"EXPERIMENTAL INVESTIGATIONS OF MAGNETIZING INRUSH CURRNT IN POWER TRANSFORMER USING A PROPOSED SYNCHRONOUS SWITCH"

BY

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ABSTRACT:

This paper introduces deep and careful theoritical and experimental investigations of the magnetizing inrush current (M.I.C.) phenomenon in single phase power transformers, All salient parameters influencing M.I.C. waveform and particushed by the control of the instant of switching the supply to the transformer, the factor which appreciably affects M.I.C.

LIST OF SYMBOLS:

θ_{sn}	is the saturation angle for nth cycle.
V	is the r.m.s. value of the supply voltage.
α_n	is the desaturation angle at which the magnetizing current is almost zero at the end of the nth cycle.
B_R	is the residual flux density in the core when the switch is closed at -180° in Tesla.
B_m	is the peak flux density in the core due to the applied sinusoidal voltage E in Tesla.
BR_L	Saturation limit of the residual flux density.
I_e	rms current for the nth cycle.
R	resistance of the primary winding.
X	Leakage reactance of the primary winding.
Z	Impedance of the primary winding.
i	instaneous value of the magnetizing current.
t	time.
w	angular frequency of the supply.

1. INTRODUCTION:

The magnetizing inrush current of a transformer is a transient phenomenon which is accasionally observed when it is being switched-on. Its magnitude and behaviour are affected considerably due to several factors of switching-on instant, amount and polarity of the actual residual magnetism in the transformer core, condition and type of the secondary loading and the addition of the circuit elements in its primary side. (1). It may reach too high values and cause a momentary dip in the voltage if the impedance of the source is considerable. It may, consequently, trip the overload or common differential relays, if they have been set too close.

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Thus, this phenomenon has a stupendous attention for investigation. Many researchers have discussed and analyized M.I.C. so that reasonable circuits are designed to help the protective system to have a discrimination between the internal fault and magnetizing inrush currents (2). In the present paper, the magnetizing inrush current(M.I.C) phenomenon with its waveform and the parameters affecting it, are deeply investigated analytically and experimentally.

In the laboratory many oscillograms are recorded for M.I.C. at all the prementioned conditions. In order to switch-on the transformer at any selected instant of the supply voltage waveform, a solid-state synchronous switch is designed, built-up and used (3).

2. THEORITICAL INVESTIGATION OF M.I.C. PHENOMENON:

2.1. Mathematical expressions of M.I.C. The Instantaneous Value:

The inrush current in the saturation region is given by (4).

$$i = \frac{2\ VX}{Z^2}\left\{\frac{R}{X}\sin wt-\cos wt+\left(\frac{R}{X}\sin\theta_{sn}+\cos\theta_{sn}\right)\exp\left(-\frac{R}{X}(wt+\theta_{sn})\right)\right\} \quad ......(1)$$

where, $n = 1,2,3,...$ is the number of cycle.

The current returns to zero value at $wt = \ _n$, which is obtained from the solution of the following equation.

$$\frac{R}{X}\sin\alpha_n-\cos\alpha_n+\left(\frac{R}{X}\sin\theta_{sn}+\cos\theta_{sn}\right).\exp\left(\frac{R}{X}(\alpha_n+\theta_{sn})\right)=0 \quad ......(2)$$

Average value of the Inrush current:

The average value during the nth cycle is (4)

$$I_{av} = (\sqrt{2}\ V/2\pi R)(\cos\alpha_n-\cos\theta_{sn}) \quad ......(3)$$

Also, the average value for n cycles is (4)

$$I_{An}=\left(\sum_{1}^{n} I_{av}\right)/n = (\sqrt{2}\ V/2\pi Rn)(\cos\alpha_n-\cos\theta_{s1}) ...(4)$$

R.M.S. value of the Inrush current (4)

For the nth cycle,

$$I_e=(V/2\sqrt{\pi}\ Z)\left(2(\alpha_n+\theta_{sn})-\sin 2\alpha_n-\sin 2\theta_{sn}-(\cos 2\alpha_n-\cos 2\theta_{sn})\left(\frac{X}{R}\right)\right) \quad ......(5)$$

where, $\alpha_n = \theta_{s(n+1)}$

For n successive cycles, the rms current is given by:

$$I = ((\sum_{1}^{n} I_s^2)/n)^{\frac{1}{2}} \qquad \ldots\ldots(6)$$

Peak value of the nth cycle

The first few cycles of the inrush current have severe peaks. The peak value of the nth cycle is as follows(4):

$$I_{pn} = (\sqrt{2}\ V/R)\ \sin B_n \qquad \ldots\ldots(7)$$

where $(-B_n)$ is the angle at which the peak value occurs and is determined from the solution of the equation.

$$\cos B_n - \frac{X}{R} \sin B_n = (\frac{R}{X} \sin \theta_{sn} + \cos \theta_{sn}) .$$
$$\exp (\frac{R}{X} (\theta_{sn} - B_n)) \qquad \ldots\ldots(8)$$

As the saturation angle of the pervious cycle, it becomes possible to determine the saturation angle for each of the successive cycles. (4, 5, 6).

2.2. Parameters Influencing M.I.C. Characteristics:

2.2.1. Equivalent resistance/reactance ratio of the transformer:

The waveforms of M.I.C. at various cases of R/X are deduced and shown in Fig.(1) for $B_R = B_m$ and B_s=1.4 B_m using the above equations (1, 4, 6, 7). The investigation of these waveforms display the following features:

(1) As R/X has a high ratio (2 or 10), M.I.C. reaches one p.u. (referring to the full load value of the transformer) after a very limited number of cycles and time, e.g. for R/X = 2, 2 cycles and for R/X = 10, only one cycle.

Also, this number of cycles are also obtained to have 0.05 p.u. for the same B_R.

(2) Regarding the peak value of the first cycle, we remark that it has a limited value as R/X ratio increases.

(3) M.I.C. has the condition of $\theta_s = 0$ (i.e. desaturation region i.e. there is no inrush current) after 160 cycles when R/X = 0.09 and $B_R = B_m$, while it reaches this value after 16 cycles at R/X = 2, and 6 cycles only when R/X = 10.

This indicates, clearly, the considerable effect of inserting an external resistance in the transformer primary circuit.

2.2.2. Initial Residual Magnetizm:

The waveforms of M.I.C. are derived and shown in Fig.(2) for different residual flux densities of B_m, 0.8 B_m, 0,-0.8 B_m and $-B_m$. The instants of having 1 p.u., 0.05 p.u. and $\theta_s \neq 0$ are determined for any constraint as revealed in Fig.(2).

Also, lower values of B_R lead to reduced I_p of the first cycle which is valid also for the other cycles for the same instant. On the other hand, the rate of decay of M.I.C. is faster in the case of higher ones.

Researching for the cases of $B_R = -0.8\ B_m$, and $B_R = -B_m$, it can be concluded that there is no saturation phenomenon that is there is no M.I.C. flowing in these cases.

2.2.3. Material of transformer laminations:

This item concerns with the effect of the core material of the transformer upon the characteristics of the M.I.C. This is accomplished by selecting different values of saturation flux density (B_s). Under each of different constraints of B_R, the M.I.C. waveforms are deduced mathematically and plotted for various conditions of B_s as shown in Figs.(3) and (4). All these cases are researched for a unified ratio of $R/X = 0.09$.

These figures explain for each case the instant of having M.I.C. to be one and 0.05 p.u., i.e., it yields the number of cycles after which M.I.C. having these values measured from the switching-on instant.

The following features are noticed out of these characteristics:

(1) The number of cycles passed to have M.I.C. be one and 0.05 p.u. is the same independent of varying the type of core material. This conclusion is obtained under the restriction of having $B_R = 0.8\ B_m$.

(2) The peak value of the first cycle of M.I.C. is inversely proportional to B_s. Thus it is recommended to have a core material of having high B_s value as practically as it is possible (like stalloy,...).

(3) Eventually, by studing the influence of B_s on M.I.C waveforms, one can conclude that under the restrictions of $B_R = -0.8\ B_m$ and high value of B_s (1.2 B_m and 1.4 B_m), there is no saturation occurred which means zero M.I.C.

However, for a core material having $B_s = B_m$ and $B_R = -0.8 B_m$, the M.I.C. peak of the first cycle reaches a reasonable value of 2 p.u., and decays till it has 1 p.u. after 12 cycles.

2.2.4. Saturation angle (θ_s) (approximately switching angle)

θ_s is the angle at which saturation occurs and is given by (4) (see Fig.(5)).

$$\theta_s = \cos^{-1} (B_s - B_R - B_m)/B_m \qquad \text{......(9)}$$

The limit of saturation stage is assigned by having, $\theta_s = 0$, which is corresponding to zero M.I.C. (as concluded from Eq.(1) by putting $wt = \theta_{sn} = 0$).

Thus, by substituting this constraint in equation (9), we have,

$$(B_s - B_{R_L}) = 2 B_m \qquad \text{......(10)}$$

The effect of θ_s on the peak value of the first cycle M.I.C. is illustrated in Fig.(6) for different core materials, R/X ratios and initial residual flux densities.

3. EXPERIMENTAL VERIFICATION:

The goal of this section is to investigatein the laboratory-M.I.C. and explore to what extent different parameters influencing its characteristics. This is necessary to have more and better understanding of M.I.C. to counteract its effect on the false operation of the tripping circuit of the differential protection system respensible for the transformer.

One of these important factors is what is called the saturation or switching angle (θ_s) measured on the supply voltage waveform.

Thus, a certain means or switch must be proposed to control this parameter. In the following item, a solid state synchronous swich will be proposed using thyristors as its main elements.

3.1. Proposed synchronous switch:

Many circuits were proposed and examined to be used as a synchronous switch, (3) these circuits are shown in Figs. (7, 8, & 9). The third circuit shown in Fig.(9) is opted design which verifies more advantages than the others. It has been used in our experimental investigation. It, mainly, consists of:

(a) A triggering circuit.

(b) A slave thyristor S_2 with its triggering circuit.

(c) A main thyristor S_1.

The thyristor S_1 triggring circuit is based on the unijunction transistor. The output of the rectifier bridge D_1 is clipped by the zener diode Z_{33} at a constant voltage level V_z, and becomes similar to a battery of emf V_z volts. The zener diode voltage in synchronizid with the supply voltage. Then the unijunction transistor and pulse transformer (P.T.) generate the trigger pulse for operating S_1. The variable resistance R_3 is used as a controller resistance for firing S_1.

Whenever S_1 conducts, C_3 charges and maintains a sufficient charge long enough during the positive half cycle to assure the firing of S_2 regardless of the loads phase shift. This design makes it possible for S_2 to trigger at zero phase shift during the negative half cycle. Also, it gives a phase control from 15° to 170° during the positive half cycle and makes the second thyristor S_2 slave for the main thyristor S_1.

3.2. Test conditions, oscillograms and discussions:

Oscillograms of M.I.C. have been recorded at various constraints of switching instant, inserting an external resistor in the transformer primary circuit (i.e. different ratios of equivalent transformer resistance to equivalent transformer reactance), different transformer types and various its secondary load types.

The switching angle (θ_s) effect is examined solely keeping otner factors be fixed. The corresponding oscillograms are (O-1) and (O-2) for θ_s = 45° and 90° respectively with the same R_{ex} and the secondary to opened. These oscillograms display the sustained peak of the first cycle of M.I.C. which demonstrates that this peak value at 90° is considerably less than that of 45°. This conclusion coincides with that attained due to the theoritical analysis.

Oscillorgrams (O-3), (O-4) and (O-5) depict the sustained peak value of the first cycle of M.I.C. in addition to its decay with R_{ex} being disconnected for different θ_s nearly 90°, (at which the peak of the positive half cycle of the supply voltage occurs) better behaviour of M.I.C. is attained.

To assess the external resistance affect the oscillograms (O-1) and (O-3) for θ_s = 45° and (O-2) and (O-5) for θ_s = 90° can be studied and the following table shows at θ_s = 45° the peak value of M.I.C. as a ratio of that of no load value for various R_{ex}:

R_{ex}	0.00	43.00
M.I.C./I_{op}	13.33	11.11

which explains the slight effect of variations in R_{ex}.

The preceding oscillograms concern with a shell type transformer of 500 VA and 220/110 V and its secondary be opened.

As the secondary has been loaded it is expected that M.I.C. has different behaviour depending on the load type. Thus, with the aid of the synchronous switch, oscillograms (O-6), (O-7, (O-8 and (O-9) display the M.I.C. characteristics with all practically possible load types for constant θ_s (15°). The peak value of the first cycle is deduced and expressed in terms of the peak of no load current (I_{op}). The results are summarizid in the following table:

Load type	no-load	Resistive	inductive	capacitive
M.I.C./I_{op}	16.67	6.67	28.89	13.33
Load type	R-C in series			
M.I.C./I_{op}	11.11			

All these ratios are derived for $\theta_s = 15°$ which is relatively far from the preferable angle (90°). It can be seen that the M.I.C. for a resistive load has a lower ratio compared with other load type. This is, of course, preferable from the practical point of view-unfortunately, the worst condition belonging to this ratio occurs at the inductive load type which is practically, the most common one. Thus, it is recommended to have a reasenable peak for M.I.C. which consequently helps the discriminative subsystem of the protective system to avoid the false tripping.

With the auto-transformer type (750 VA, 220/110 V), the preceding steps of investigation for shell type are repeated. Thus, the switching angle (θ_s) effect is fully investigated and its corresponding oscillograms (O-10), (O-11) and (O-12) are shown. Numerically, the differentiation of peak value of M.I.C. of the first cycle between these cases is demonstrated as a ratio of peak of no load current as follows:

θ_s^o	15°	45°	90°
I_{p1}/I_{op}	72.02	52.00	12.00

It is unequivocal, that a considerable reduction of M.I.C. is achieved at $\theta_s = 90°$ at which its ratio reaches only about 16.7% of that at $\theta_s = 15°$.

Oscillograms (O-11), (O-13) and (O-14) explain the advantageous effect of inserting an exteranal resistance for auto-transformer than that for the shell-type.

The specified ratios of the M.I.C. peak are illustrated as

R_{ex}( )	0.00	21.50	43.00
I_{pl}/I_{op}	52.00	23.33	10.00

Thus, for auto type transformer, an appreciable reduction in M.I.C. peak value is obtained by inserting an external resistance. This reduction has an amount at 43.0 of one fifth of that at the case of having no resistor.

It is recommended to disconect an external resistor at the steady state condition to have the full applied voltage keeping it only at the switching-on instant.

4. CONCLUSIONS:

M.I.C. is a transient phenomenon which affects the correct and sensitive tripping of the differential protection for the power transformer. Thus,it has a stupendous attention for investigation.

This paper presents a deep and careful analytical and experimental investigation for M.I.C. in single phase power transformer.

General mathematical expressions are given for computing the characteristics of M.I.C. for any constraints including time, θ_s, R/X, B_R and B_s.

A proposed synchronous switch is designed and constructed for the experimental work. By the aid of this switch, switching angle could be controlled in a simple manner of this work. The significant conclusions can be stated as follows:

1. For both types of transformers, it is advantageous to have the switching instant to be occurred as close as possible to the instant of peak voltage ($\theta_s = 90°$). The M.I.C. peak at this instant reaches about 16.7% of that at $\theta_s = 15°$.

2. For shell-type, a slight effect, regarding M.I.C. is noted on changing R_{ex} by a consider-able amount. On the contrary, an appreciable reduction in M.I.C. is attained for auto-transformer, with specified values, it has, at $R_{ex} = 43$ one fifth of that at $R_{ex} = 0\,\Omega$. This is ascribed by the fact of having smaller impedance angle(λ) for the former than that of the auto one.

3. The switching inrush current (S.I.C.) with a resistive load has the smallest ratio to I_{op} in comparison. With other types. However, the worst condition occurs for the inductive one: it has 5 times of the S.I.C. at the resistive load.

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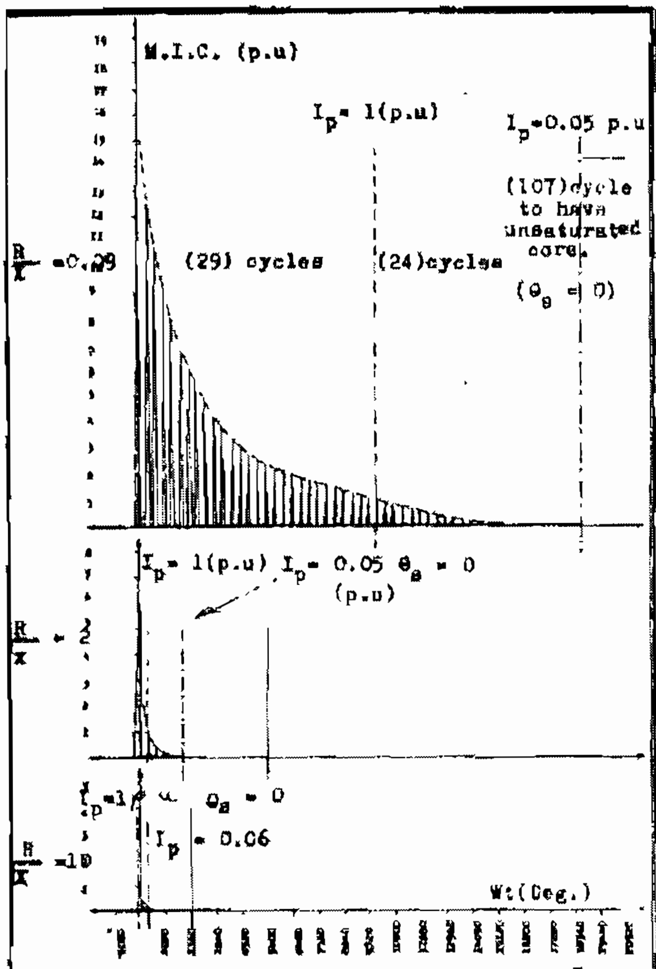


Fig.(1) M.I.C. waveforms for different $\frac{R}{X}$ ratios

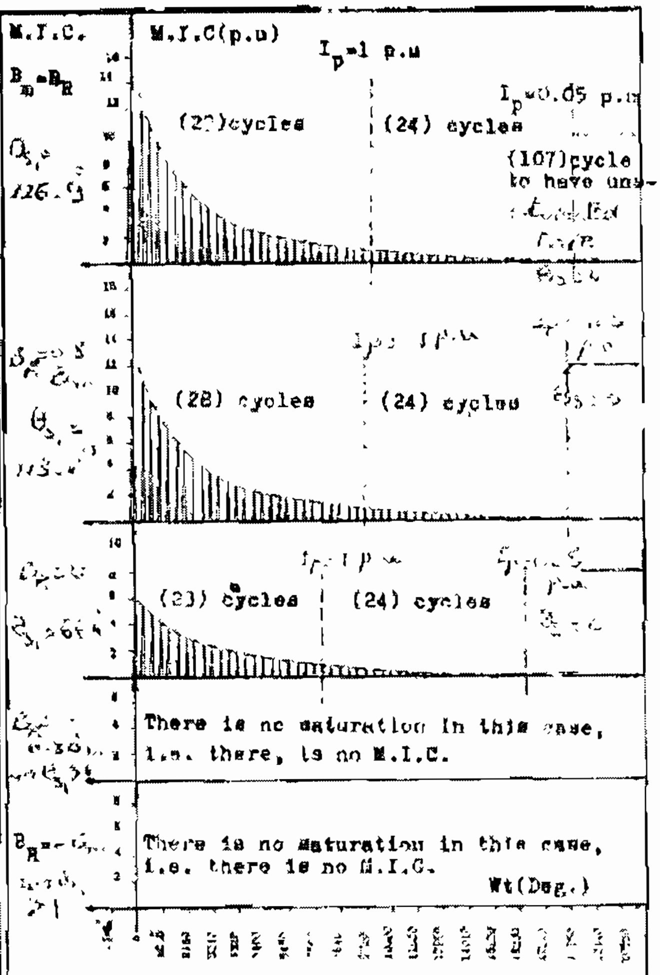


Fig.(2) M.I.C. waveform for various B_R, B_s = 1.4 B_m

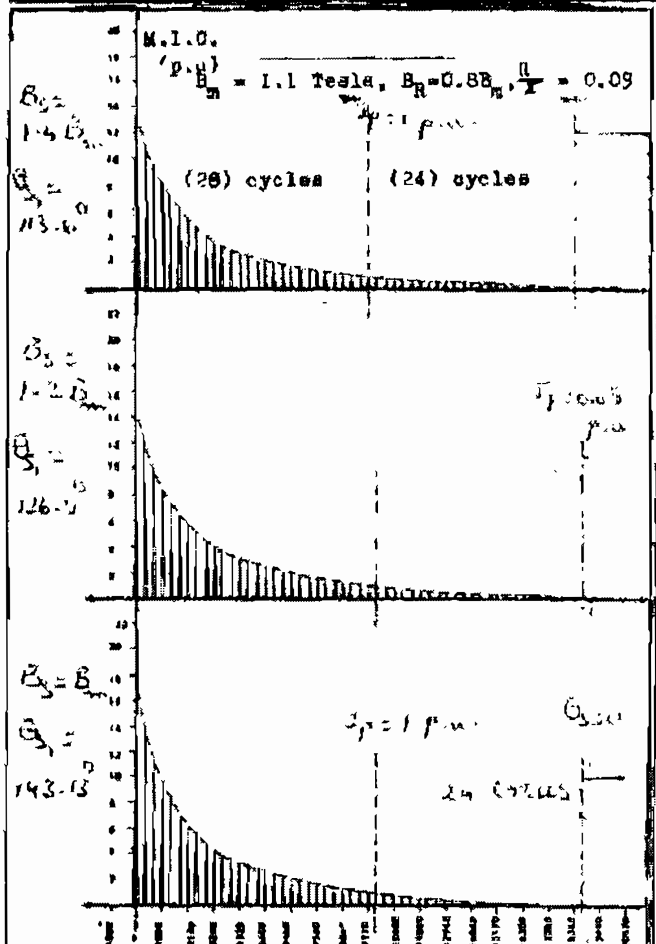


Fig.(3) The influence of the core material type of the transformer on M.I.C.

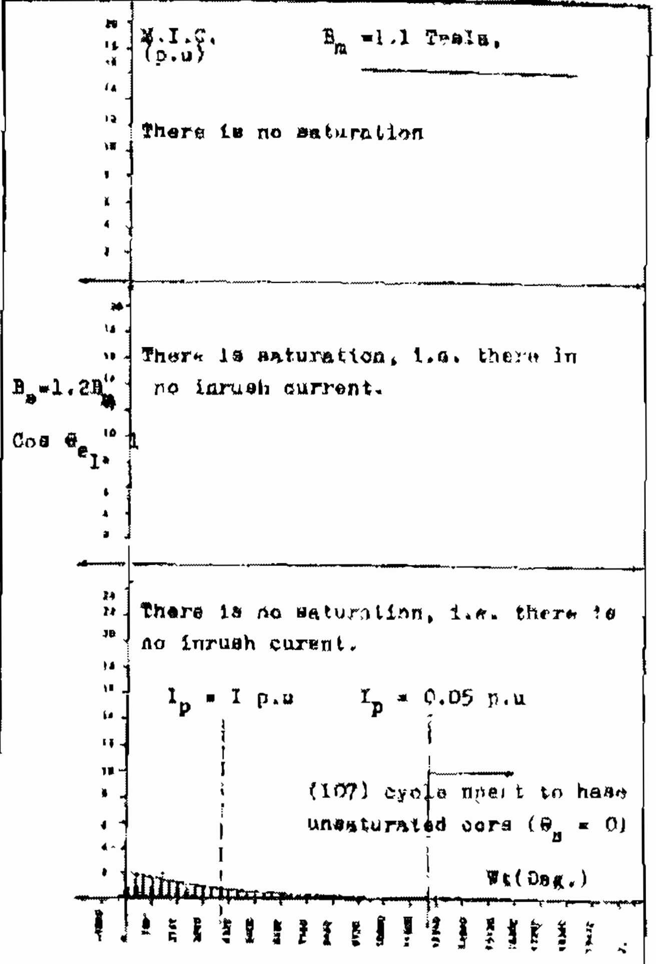


Fig.(4) The influence of core material type of the transformer on M.I.C.

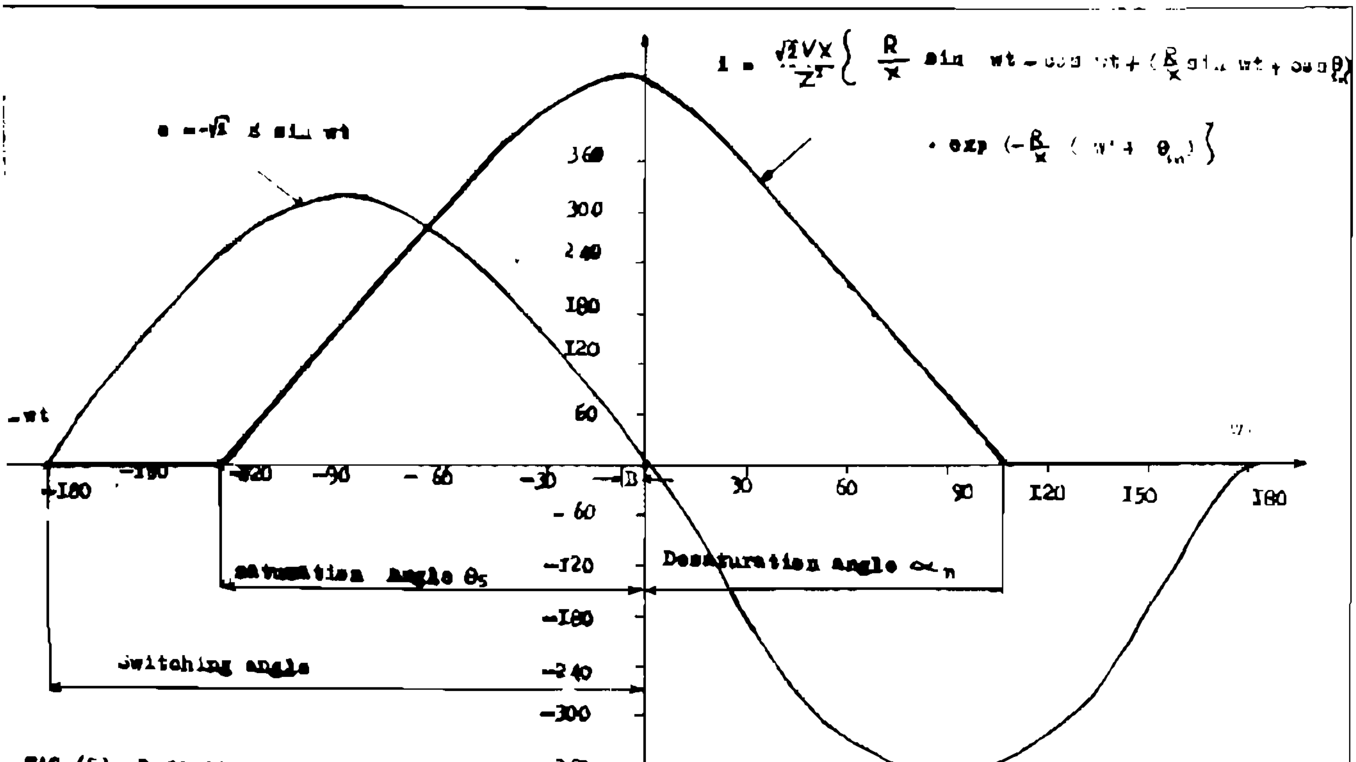
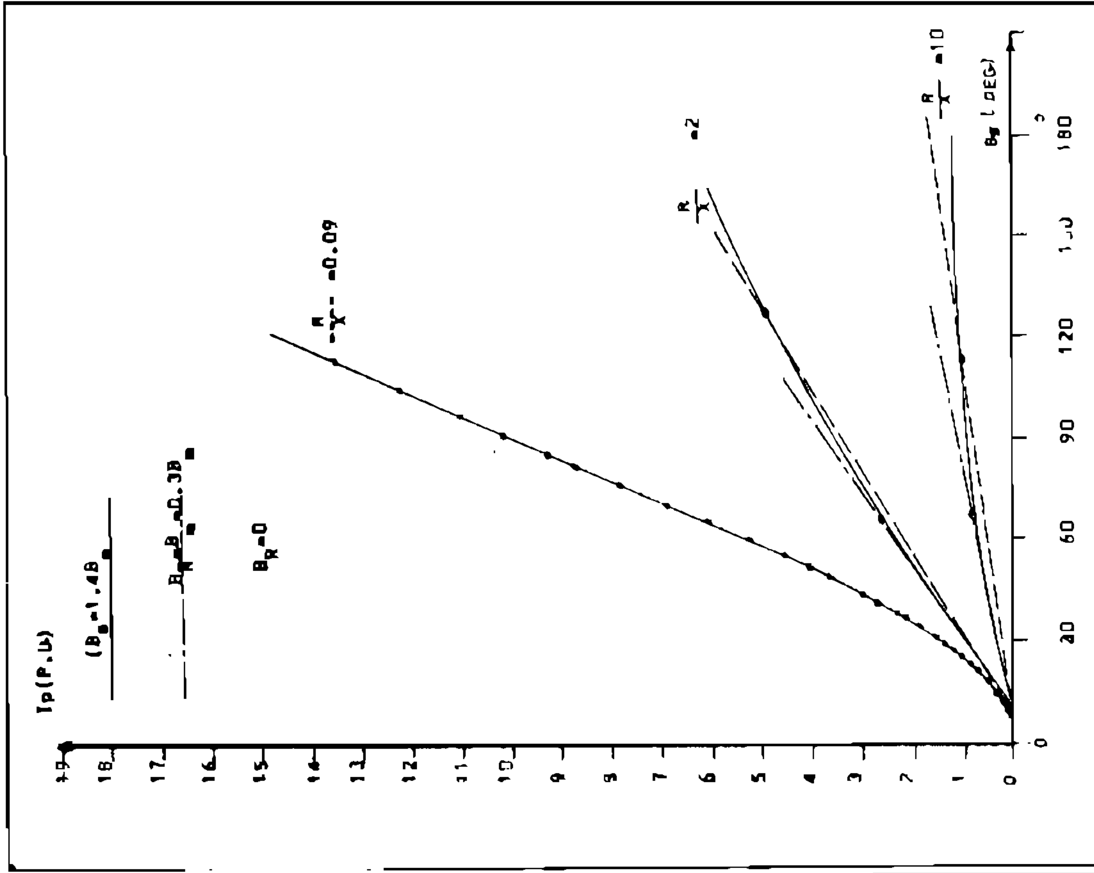


FIG.(5). Definition of Saturation and Switching angle.

Switching-on occurs at -180° on the supply voltage waveform

(practically Switching angle = Saturation angle θs)



Fig(6) Peak current (Ip) versus the saturation angle (θ_s).

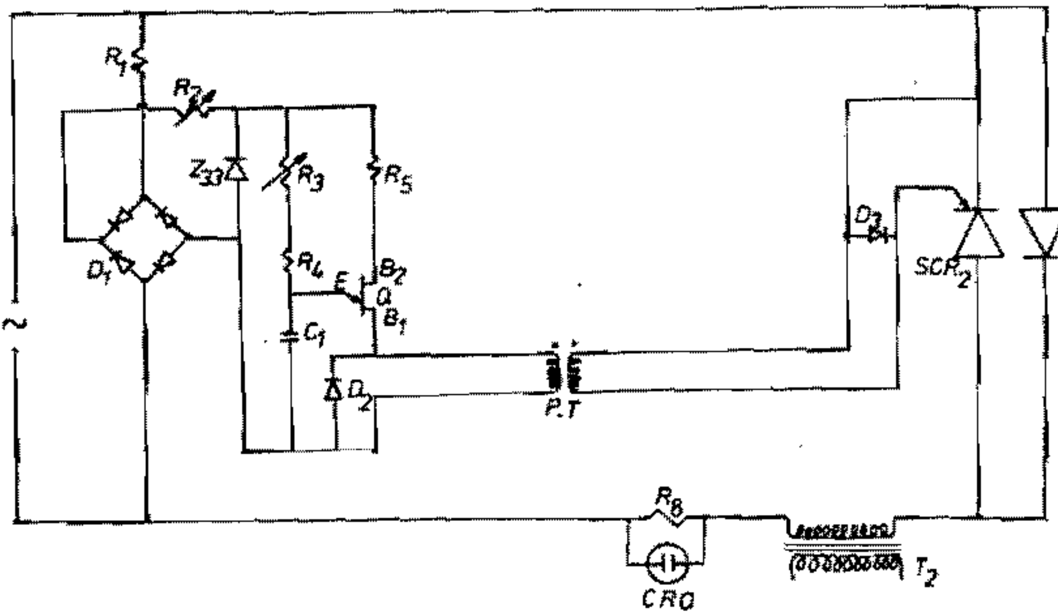


Fig. (8) Circuit 2 as a synchronous switch

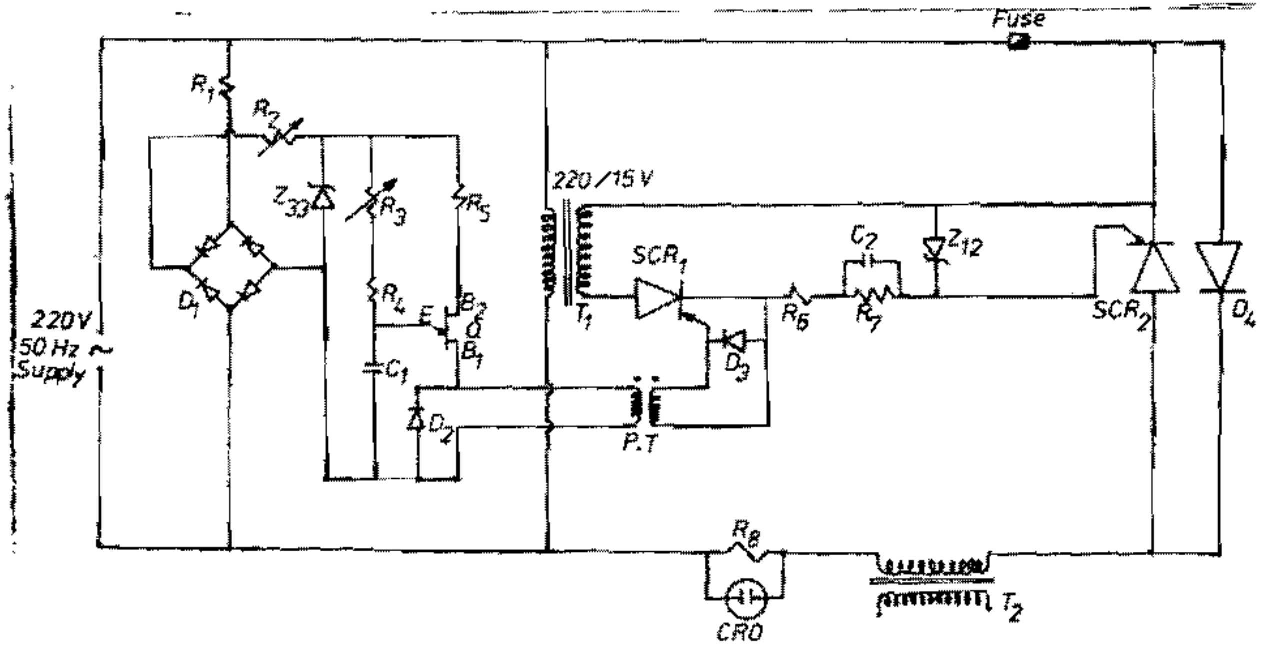


Fig. (7) Circuit 1 for a synchronous switch.

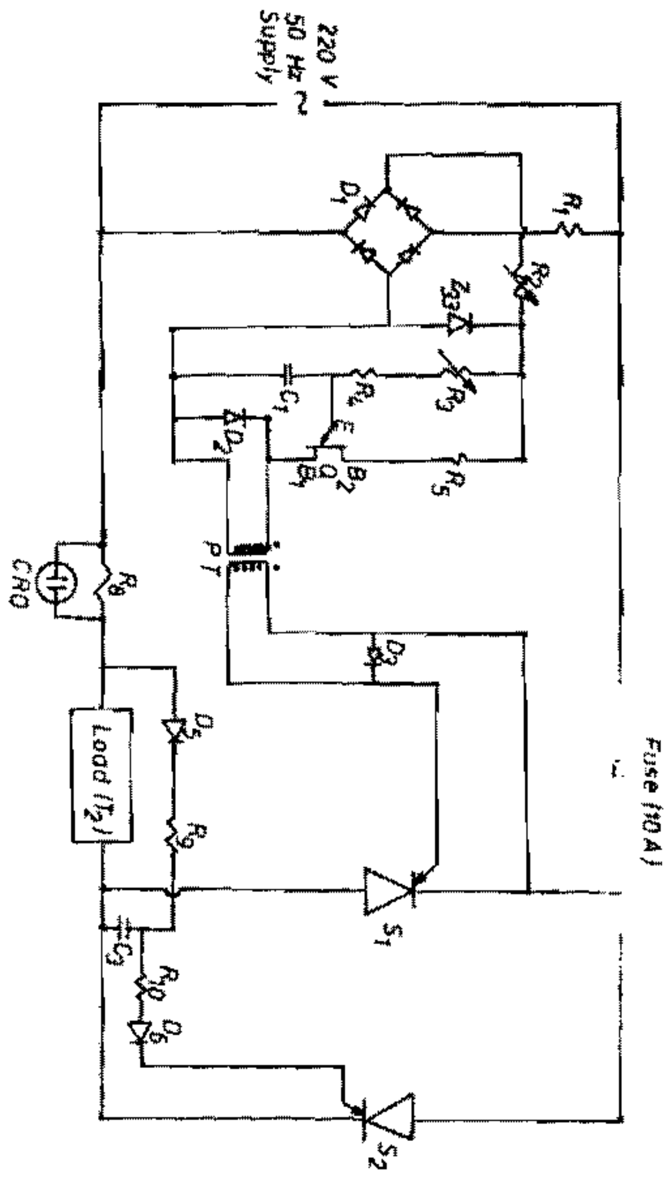
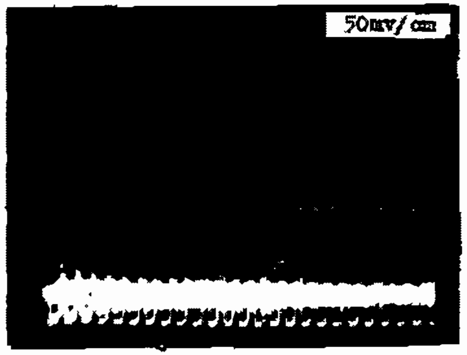
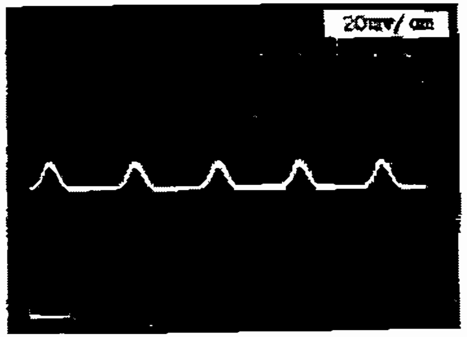


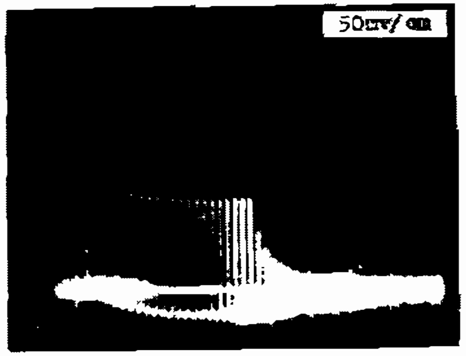
Fig. (9) Circuit 3 as a synchronous switch.



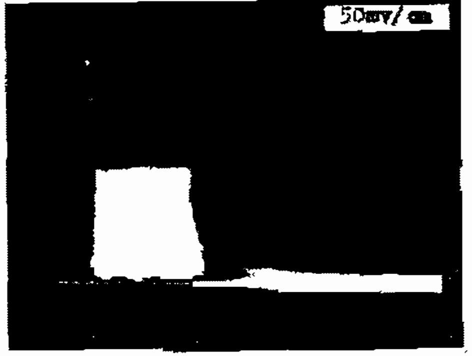
(0 - 1) Sustained peak of the first cycle of the M.I.C. (secondary - open) $R_{ex.}$ = 43 - ohm in series with the primary θ_s = 45°



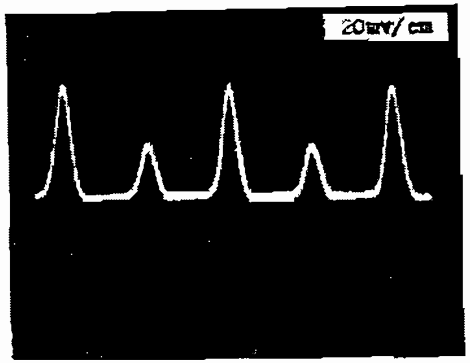
(0 - 2) Sustained peak of the first cycle of the M.I.C. with $R_{ex.}$ = 43 - ohm in series with primary. θ_s = 90° (slave thyristor not conduct)



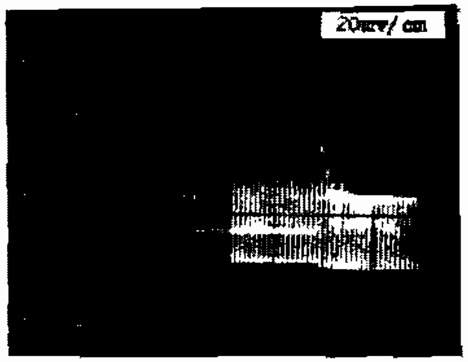
(0 - 3) Sustained peak of the first cycle of the M.I.C. and its decay, for θ_s = 45° (secondary - open) There is no any $R_{ex.}$ with the primary.



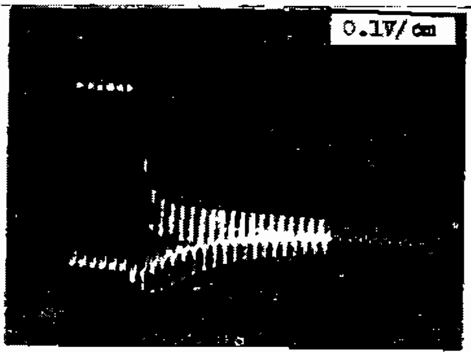
(0 - 4) Sustained peak of the first cycle of the M.I.C. and its decay, for θ_s =15° (secondary - open, without $R_{ex.}$)



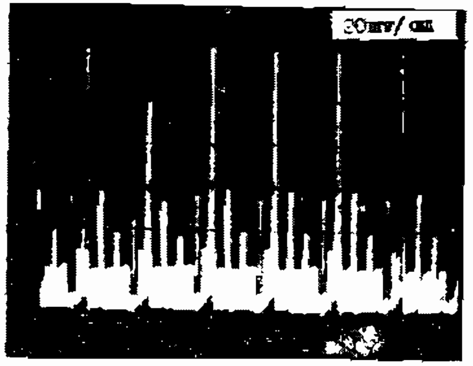
(0 - 5) Sustained peak of the first cycle of the M.I.C. (secondary - open) with $R_{ex.}$ = 21.5 ohm, θ_s = 90° (slave thyristor not -conduct)



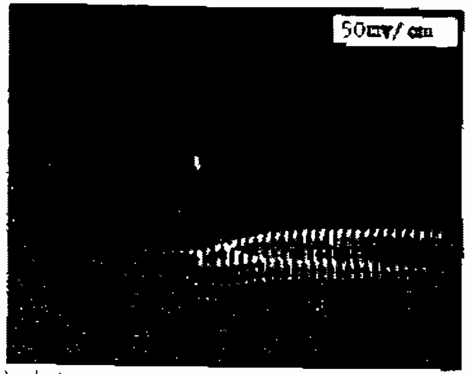
(0 - 6) Switching inrush current of the secondary resistive load. (R_{load} = 43 - ohm), θ_s = 15°



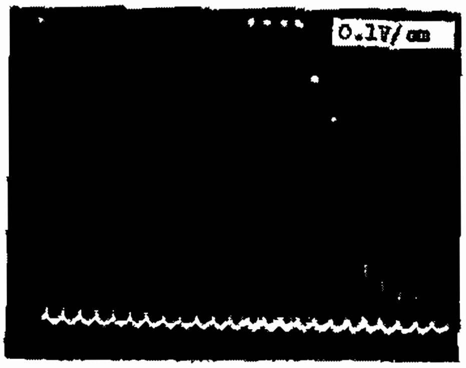
(0 - 7) Switching current of the secondary inductance load. θ_s = 15°



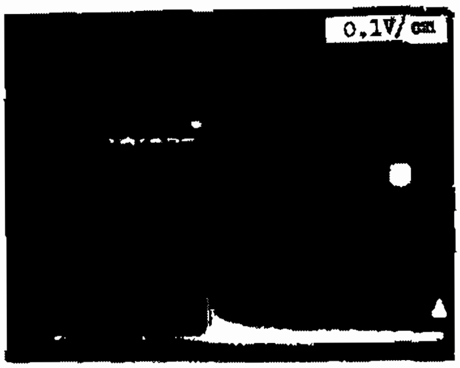
(0 - 8) Switching inrush current of the secondary capacitive load. θ_s = 15°



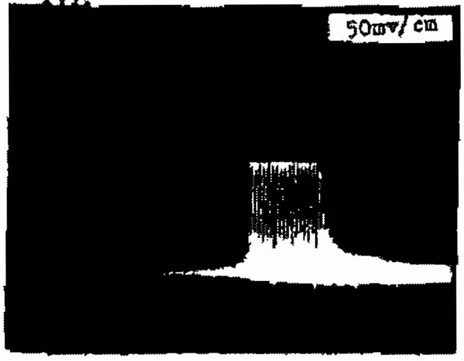
(0 - 9) Switching inrush current of the secondary (R - C)-in series - load. (R = 43 - ohm, C = [illegible] μF)



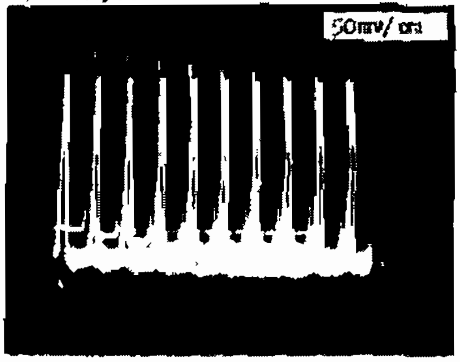
(O - 10) Sustained peak of the first cycle of the M.I.C. and its decay for θ_s = 15°. (without $R_{ex.}$ in series with the primary).



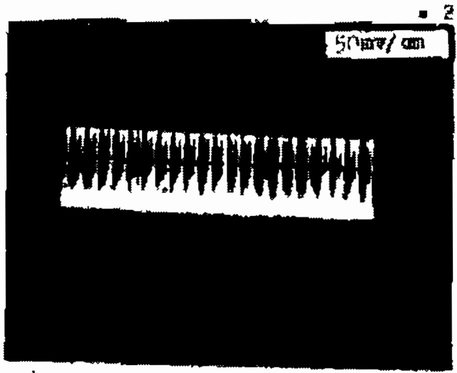
(O - 11) Sustained peak of the first cycle of the M.I.C. and its decay. θ_s = 45°. (without $R_{ex.}$ in series with the primary).



(O - 12) Sustained peak of the first cycle of the M.I.C. for θ_s = 90°, (without $R_{ex.}$)



(O - 13) Sustained peak of the first cycle of the M.I.C. θ_s = 45° ($R_{ex.}$ in series with the primary = 21.5 - ohm).



(O - 14) Sustained peak of the first cycle of the M.I.C. θ_s = 45° ($R_{ex.}$ in series with the primary = 43 - ohm).