio $R = 2.143$ filled with stainless beads saturated with air of $Pr = 0.7$. The inner cylinder was exposed to constant heat flux 4-324W with $10 \leq Ra \leq 550$ while the outer one was maintained at constant temperature. The beads diameter is changed from 3.11 to 3.99, to 6.35 with porosities of 0.42, 0.43, and 0.45 respectively. The effect of porosity of the medium on the heat transfer characteristics is studied and the following are concluded:

The existence of the porous media causes a great reduction of the inner cylinder wall temperatures and gives the chance to increase the heat power generated from the inner cylinder.

for non-horizontal annulus, as θ increases, an existence of two cells of flow inside the annulus occurs and for non-vertical annulus, the flow separation may occur near $\phi = 150^\circ$, the maximum temperature and minimum Nu occur at $\phi = 150^\circ$.

The heat transfer coefficient h is higher for porous annulus than that for non-porous annulus by nearly 15 times in the studied case. With the increase of porosity, the conduction heat transfer coefficient decreases and the convection heat transfer coefficient increases. With increase of the input power, h increases due to the increase of the convective part of heat transfer.

By $Q < 230W$, a conduction dominated heat transfer region exists and with the increase of porosity \bar{t} and t_{max} increase, h decreases and it is preferable to use lower porosity medium.

By $Q > 230W$, a convection dominated heat transfer region, with the increase of porosity \bar{t} and t_{max} decrease and h increases and it is preferable to use higher porosity medium.

\bar{Nu} takes nearly constant value for $Ra \leq 40$ that is the one of purely conduction. For $Ra > 40$, \bar{Nu} increases as Ra increases expressing the existence of convective heat transfer, therefore, \bar{Nu} increases with the increase of the porosity. \bar{Nu} is correlated with inclination angle, Rayleigh number and porosity as follows:

$$\bar{Nu} = 1.497 + 3.18 \times 10^{-4} (0.695 + \sin\theta) Ra + 5.35 \times 10^{-5} Da^{0.515} Ra^2$$

for $8.33 \times 10^{-5} \leq Da \leq 4.583 \times 10^{-4}$

NOMENCLATURES

A	aspect ratio = L / δ	q	heat flux, W/m^2
c	specific heat of fluid, J/kgK	Q	input power, W

d	diameter of bead balls , m	r_i, r_o	inner and outer cylinder radii
Da	Darcy number, K/δ^2	R	radius ratio r_o/r_i
g	gravity acceleration, m/s^2	Ra	Rayleigh number
h	heat transfer coefficient, W/m^2K	t, T	temperature C, K
h_c	conduction part of heat transfer coefficient, W/m^2K	x	longitudinal length, m
k_m	effective thermal conductivity, W/mK	β	thermal expansion coefficient, K^{-1}
K	permeability of porous medium, m^2	ν	kinematic viscosity of fluid, m^2/s .
L	effective length of test section, m	ρ	density of fluid, m^3/kg .
Nu	local Nusselt number	ϵ	Porosity
\bar{Nu}	mean Nusselt number	δ	gap width = $(r_o - r_i)$, m.
Pr	Prandtl number	θ	Inclination angle.
		ϕ	angular inclination angle

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