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THE OPTIMUM UTILITY OF UNDER GROUNDWATER IN EGYPTIAN BASIN

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The groundwater is the most important source of the river Nile of water sources in Egypt. It is very important to satisfy the water demand for development for many purposes. In this paper we build a mathematical model for the stochastic utility of groundwater by using the stochastic inventory theory (4) and applying that model to deduce the optimum utility of groundwater in Egyptian basin. This objective would increase the use of groundwater, decrease of cost pumping, and decrease fresh water discharges to the sea.

Key words : Stochastic processes - Inventory models - Stochastic inventory models - Statistical decision theory.

INTRODUCTION

Geographically Egyptian basin is divided into main two aquifers; Nile Delta and valley aquifers. Recently, the geologists began to plan for use any water aquifer. They put the volume of any of the aquifer as a buffer stock to balance with the salt water from Red and Mideterranian seas and the extraction of water must equal to inflow water annually in order to decrease the outflow water and the excess in storage (6). The problem in the liture (1); how much can we pump the aquifer before "significant" piezometric head lowering and sea water intrusion occurs?. This is an issue involving what is often called "Safe Yield". Here we treated this problem as an inventory problem (4) with the assumptions that the demand water is a continuous random variable and the optimal inventory of any aquifer with the minimization of any outflow water and the excess in the storage is introduced. This objective would increase the use of groundwater, decrease of cost pumping, and decrease fresh water dischargers to the sea.

Data analysis :

In this section we shall select the distribution of the demand and how well this distribution fits the observed demand data. Departures from stationarity might be of two different types trend and seasonal variation. Now we have to test for the existence of trend and seasonal variation. One way to do so, is to use a one way analysis of variance classifying our observations by months. We found that the demand is stationary in Nile valley and Delta regions. Its appears from the charts of demand that the normal distribution is an approximated distribution of demand for all demand regions.

Hence the normal distribution was fitted to two region distributions and the Chi-Square test was applied to each of the fitted distributions (11). There is a considerable variation in the goodness of fit. The results are :

Nile Delta region

Demand	Expected	Observedd
180 - 210	3.5028	3
210 - 240	9.0612	14
240 - 270	11.6700	6
270 - 300	7.7900	8
300 - 330	2.6850	5

χ^2 (2) = 7.5190629, P = 0.99, consequently the fit of data is very good at 0.01 level of significance.

Nile Valley region

Demand	Expected	Observedd
70 - 90	3.2856	8
90 - 110	4.4256	5
110 - 130	4.9032	3
130 - 150	4.1712	2
150 - 170	2.6064	2
170 - 190	1.2960	4

χ^2 (3) = 14.490742, P = 0.999, consequently the fit of data is very good at 0.001 level of significance.

3. Problem formulation

In this section we introduced the proposed model and it's solution with normal distribution demand.

3.1. The proposed model

In our problem, we assume that the demand occurs continuously during the period. Also, it is assumed that the stocks replenishment occurs continuously. Thus depending on the amount ξ, the inventory position right after demand occur may be either positive which is lost (outflow. or increase in storage) or negative (shortage in the Buffer stock). These two cases are shown in Figure 3.1, which are a shortage in water.

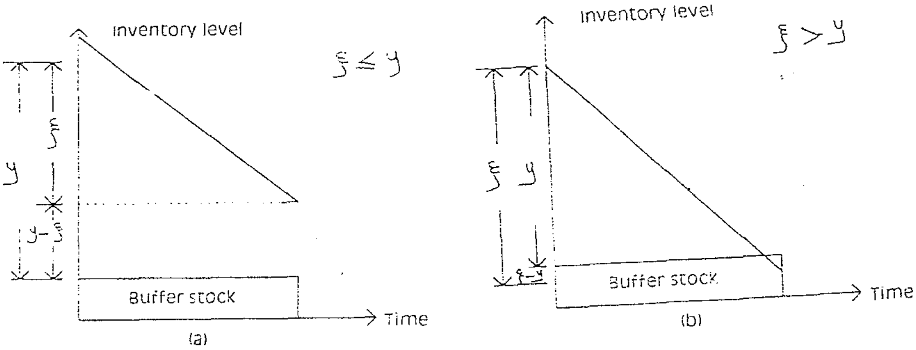

Figure (3.1)

From the figure, given y, the amount on hands after an order is received, the function of the inventory system is generally given by

$$G(y) = \begin{cases} y - \xi, & \text{for } \xi \leq y \\ \xi - y, & \text{for } \xi > y \end{cases} \qquad (3.1)$$

Our problem coincides with the single period inventory model, i.e. a single critical number policy is applicable or optimal.

3.2. Solution of the model with normal distribution demand.

consider the assumptions

1. There is no set up cost for an order.
2. There is no holding cost for inventory.
3. Continuously demand with probability distribution $\varphi(\xi)$ which is a normal distribution with mean μ and standard deviation σ.
4. The shortage of development due to lost water is p per unit cubic meter.
5. Production cost is c per unit cubic meter and initial inventory is zero.

6. The shortage cost due to the abstraction from the buffer stock equals (28 p + c) per unit cubic meter which means that it has 28 cubic meter for a treatments and increase the production cost by c over the production of y (7).

From the previous assumptions, The optimal inventory level will be derived based on the minimization of expected inventory costs, which include the production, the shortage of development due to the lost water which is given by

$$E\{L(y)\} = \int_{0}^{y} (y - \xi)\, \varphi(\xi)\, d\xi \quad \text{..........} \quad (3.2)$$

While the shortage in the buffer stock cost is

$$E\{B(y)\} = \int_{y}^{\infty} (y - \xi)\, \varphi(\xi)\, d\xi \quad \text{..........} \quad (3.3)$$

Then the inventory equation has the following form :

$$E\{C(y)\} := cy + E\{L(y)\} + E\{B(y)\}$$

$$= cy + p\int_{0}^{y} (y - \xi)\, \varphi(\xi)\, d\xi + (28p + c)\int_{y}^{\infty} (\xi - y)\, \varphi(\xi)\, d\xi \quad \text{......} \quad (3.4)$$

The following lemma is needed in the sequel for computation of the cost.

Lemma 3.1 : The inventory equation has the following form :

$$E\{C(y)\} = cy + (\mu - y)(28\,p + c) + y\,(29\,p + c)\int_{0}^{y} \varphi(\xi)\, d\xi$$

$$- (29\,p + c)\int_{0}^{y} \xi\, \varphi(\xi)\, d\xi \quad \text{..........} \quad (3.5)$$

Lemma 3.2 : The inventory equation E{C (y)} has a global minimum.

Proof : By taking the first derivative of (3.4) we get

$$\frac{\partial E\{C(y)\}}{\partial y} = c - (28\,p + c) + (29\,p + c)\int_{o}^{y} \varphi(\xi)\, d\xi$$

$$+ \left[(29\,p + c)\, y\, \varphi(\xi)\, (d\xi/dy)\right]_{o}^{y} - \left[(29\,p + c)\, \xi\varphi(\xi)\right]_{o}^{y}$$

$$= -28p + (29\,p + c)\int_{o}^{y} \varphi(\xi)\, d\xi \qquad \text{...} \qquad (3.6)$$

Then, by taking the second derivative of (3.6) we have

$$\frac{\partial E\{C(y)\}}{\partial y^2} = (29\,p + c)\, \varphi(y) > 0$$

Which proves that y corresponds to a minimum point, the function E {C (y)} is convex and hence y is unique, it must hold a global minimum.

We define the following :

y^* = the optimum order of inventory through a month.

$$C_O(y) = \text{Min } y \geq C_O(y) = C(y^*) \qquad \text{..............................} \qquad (3.7)$$

Hence according to above lemma the optimum order level is determined by the equation :

$$\int_{o}^{y} \varphi(\xi)\, d\xi = \frac{28\,P}{2\,qp + c} \qquad \text{..............................} \qquad (3.8)$$

The value of y^* is selected such that the probability $\xi \leq y^*$ is equal to

$$q = \frac{28\,P}{2\,qp + c} \qquad \text{...} \qquad (3.9)$$

Consequently the optimal policy is as follows

order y^* if $y^* >$ zero

not order if $y^* \leq$ zero .... (3.10)

Nor, we have to solve the integral equation (3.8) with the normal distribution;

$$\varphi(\xi) = \frac{1}{\sqrt{2}\,\pi\sigma} e^{-(\xi-\mu)^2/2\sigma^2}, \; -\infty < \xi < \infty \qquad (3.11)$$

then equation (3.8) yields to :

$$q = \frac{1}{\sqrt{2}\,\pi\sigma} \int_{0}^{y^*} e^{-(\xi-\mu)^2/2\sigma^2} d\xi \qquad (3.12)$$

Since the demand has the normal distribution, then from equation (3.9) this integral equation (3.12) has the solution

$$y_N^* = \mu + \sigma z \text{ and } z = \Phi_N^{1}(q), \qquad (3.13)$$

where $\Phi_N(z)$ is the standard normal distribution function

Now, we shall find a numerical solution for that integral equation (3.8) with the normal distribution.

Let us put $\zeta = \frac{\xi - \mu}{\sqrt{2}\sigma}$, then $\sqrt{2}\sigma \, d\zeta = d\xi$,

Then at $\xi = 0$ implies $\zeta = \frac{-\mu}{\sqrt{2}\sigma}$, and at $\xi = y^*$ implies $\zeta = (y^* - \mu)/\sqrt{2}\sigma$

Hence

$$q = \frac{1}{\sqrt{\pi}} \int_{-\mu/\sqrt{2}\sigma}^{y^* - \mu/\sqrt{2}\sigma} e^{-\zeta^2} d\zeta$$

$$= \frac{1}{\sqrt{\pi}} \left\{ \left[\frac{y^* - \mu}{\sqrt{2}\sigma} - \frac{(y^* - \mu)^3}{3(\sqrt{2}\sigma)^3\, 11} + \frac{(y^* - \mu)^5}{5(\sqrt{2}\sigma)^5\, 21} \cdots\cdots\cdots \right] \right.$$

$$\left. + \left[\frac{\mu}{\sqrt{2}\sigma} - \frac{\mu^3}{3(\sqrt{2}\sigma)^3\, 21} + \frac{\mu^5}{5(\sqrt{2}\sigma)^5\, 21} \cdots\cdots\cdots \right] \right\}$$

We let $s = \sqrt{2}\sigma$ we get

$$q = \frac{1}{\sqrt{\pi}} \sum_{k=0}^{\infty} \left\{ \frac{(-1)^k \left[\left(\frac{y - \mu}{s} \right)^{2k+1} + \left(\frac{\mu}{s} \right)^{2k+1} \right]}{k!\,(2k+1)} \right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3.14)$$

In the next section (4) we shall find the numerical results for equation (3.14) to deduce the optimal value of y^*.

4. The numerical outcomes

In this section, we introduce the results of all region which obtained by using computer program (12). In every iteration we selected the only solution which associated to the minimum cost. Also, we took the absolute value of the imaginary solutions if there exist. Finally we discuse these results and stated our recommendations.

4.1. The Nile Delta region

The data of this region are as following :

(i) $q = 0.9567949$ (no dimension)

(ii) $\mu = 251.9$ (c.m.)

(iii) $s = 51.1$ (c.m.)

(iv) $a = \{(y - \mu)/s\}^{(2k+1)}$ .... (4.1)

(v) $b = (\mu/s)^{(2k+1)}$ .... (4.2)

(vi) $d = (-1)^k\,(2k + 2)k!$ .... (4.3)

(ix) $\sum_{k-0}^{\infty} \left(\frac{a+b}{d} \right) - q\sqrt{\pi} = 0$ .... (4.4)

[Note that (iv) → (ix) represent equation (3.14) above] and equation (3.5) above :

(x) $E\ \{C(y)\} = cy + (\mu - y)\ (28\ p + c) + y\ (29\ p + c)_O \int^y \xi\varphi\ (\xi)\ d\xi +$ $(29\ p + c)_O \int^y \quad \xi\varphi\ (\xi)\ d\xi$

Iteration 1 :

$$y - sq\ \sqrt{\pi = 0} \qquad \text{..} \qquad (4.5)$$

y = 86.659168 and the associated cost is 4.222 (10^9) (bound).

Iteration 2 :

$$\sum_{k=0}^{1} \left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \qquad \text{....................................} \qquad (4.6)$$

y = 430.476 and the associated cost is 2.59728 (10^8) (bound).

Iteration 3 :

$$\sum_{k=0}^{2} \left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \qquad \text{.................................} \qquad (4.7)$$

y = 284.849 and the associated cost is 1.88656 (10^8) (bound).

Iteration 4 :

$$\sum_{k=0}^{3} \left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \qquad \text{...............................} \qquad (4.8)$$

y = 209.962 and the associated cost is 1.1726 (10^9) (bound).

Iteration 5 :

$$\sum_{k=0}^{4} \left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \qquad \text{.................................} \qquad (4.9)$$

y = 3144242, the associated cost is 1.4474 (10^8) (bound).

Iteration 6 :

$$\sum_{k=0}^{5}\left(\frac{a+b}{d}\right) - q\sqrt{\pi} = 0 \quad \text{.............................} \quad (4.10)$$

y =373.522, the associated cost is 1.95464 (10^8) (bound).

Iteration 7 :

$$\sum_{k=0}^{6}\left(\frac{a+b}{d}\right) - q\sqrt{\pi} = 0 \quad \text{............................} \quad (4.11)$$

y =325.121, the associated cost is 1.482 (10^8) (bound).

Iteration 8 :

$$\sum_{k=0}^{7}\left(\frac{a+b}{d}\right) - q\sqrt{\pi} = 0 \quad \text{..........................} \quad (4.12)$$

y =286.825, the associated cost is 1.81844 (10^8) (bound).

Iteration 9 :

$$\sum_{k=0}^{8}\left(\frac{a+b}{d}\right) - q\sqrt{\pi} = 0 \quad \text{..........................} \quad (4.13)$$

y =330.565, the associated cost is 1.5179 (10^8) (bound).

Iteration 10 :

$$\sum_{k=0}^{9}\left(\frac{a+b}{d}\right) - q\sqrt{\pi} = 0 \quad \text{.........................} \quad (4.14)$$

y =300.091, the associated cost is 1.52695 (10^8) (bound).

Iteration 11 :

$$\sum_{k=0}^{10}\left(\frac{a+b}{d}\right)-q\sqrt{\pi=0} \qquad (4.15)$$

y =333.784, the associated cost is 1.5435 (10^8) (bound).

Iteration 12 :

$$\sum_{k=0}^{11}\left(\frac{a+b}{d}\right)-q\sqrt{\pi=0} \qquad (4.16)$$

y =308.418, the associated cost is 1.45852 (10^8) (bound).

Iteration 13 :

$$\sum_{k=0}^{12}\left(\frac{a+b}{d}\right)-q\sqrt{\pi=0} \qquad (4.17)$$

y =335.827, the associated cost is 1.56033 (10^8) (bound).

Iteration 14 :

$$\sum_{k=0}^{13}\left(\frac{a+b}{d}\right)-q\sqrt{\pi=0} \qquad (4.18)$$

y = 314.038, the associated cost is 1.44734 (10^8) (bound).

This solution is illustrated graphically in Figure (4.1) and y^* in the last iteration is the optimal order size in this area.

4.2. The Nile valley region ;

In this region we listed only the new data and the remaining data from (vi) to the Nile Dellta region (x) are similar to above.

(i) q = 0.9567949. (no dimension)

(ii)) μ = 116.48. (c.m.)

(ii) s = 53.74011537. (c.m.)

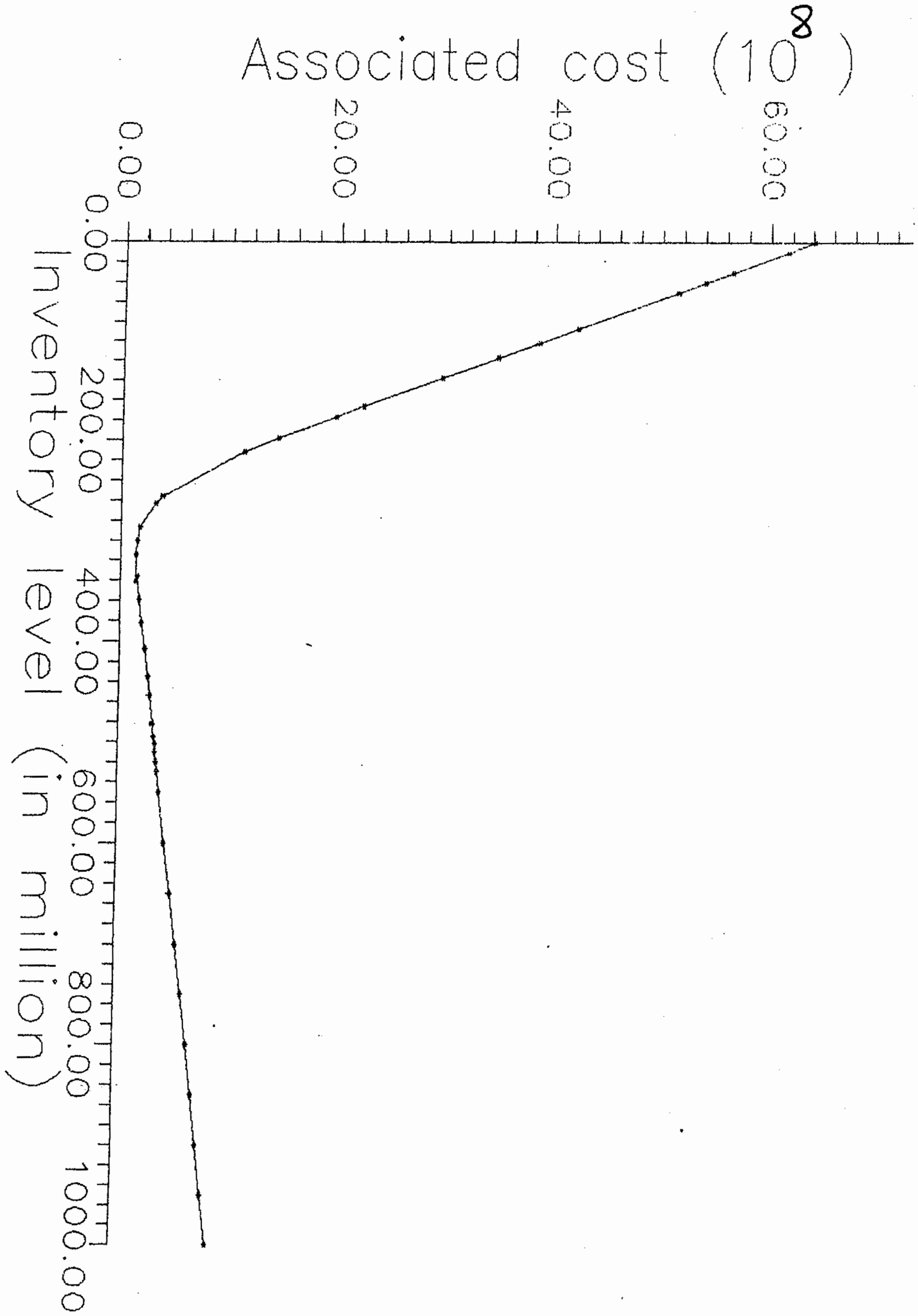
Associated cost (10^8)
0.00
20.00
40.00
60.00
0.00
200.00
400.00
600.00
800.00
1000.00
Inventory level (in million)

Figure (4.1)

Iteration 1 : equation (4.5)

$y = 91.1365$, the associated cost is $8.136\ 10^8$ (bound).

Iteration 2 : equation (4.6) above

$y = 198.585$, the associated cost is $1.1904\ 10^8$ (bound).

Iteration 3 : equation (4.7) above

$y = 92.9526346$, the associated cost is $7.8059\ 10^8$ (bound).

Iteration 4 : equation (4.8) above

$y = 176.7766$, the associated cost is $1.1355\ 10^8$ (bound).

Iteration 5 : equation (4.9) above

$y = 205..5099$, the associated cost is $1.244\ 10^8$ (bound).

Iteration 6 : equation (4.10) above

$y = 175.2078$, the associated cost is $1.14248\ 10^8$ (bound).

Iteration 7 : equation (4.11) above

$y = 204.079$, the associated cost is $1.232\ 10^8$ (bound).

Iteration 8 : equation (4.12) above

$y = 183.374$, the associated cost is $1.12599\ 10^8$ (bound).

Iteration 9 : equation (4.13) above

$y = 151.3$, the associated cost is 10^8 (bound).

Iteration 10 : equation (4.14) above

$y = 219.759$, the associated cost is $1.379\ 10^8$ (bound).

Iteration 11 : equation (4.15) above

$y = 173.694$, the associated cost is $1.151\ 10^8$ (bound).

Iteration 12 : equation (4.16) above

$y = 186.179$, the associated cost is $1.126\ 10^8$ (bound).

Iteration 13 : equation (4.14) above

y = 180.859, the associated cost is $1.126\ 10^8$ (bound).

Iteration 14 : equation (4.14) above

y = 182.515, the associated cost is $1.125\ 10^8$ (bound).

Iteration 15 :

$$\sum_{k=0}^{14}\left(\frac{a+b}{d}\right)-q\sqrt{\pi}=0 \qquad \text{.........................} \quad (4.19)$$

y = 181.958, the associated cost is $1.125\ 10^8$ (bound).

Iteration 16 :

$$\sum_{k=0}^{15}\left(\frac{a+b}{d}\right)-q\sqrt{\pi}=0 \qquad \text{.........................} \quad (4.20)$$

y = 182.154, the associated cost is $1.125\ 10^8$ (bound).

Iteration 17 :

$$\sum_{k=0}^{16}\left(\frac{a+b}{d}\right)-q\sqrt{\pi}=0 \qquad \text{........................} \quad (4.21)$$

y = 182.121, the associated cost is $1.125\ 10^8$ (bound).

Iteration 18 :

$$\sum_{k=0}^{17}\left(\frac{a+b}{d}\right)-q\sqrt{\pi}=0 \qquad \text{.......................} \quad (4.22)$$

y = 182.094, the associated cost is $1.125\ 10^8$ (bound).

Iteration 19 :

$$\sum_{k=0}^{18}\left(\frac{a+b}{d}\right)-q\sqrt{\pi}=0 \qquad \text{.......................} \quad (4.23)$$

$y = 182.091$, the associated cost is $1.125\ 10^8$ (bound).

Iteration 20 :

$$\sum_{k=0}^{19}\left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \quad \text{......................... (4.24)}$$

$y = 182.091$, the associated cost is $1.125\ 10^8$ (bound).

Iteration 20 :

$$\sum_{k=0}^{20}\left(\frac{a+b}{d}\right) - q\sqrt{\pi = 0} \quad \text{......................... (4.25)}$$

$y = 181.092$, the associated cost is $1.125\ (10^8)$ (bound).

This solution is illustrated graphically in Figure (4.2) and y^* in the last iteration is the optimal order size in this area which associated to the minimum cost.

5. Discussion and recommendations

In this section we introduced the validations in the real situations of the results obtained in the previous sections and stated our recommendations on this work. In that sections we see that from Table 5.1 the results obtained

Region	y_c*	y*
Nile Delta	313.992	314.038
Nil Vally	181.631	181.689

A comparison of central limit and numerical solutions. Table 5.1

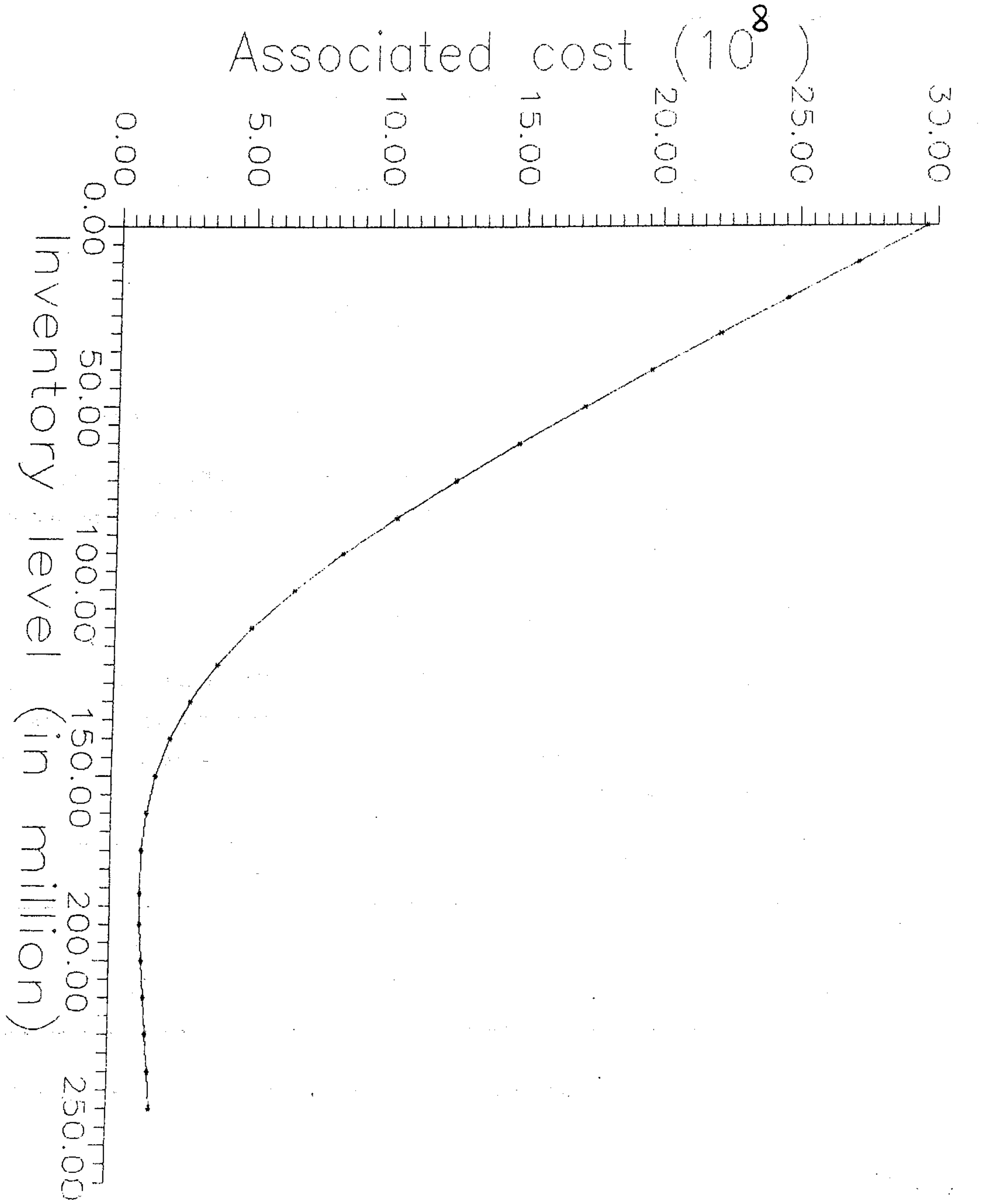
Associated cost (10^8)
0.00
5.00
10.00
15.00
20.00
25.00
30.00
Inventory level (in million)
0.00
50.00
100.00
150.00
200.00
250.00

Figure (4.2)

by central limit theorem dented by y_C^* and numerical outcomes denoted by y^* are comparable and hence the optimal policy in every region is to order y^* every month in order to its demand to be satisfied. That is the optimal policy is to order about 413 million cubic meter of water every month or 3768 million cubic meter of water annually to satisfy our demand in the Nile Delta region, about 2180 million cubic meter of water annually to satisfy our demand in the Nile Valley region.

Consequently, in order to put these results under validations we need to compare this order quantity with the inflow water quantity to determine the overage or shortages in the inflow water quantity in every aquifer. The validations of these results under the optimal policy or under the minimization of effected costs are tabulated in Table 5.2. In this table we deduced that, in every region there is an overage of the inflow water which is going directly or indirectly lost to the sea. Which means that there is about 727 million cubic meter of water in the Nile Delta aquifer, about 868 million cubic meter of water in the Nile Valley aquifer lossing directly or indirectly in the sea annually. The results of the future pumage ares differs from geologest man results. This difference when occurs yield pollution by the salted water from the sea and the result of this model will prevent this pollution if it is valid.

Region	Nile Delta	Nile Delta
Mean abstraction (million c.m./yar)	3022.8	1397.76
The result of the model (million c.m./yar)	3768	2180
Inflow watr (million c.m./yar)	4495	3048
Th eexcss of future pumpage (million c.m./yar)	727	868

Safe pumpage for Nile Delta and Valley aquifers Table 5.2

RECOMMENDATIONS

1. The optimal usage of the Nile Delta region is that the extraction can be up to 312.5 million cubic meter, hence, the extraction can be increased to the rate of 727 million cubic meters distributed uniformly on twelve month.

2. The optimal utility of the Nile Valley region is that the extraction can be up to 188.78 million cubic meter, hence, the future pampage can be increased to the rate of 868s million cubic meter distributed uniformly on the twelve month.

3. One of the main sources of water in Egypt is the ground water, which has the same importance as the river Nile's water, and to control the storage we have to measure weakly the inflows and outflows quantities.

4. It is easy to recompute the parameters in this model, in case if the demand exceeds that in the model from Recommendations 1 - 2, to avoid unused water to keep to from throwing it in the medeterenean sea yearly.

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الملخص العربى

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تعتبر المياه الجوفية فى مصر مصدرا هاما من مصادر نهر النيل. ويرجع ذلك إلى زيادة الطلب على المياه للتنمية فى العديد من الأغراض .

فى هذا البحث قمنا ببناء نموذج رياضى للإستخدام العشوائى للمياه الجوفية وذلك بإستخدام نظرية المخزون العشوائى لإستنتاج الإستخدام الأمثل للمياه الجوفية فى حوض نهر النيل .

وهذا النموذج سوف يزيد من إستخدام المياه الجوفية فى هذه المنطقة وتقليل تكاليف السحب وأيضا تقليل كميات المياه المهدرة بطريق مباشر أو غير مباشر إلى البحر وأيضا قمنا بإعطاء بعض التوصيات ، منها أنه يجب مراعاة حسابات هذا النموذج إذا زاد الإستخدام عن الحد الأقصى الذى حدده هذا النموذج.