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INVESTIGATION OF THE ACCURACY OF EARTH PRESSURE VALUES OBTAINED USING RANKNE THEOREM

دراسة مدي دقة حساب ضغط التربة الجانبي باستخدام نظرية رانكين

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الخلاصه : تحوز نظرية رانكين علي أفضلية كبيرة من الباحثين ومهندسي التصميم نظرا لسهولة تطبيقها وعدم الحاجة إلي عدد كبير من المعاملات الخاصة بخواص التربة، ويهدف هذا البحث إلي تقييم النظرية من ناحية دقة النتائج التي يتم الحصول عليها من جراء استخدامها، ولتنفيذ ذلك تم اللجوء إلي حل المعادلات التفاضلية الخاصة بربط جهد التربة بالانفعال، وقد تم حل هذه المعادلات عدديا بواسطة نموذج رياضي تطبق فيه طريقة العناصر المحددة من خلال برنامج ثلاثي الأبعاد PLAXIS3 حيث امكن استخدام جميع المعاملات الهامة في معالجة حائط ساند علي هيئة حرف L ، وقد قورنت النتائج مع نتائج استخدام نظرية رانكين، ونتج من المقارنة أن نتائج نظرية رانكين تقود إلي قيم انفعال تقل كثيرا عما تم استنتاجه من النموذج الرياضي، وفي نفس الوقت تم إجراء تجربة عملية باستخدام جهاز المحاكاة بالمركزيات لنفس الحالة التي تم حسابها عن طريق النموذج الرياضي ونظرية رانكين، وقد تبين من المقارنة تطابق نتائج النموذج الرياضي مع التجربة المعملية في حين كانت نتائج استخدام نظرية رانكين تقل عنهما بنسب تصل أحيانا إلي أكثر من ٥٠%.

ABSTRACT: The application of Rankine theorem to calculate the earth pressure forces on earth retaining structures is widely used due to its simplicity. In this work, the theorem results are tested for a cantilever retaining wall in pure dry sandy soil. The tests included the comparison with a more sophisticated numerical model as well as with results of centrifuge simulation works. The study included the investigation of the effect of retaining wall dimensions on wall and soil behavior. Bending moments as calculated using the Rankine's theory are tested against the other methods. Rankine's theory provided generally low straining action values as compared with the finite element and the centrifuge testing results.

1. INTRODUCTION

The Rankine theorem is probably the most widely used method to calculate the earth pressure forces on retaining structures. The reason for that is its simplicity and the minimum number of parameters needed for computation. Many seemingly important factors such as the wall and foundation dimensions . play practically no role on the computation procedure.

Coupling between behavior of soils and structures needs to be considered in the analysis and design of structures founded on and in soils. It is recognized that numerical methods that are build on properly chosen soil stress-strain models can provide realistic and satisfactory solutions for many static and dynamic problems involving coupling or interaction between soils and structures. Among the numerical methods used to solve equations built on these models is the Finite element method. It has been a prominent procedure used successfully for solution of a wide range of problems (Desai, 1977 [4]). Some of these problems are footings, piles, retaining structures, locks and many others structures.

The stability analysis of the earth retaining structures requires a proper prediction of the applied earth pressure on these structures. Several methods have been adopted to calculate the pressures on walls. The most popular of these methods are the Rankine and Coulomb. In this work, it is tried to spotlight on the convergence between the results of using Rankine procedure on one side and those of numerical and experimental approaches on the other. The comparison is conducted for different retaining wall and foundation dimensions.

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2. PROBLEM DESCRIPTION AND MODEL DETAILS

A series of numerical tests on a prototype retaining wall was carried using the finite element method. The analysis was performed using the PLAXIS 3D Tunnel software package (version 1, Brinkgreve and Vermeer 2001[3]). The geometry of the retaining system is a 5.0 m. high cut in cohessionless soil (pure dry medium sand) and is retained by a concrete cantilever retaining wall with a horizontal leg. The vertical modeled boundaries are located at three times the cut height away from the retaining wall location and they were assumed to be free vertically and constrained horizontally, while the lower boundary was located four times the cut height underneath the retaining wall leg and it is assumed as fully fixed as illustrated in Figure 1.

The concrete retaining wall is modeled as non-porous material with E_{conc} = 2.6 e^7 KN/m^2 and $\nu = 0.2$.

The soil elements are taken as 15-node wedge (3D) containing of 6-node triangles in x-y direction and 8-node quadrilaterals in z-direction. Moreover, 16-node elements are used to simulate soil-structure interaction. Figure 1includes the retaining system, the generated mesh for all elements (retaining wall, soil and interface) and the boundary conditions.

For simulating the soil, the hardening soil model (Isotropic hardening) is chosen. The hardening-Soil model is an advanced model for simulating the behavior of different types of soil, both soft and stiff soils, (Schanz, 1999 [9]). When subjected to primary deviatoric loading, soil shows a decreasing stiffness and simultaneously irreversible plastic strains develop. In the special case of a drained triaxial test, the observed relationship between the axial strain and the deviatoric stress can be well approximated by a hyperbola (Figure 2)

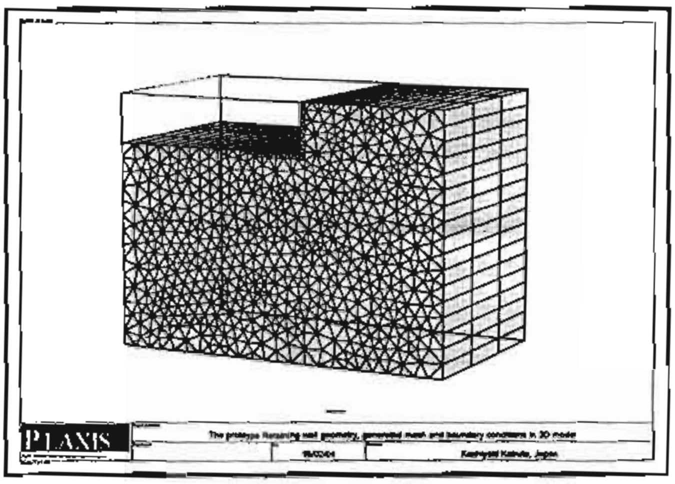


Fig. 1 The retaining system the generated mesh for all elements (retaining wall, soil and interface)

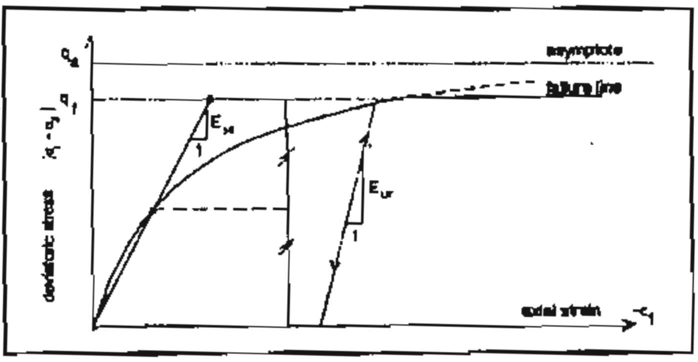


Fig. 2 Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test

Such a relationship was first formulated by Kondner 1963 [7] and later used in the well-known hyperbolic model (Duncan & Chang, 1970 [5]). The hardening-soil model (HS), however, supersedes the hyperbolic model by far. Firstly, by using the theory of plasticity rather than the theory of elasticity; secondly, by including soil dilatency and thirdly, by introducing a yield cap. Some basic characteristics of the HS model are as shown in Table1.

The advantage of the Hardening-Soil model over the Mohr-Coulomb model is not only the use of a hyperbolic stress-strain curve instead of a bi-linear curve, but also, the control of stress level dependency. When using the Mohr-Coulomb model, the user has to select a fixed value of Young's modulus whereas for real soils this stiffness depends on the stress level. It is therefore necessary to estimate the stress levels within the soil and use these to obtain suitable values of stiffness(Potts [8]. With the Hardening-Soil model, however, this cumbersome selection of input parameters is not required. Instead, a stiffness modulus E_{50}^{ref} is defined for a reference minor principal stress of σ_3. In contrast to the Mohr-Coulomb model (Gerham[6]), the transition from elastic behavior to failure is much more gradual when using the Hardening-Soil model. In fact, in the HS model, plastic straining occurs from the onset of loading.

Where:

$$E_{50} = E_{50}^{ref}\left(\frac{c\cot\phi - \sigma_3'}{c\cot\phi + p^{ref}}\right)^m$$

$$E_{ur} = E_{ur}^{ref}\left(\frac{c\cot\phi - \sigma_3'}{c\cot\phi + p^{ref}}\right)^m$$

$$E_{oed} = E_{oed}^{ref}\left(\frac{c\cot\phi - \sigma_1'}{c\cot\phi + p^{ref}}\right)^m$$

Table 1: Parameters of the Hardening-Soil model.

Parameter	**Definition**	**Unit**	**Values (for medium sand)**
Failure parameters as in Mohr-Coulomb model			
c	*Effective cohesion*	KN/m^2	0.0
ϕ	Effective angle of internal friction	Degree	37
Ψ	Angle of dilatency	Degree	5
Basic parameters for soil stiffness			
E_{50}^{ref}	Secant stiffness in standard drained triaxial test	KN/m^2	30000
E_{oed}^{ref}	Tangent stiffness for primary oedometer loading	KN/m^2	30000
m	Power for stress-level dependency of stiffness	---	0.5
Advanced parameters			
E_{ur}^{ref}	***Unloading/reloading stiffness***	KN/m^2	(default $E_{ur}^{ref} = 3 E_{50}^{ref}$) = 90000
ν_{ur}	Poisson's ratio for unloading/reloading [1]	---	(default ν_{ur}=0.2)
p^{ref}	Reference stress for stiffness	KN/m^2	(default p^{ref} =100 stress unit)
$K_0^{n.c}$	K_0 – value for normal consolidation	---	(default K_0 =1- sin ϕ)
R_f	Failure ratio	---	(default R_f = 0.9)
$\sigma_{tension}$	Tensile strength	KN/m^2	(default $\sigma_{tension}$ = 0 stress unit)
$c_{increment}$	As in Mohr-Coulomb model	KN/m^2	(default $\sigma_{tension}$ = 0)

3. CENTRIFUGE MODELING

The used centrifuge testing included the simulation of an L-shaped wall with the same height as that represented in the numerical study and a thickness of 0.5, and a leg length of 4.5m. It is found that suitable model dimensions that fit the centrifuge basket are those corresponding to rotational gravity of 30g (Allersma [1]).

Clean sand is used in the test. Different types of tests were conducted to determine the sand physical and mechanical properties (Barja [2]). For each test, 30 samples were treated to determine the average values of relevant properties. It is found that the dry density γ=1736 kg/m^3, φ= 37° while the cohesion has a zero value. The container material and dimensions are chosen after conducting the necessary calculations to guarantee that the body can stand severe stresses due to high gravity forces without influential strain.

Figure 3 a& b illustrate the model condition before and during the experimentation process.

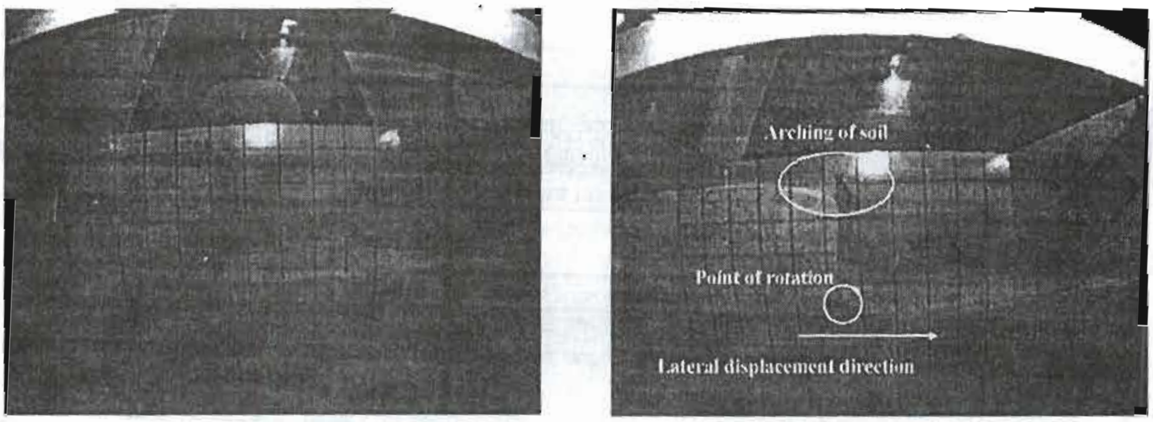
4. ANALYSIS AND RESULTS

The results of the application of Rankine theorem on retaining walls are compared with that of numerical analysis using PLAXIS software for retaining walls with different dimensions and that of the centrifuge modeling for the retaining wall with the dimensions indicated in section 3. It is found that there are almost coinciding results between the numerical and experimental results while the comparison with the Rankine theorem is illustrated in the following sections

The effect of retaining wall thickness

The earth Force on the wall of height H, from Rankine theorem is $0.5\gamma KH^2$ while the bending moment $M_{th} = \gamma.K.H^3/6$ no matter how much is the wall thickness

A series of calculations using the numerical model with retaining wall thickness (thr) of 0.25, 0.50, 0.75 and 1.00m was carried out. Figures 4 and 5 illustrate respectively the total displacement and stresses distribution in the soil medium for a wall thickness 0.75m. The comparison of the maximum bending moment on the wall as calculated from Rankine theorem (M_{th}) and that from the Numerical model M_{cal} is found in Figure6.



a) before test

b) during test

Fig. 3 Centrifuge Model Testing

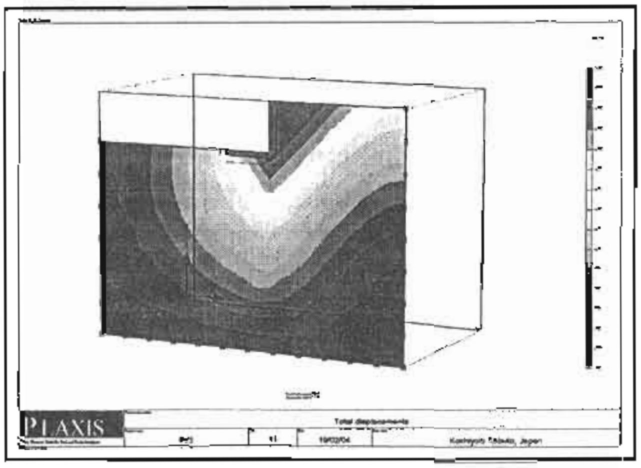


Fig. 4 Total displacement (retaining wall thickness = 0.75 m)

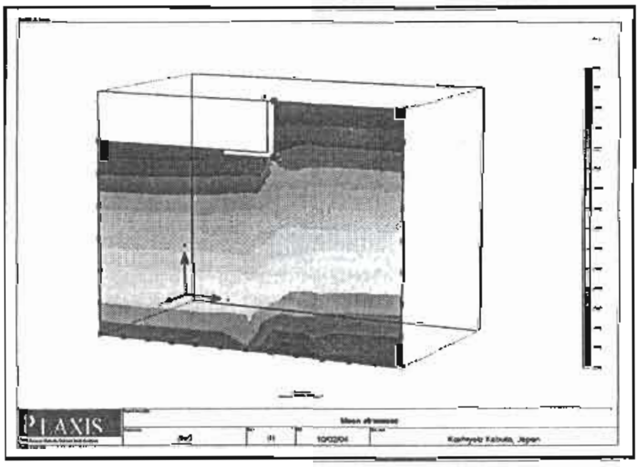


Fig. 5 Mean stresses (retaining wall thickness = 0.75 m)

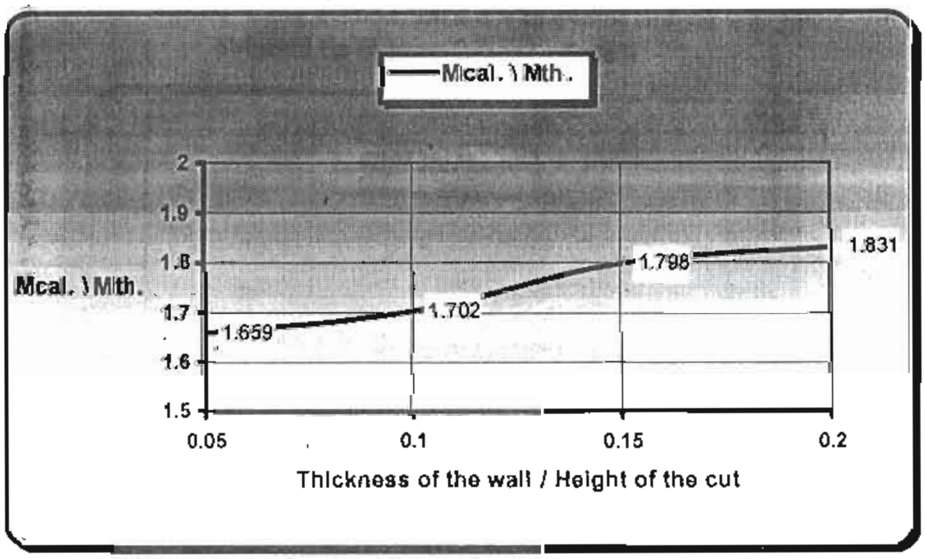


Fig. 6 Relation between Rankine Theory and Numerical Model Results for different Wall Thickness

Generally the obtained results reflect the following remarks:

a) The bending moment calculated by Rankine theorem is less than that calculated from numerical by a margin ranging from 66% to 83%
b) The increase in the retaining wall thickness (keeping other factors constant), leads to a slight increase in the bending moment values. However, the rate of this increase decreases gradually as the thickness increases.

The effect of retaining wall height

The second series of calculations are devoted to retaining walls with height (d) of 5.50, 6.00, 6.50, 7.00 and 7.50m. Figure 7 illustrates the cumulative relation between the dimensionless ratios M_{cal} / M_{th} and the ratio of retaining wall height with the original height of the cut(5.00m).

Also, in this series of calculations, similar remarks to those noticed in the case different wall thicknesses are found

a) The bending moment calculated by Rankine theorem is less than that calculated from numerical by a margin ranging from 70% to 150%
b) The increase in the retaining wall height(keeping other factors constant) leads to a slight increase in the bending moment values.

The effect of retaining wall horizontal leg thickness

The third series includes the effect of the change of retaining wall horizontal leg thickness (thl) from 0.25m to 1.00m. Fig 8 illustrates the graphical presentation of the obtained results.

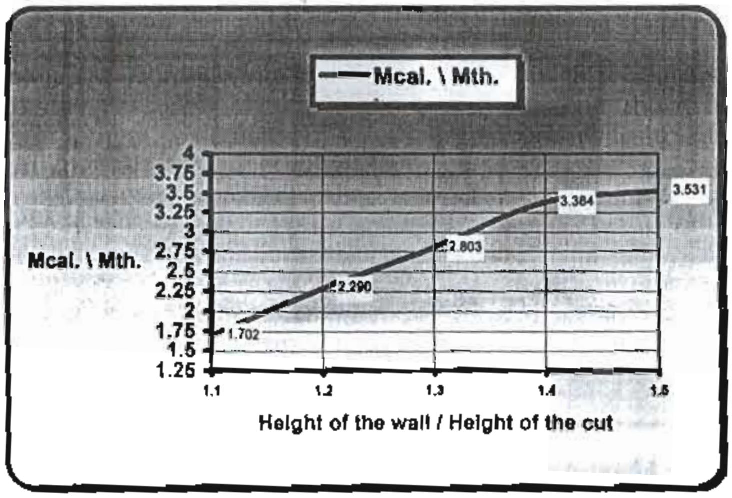


Fig. 7 Relation between Rankine Theory and Numerical Model Results for different Wall Heights

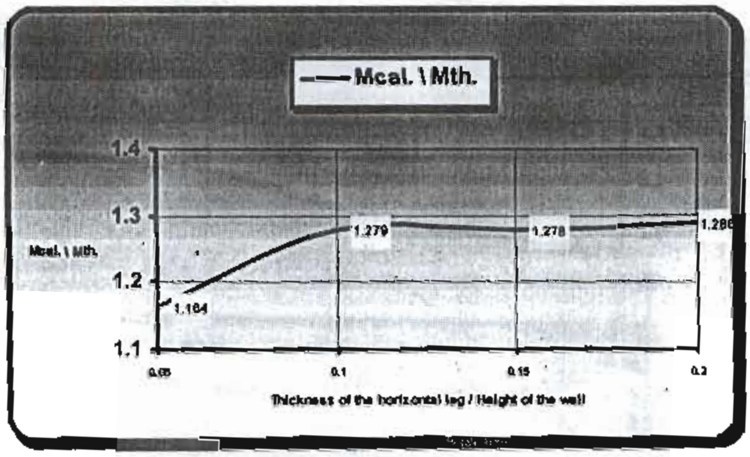


Fig.8 Relation between Rankine Theory and Numerical Model Results for different Leg Thickness

The effect of retaining wall horizontal leg length

The last series of computation includes change in bending moment duo to the variation of retaining wall horizontal leg length. It includes the walls with leg lengths of 4.00, 4.50, 5.00, 5.50 and 6.00m. The summary of the results are illustrated in Figure 9.

Also, in this case, the increase of leg length increases the relative difference between the results of the numerical modeling and those obtained from Rankine theorem by a big margin that reaches about 72% for the case of an increase of 20% in leg length.

5. CONCLUSION

A numerical analysis is conducted to test the validity of Rankine Theorem in a case in which its application is most favorable. In the numerical work, soil hardening condition was adopted to correlate the soil stress strain relation. In order to make sure of the accuracy of the analytical model, a centrifuge test was conducted which lead to almost similar results.

The analysis of the obtained results shed the light on several aspects. Generally, the following major conclusions may be drawn:

- The Results obtained from the hardening –finite element model differs greatly from that obtained from Rankine theorem
- The Rankine theorem under-estimates greatly the Design Bending Moment for all the studied cases.
- The centrifuge test results implied a confidence on the numerical modeling results as the values of the lateral displacement of the top ground level obtained from the centrifuge test and the numerical analysis are almost equal and has the same direction and shape.
- The different considered parameters such as the thickness of the stem or leg of the retaining wall, the length of the wall and its leg cause an increase in bending moment value . A phenomenon that is not remarkable in the results of Rankine Theorem as they do not appear in the applied equations.

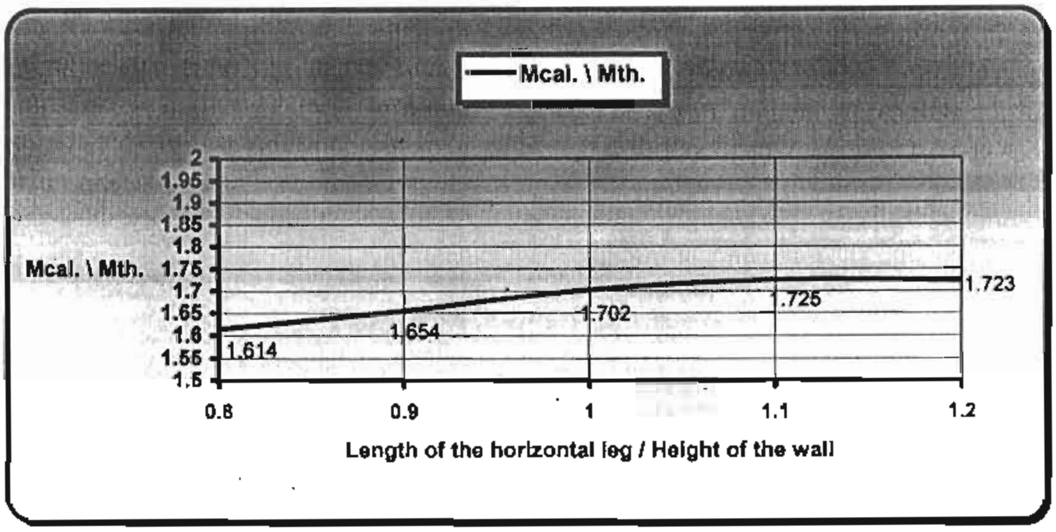


Fig. 9 Relation between.Rankine Theory and Numerical Model Results for Different Leg Wall Lengths

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