MINIMUM WEIGHT DESIGN OF PITCHED ROOF STEEL FRAMES

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(Received Oct. 18, 1987, accepted Dec. 1987)

الخلاصـــــة :

تختص هذه الدراسة بموضوع التصميم المرن الأمثل للاطارات المعدنيـــة ذات الاسطح المائلة ولقد تحقق تطور فى موضوع التصميم الانشائى عن طريق استعمـــال البرمجة الرياضية ولذلك فان المدخل الطبيعى للتصميم الانشائى الأمثل يتم عن طريق تحويل مسألة التصميم الانشائى لمسألة برمجة رياضية قابلة للحل .

استخدمت فى هذه الدراسة طريقة التكرار للتعبير عن القيود المطلوبة عـــن طريق استخدام الحد الاول من ملكوك تيلور للدوال . والغرض من هذه الدراسة هـــو تعيين أمثل مساحات لقطاعات أعضاء المنشأ المختلفة مع الاخذ فى الاعتبار قيـــود الاجهادات والترخيم مع تقديم منحنيات تصميمية لاطارات ذات أسطح مائلة .

ABSTRACT

The optimum elastic design of pitched roof frames has been studied . An iterative method which uses Taylor's first order series expansion to express the various required constraints is employed. The function of the problem is linear in the design variables, which enables the simplex linear programming algorithm to be used to solve the problem.

An example of rectangular hall has been investigated. The hall is covered by pitched roof frames with different spans, spacing and slopes.

Stress and deflection limitations are taken into consideration and the optimum cross-sectional area of the various members of the structure has been obtained.

Design charts to determine the minimum weight of pitched roof frames, with different spans, spacing and slopes, covering a rectangular hall are given.

1. INTRODUCTION

The minimum weight design is one of the interesting research fields during the past two decades. The usual approach to structural optimization is to transfar the structural design problem to mathematical optimization problem. The concept of mathematical optimization is to seek the maximum or the minimum for a function that has a number of variables while satisfying the imposed constraints. Structural optimization seeks the selection of the best design variables for a structure with certain geometry and under a certain loading condition to achieve the objective of minimum weight and to satisfy the limits placed on the behaviour of the structure.

The optimum design of pitched roof frames with deflection limitation imposed by BS 449 has been studied by Elliot et al (1) using non-linear programming. They consider two cases of loading and obtain a non-linear inequalitites in the design variables for the deflection constraints . The dynamic search method has been used to solve the problem.

In 1974 Majid (2) used the graphical technique of the non-linear programming for solving simple frames. The constraint vector consists of the stiffness, stress and deflection constraint. This graphical technique is limited to problems with only two variables.

Anderson and Salter (3) proved that the non-linear programming used by Elliott (1) may locate a local optimum more than a global one. In their analysis, they considered plane steel frames under deflection constraints. Using the first order Taylor series expansion to express this

deflection, design charts for fixed base pitched roof frames have been obtained.

Katkuda (4) used the penalty function of 1967 coded version of SUMT. The SUMT algorithm has been developed to solve non-linear functions of independent variables but the equality constraints must be linear function if convergence to the solution is to be guaranteed (5).

El-Desoky (6, 7) studied the linear optimum design of single-bay steel frame. An iterative method based on Taylor's first order series expansion to express the various required constraints has been used.

2. STRUCTURAL OPTIMIZATION

The continuous solution of the optimum design problem assumes that the field of the available member size is continuous. In practice, the choice of member size is always limited to a discrete set. One way of overcoming this difficulty is to round off the resulting design to those of the available sections. But this operation increases the cross-sectional area of all the elements and lead to a design which sometimes is not the optimal. The discrete optimum design (2, 4, 8, 9) may reduces the section of some of the members while increases the others which may lead to a better solution.

The iterative method used to determine the optimum elastic design of pitched roof steel frame is based on stress and deflection constraints. The design constraints are formulated by using the first order Taylor expansion of functions (6, 7, 10, 11). In the present analysis the continuous solution is used, then the results are modified to a discrete set of member sizes.

3. RESULTS AND DISCUSSION

The optimum weight of pitched roof steel frames (Fig.1) covering a rectangular hall L x 8 ms has been obtained. The roof slope (α) , spans of frames (L), and spacing between them (S) are the main variables. The effect of vertical deflections and horizontal sway on the design of frames are considered.

The span (L) are chosen from 12.0 ms to 32.0 ms with 4.0 ms step and the clear height of column (H) assumed to be constant and equal 6.0 ms. The spacing between the frames (S) varies from 4.0 ms to 10.0 ms. The roof slope (θ) ranges from 0.05 to 0.50 in a 0.05 step. The covering material is corrugated steel sheets supported on steel purlins spaced 2.0 ms apart.

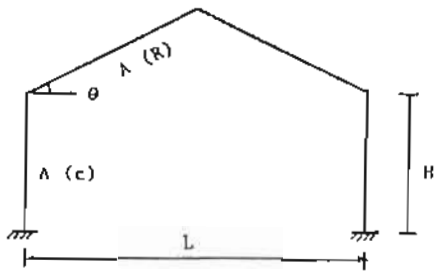

Fig. 1. Pitched roof frame.

The applied loads considered in the analysis are :

1. Dead load from own weight of steel structure (=25 kg/m²) and from roof covering (=15 kg/m²).
2. Live load are taken according to ESS for inaccessible roofs.
3. Wind loads for an inclined roof by angle θ with the horizontal.

For windward side = $75(1.2 \sin\theta - 0.4)$ kg/m²
For leeward side = 30 kg/m²

The frame is divided into two groups of members. The first group contains the columns and the second group contains the rafters. The material of construction is normal mild steel (Steel 37). The modulu s of elasticity is assumed to be 2100 t/cm² and the ultimate strength is equal to 3700 kg/cm².

The optimum design of frames requires the determination of their optimum slopes and optimum spacing to obtain the optimum cross-sectional areas of the various groups of members and to satisfy both the stress and deflection constraints. The maximum vertical deflection is restricted to L/360 while the horizontal sway is restricted to H/325 where L and H are the span and clear height respectively. The stress in any section of the frame is limited to the permissible compressive stress.

Two cases of loading, Case A (dead load and live load) and Case B (dead load, live load and wind load), have been considered. The load Case A is used as the lower bound for the load Case B to ensure that both load conditions are satisfied.

The relations between the optimum cross-sectional areas (columns and rafters) and the slope of pitched roof frame are shown in Fig. 2. It is clear that the area of cross section for rafter - A (R) - decreases with the increase in the roof slopes while the area of cross section of column A (C) decreases till a minimum value at certain slopes, then it increases with the decreases in slope. This is because as the slope increases the behaviour of the pitched roof frame becomes closer to the arch action. At a certain slope the moment at the fixed base of the column starts to increase rapidly as the slope increases as shown in Fig. 3 and thus the optimum area of the cross section of column, A (C), increases. By knowing span, slope and spacing between frames, the cross sectional area of column and rafter can be obtained.

Figure 4 shows the relation between the optimum weight of steel frame (rafter and columns) and spacing between them . For all roof slopes and different spans this relation is approximately linear. As the spacing between frames increases, the weight of steel increases but the rate of increase in spacing is bigger than the rate of increase in steel weight . By increasing the frame span, these two rates become closer. .

Figure 5 gives the relationship between the optimum weight of steel

frame and slopes of the pitched roof while in Fig. 6 the same relation is shown for the weight of steel per square meter. From these figures, it is clear that the optimum slope depends on the span and the spacing between frames. The optimum slopes vary between 0.1 - 0.20, 0.20 - 0.30, and 0.25 - 0.40 for spans 12, 16 and 20 ms respectively. As the spacing increases the optimum slope increases and for span 20.0 ms and more, the wight of steel frame decreases as the slope increases. The relations between the optimum slopes and spans are shown in Fig. 7.

4. CONCLUSIONS

Minimum weight design of pitched roof steel frames has been studied . Design charts which give the optimum cross-sectional area of column and rafter for a wide range of spans, slopes and spacing between frames have been obtained. From these results, it is clear that :

1. The optimum slope for the optimum design of pitched roof frame increases with the increase of the spacing and/or the span.
2. In general,Case A of loading governs the design of rafters while the design of columns is governed by Case B of loading at large slopes and Case A of loading at small slopes.
3. Increasing the slope, the optimum cross sectional area of rafter decreases while that of column decreases up to a certain slope after which the latter starts to increase.
4. There is a linear relation between the optimum weight of steel farmes and the spacing between them.

ACKNOWLEDGMENTS

The authors are gratefully acknowledged to Prof. A. Mahdy, Faculty of Engineering, Zagazig University, for his encouragement during this investigation.

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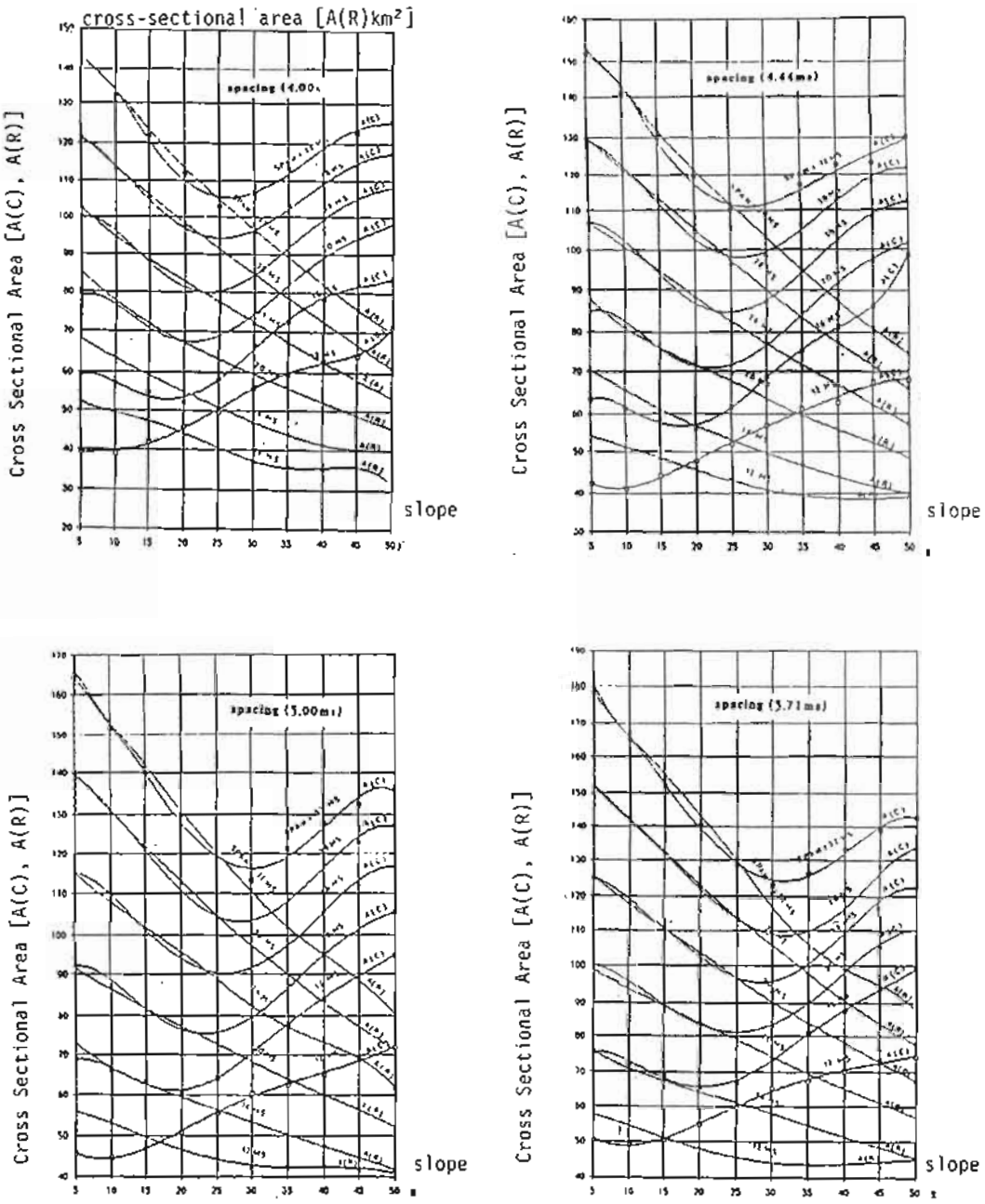


Figure 2. (Cont...)

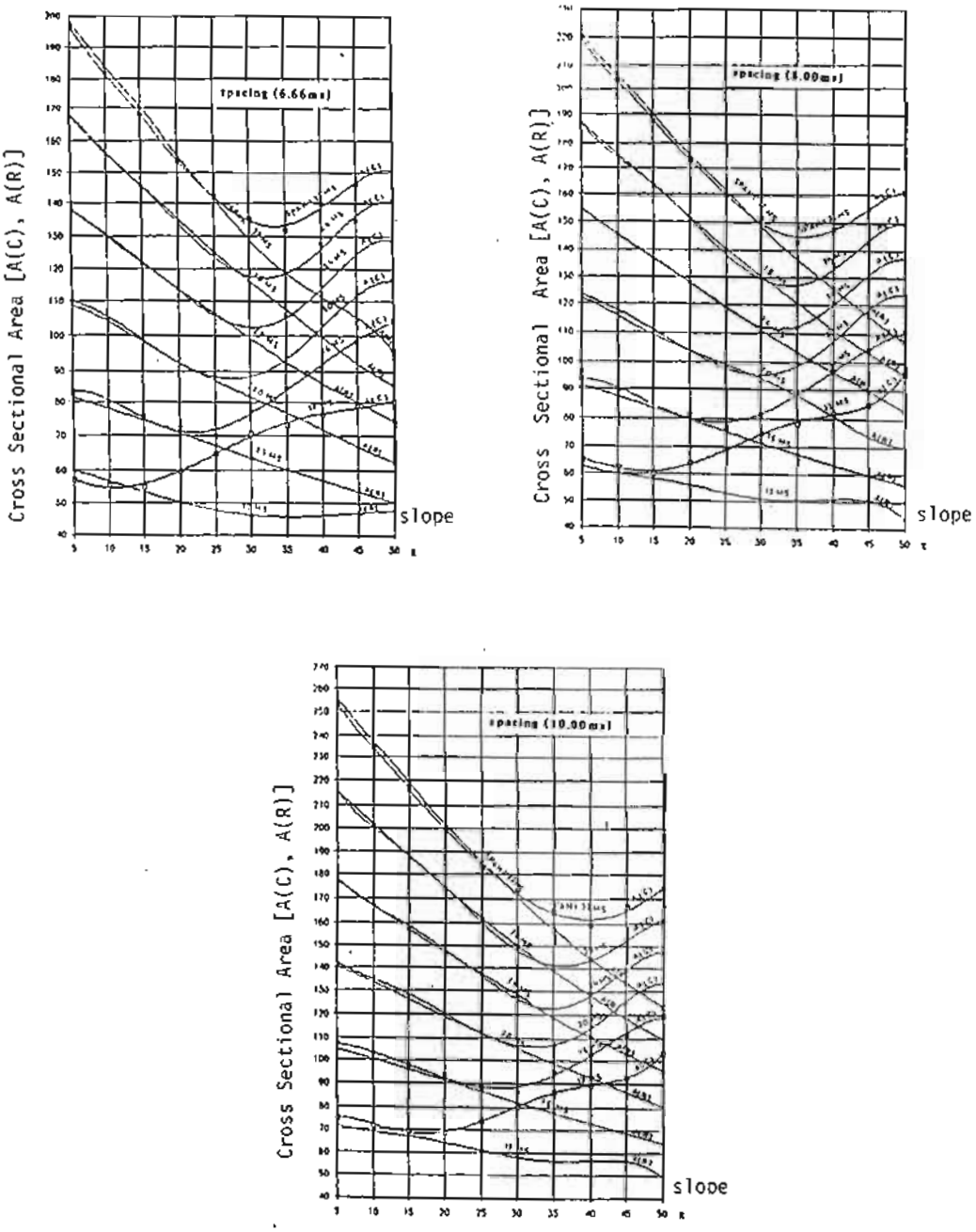


Figure 2. Reltaion between the optimum cross sectional area (Column and Rafter) and slopes of pitched roof frame.

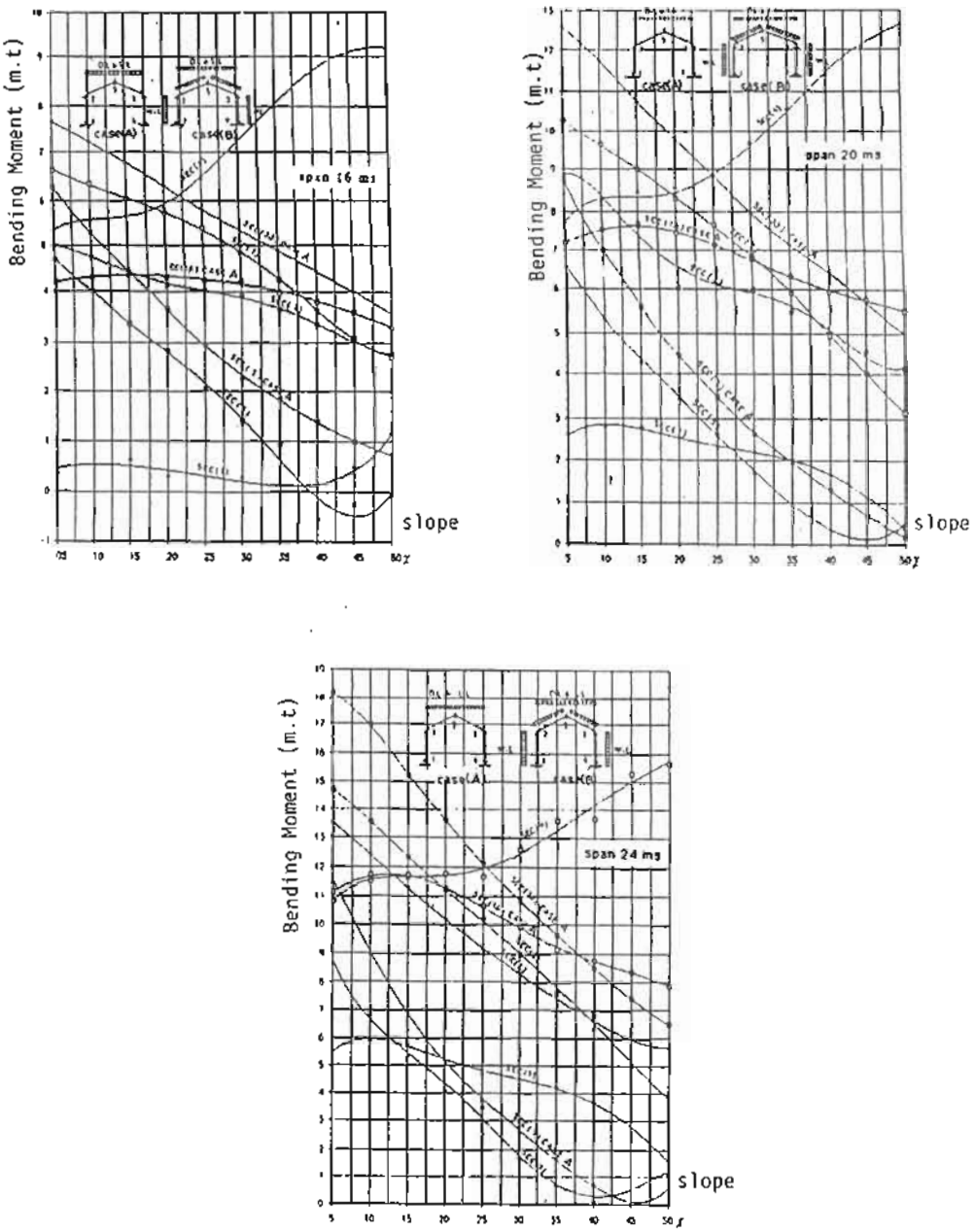
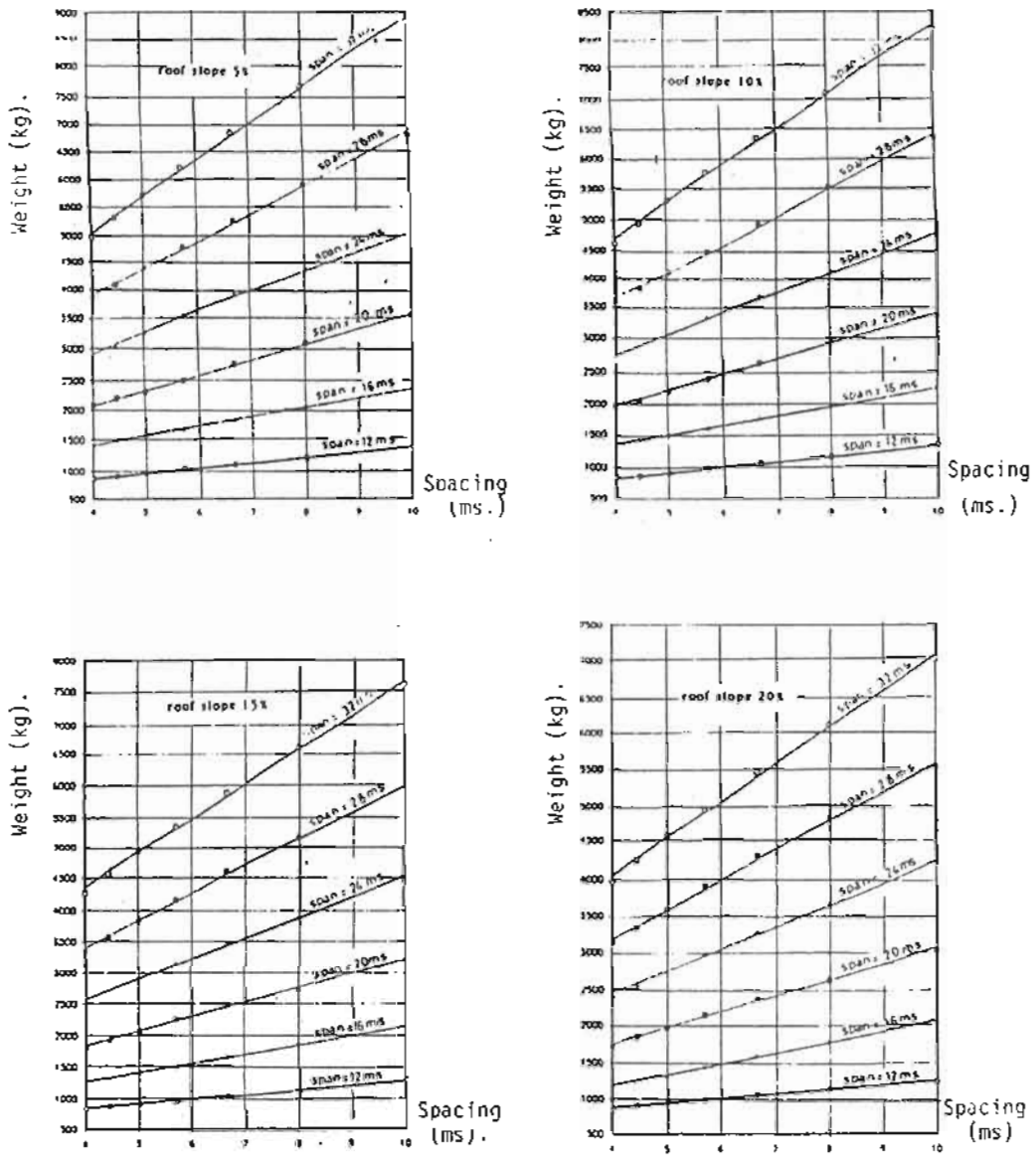


Figure 3. Bending moment at critical sections of pitched roof frame with spacing 4.0 ms.



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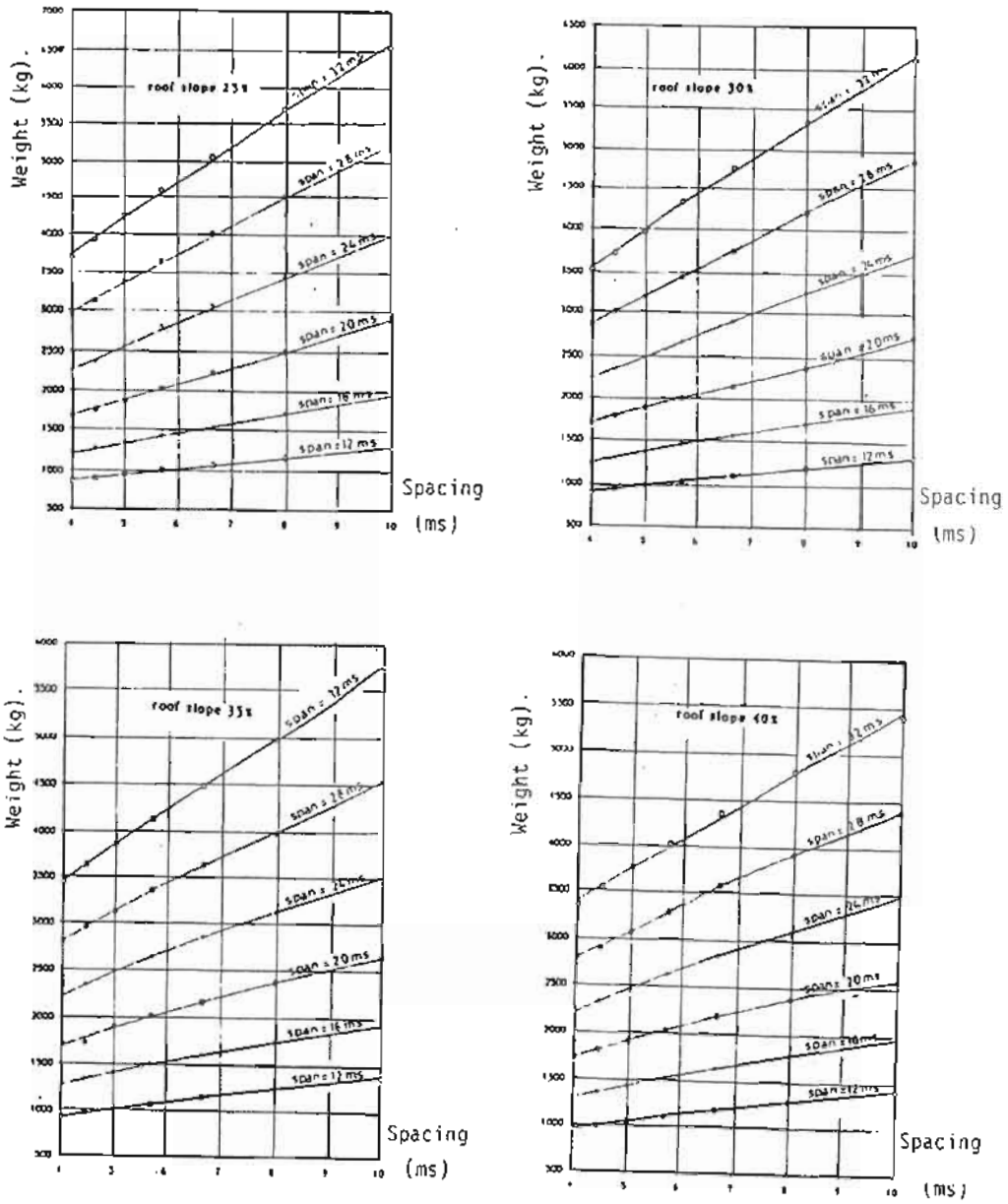

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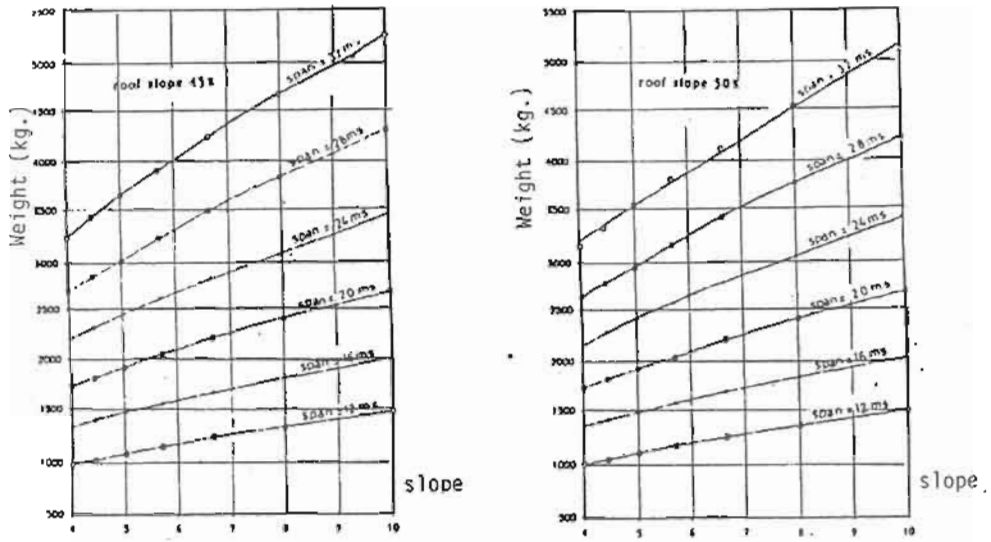
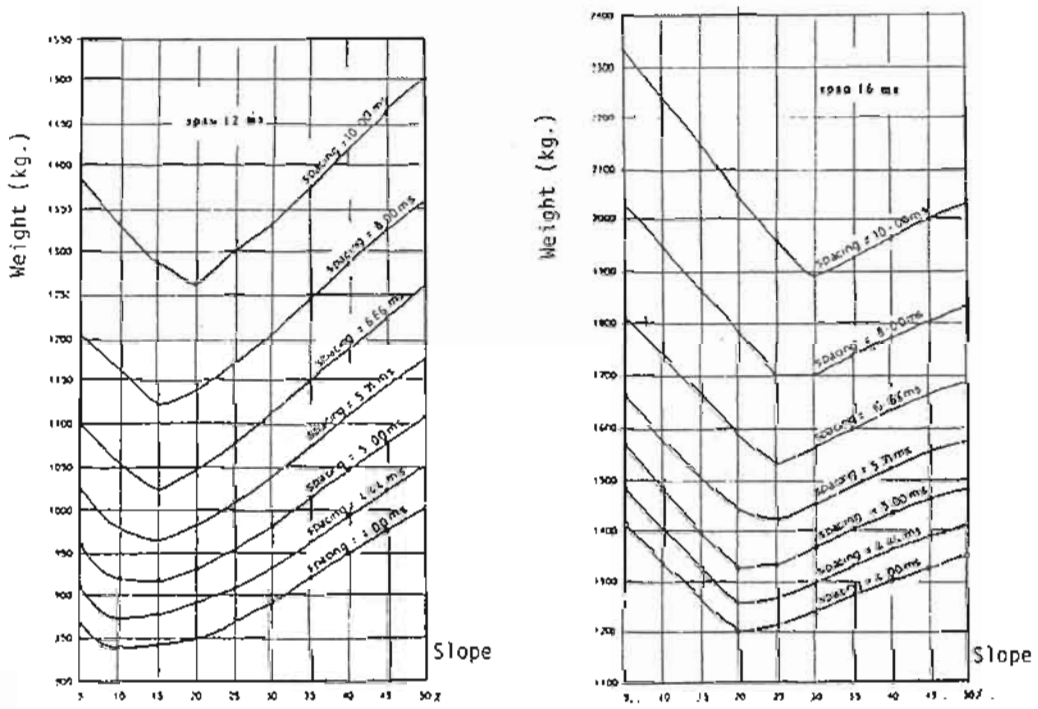


Figure 4. Relation between optimum weight of steel frame and spacing between frames.



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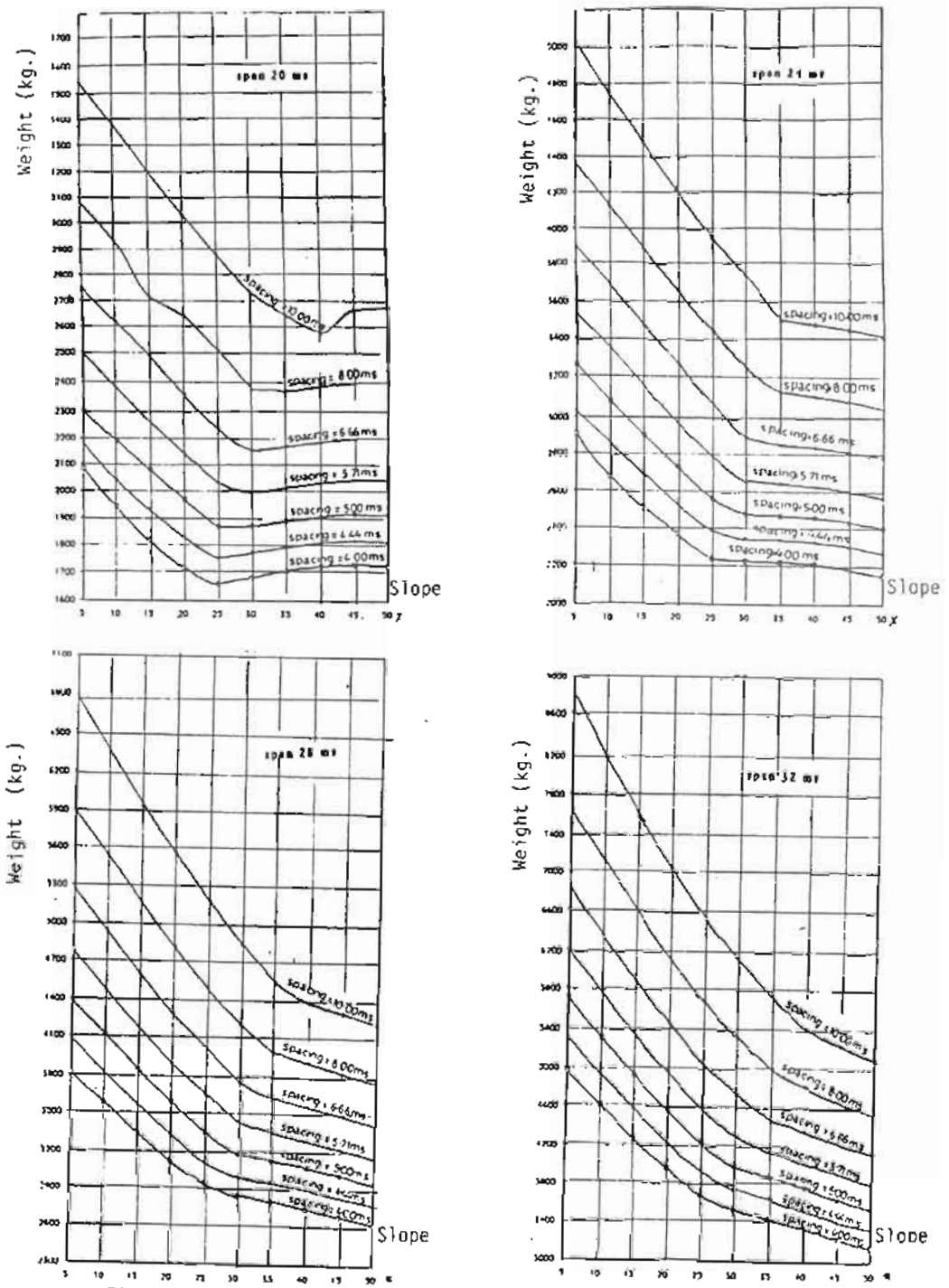

Figure 5. Relation between optimum weight and slops of pitched farme.

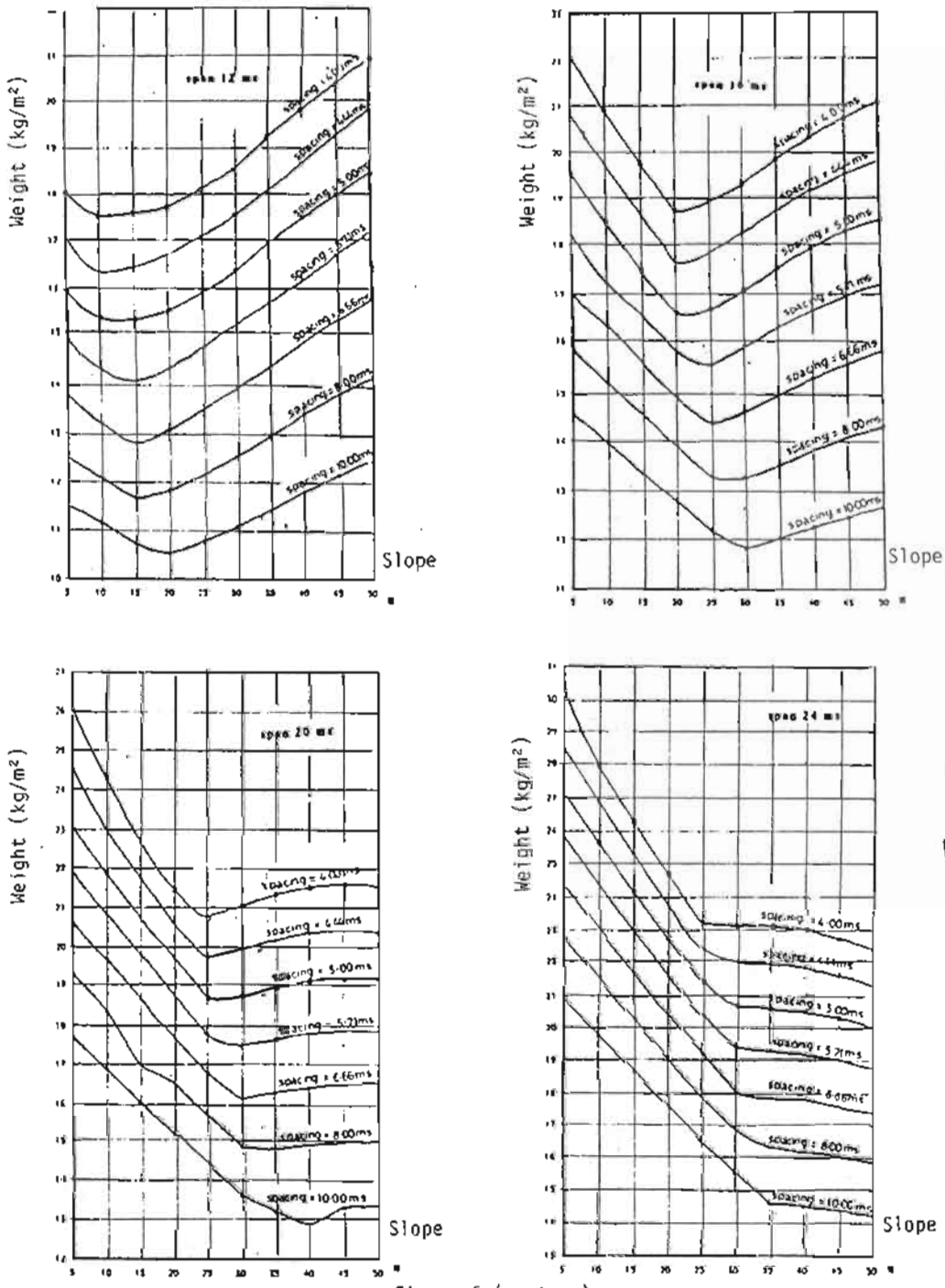

Figure 6.(cont...)

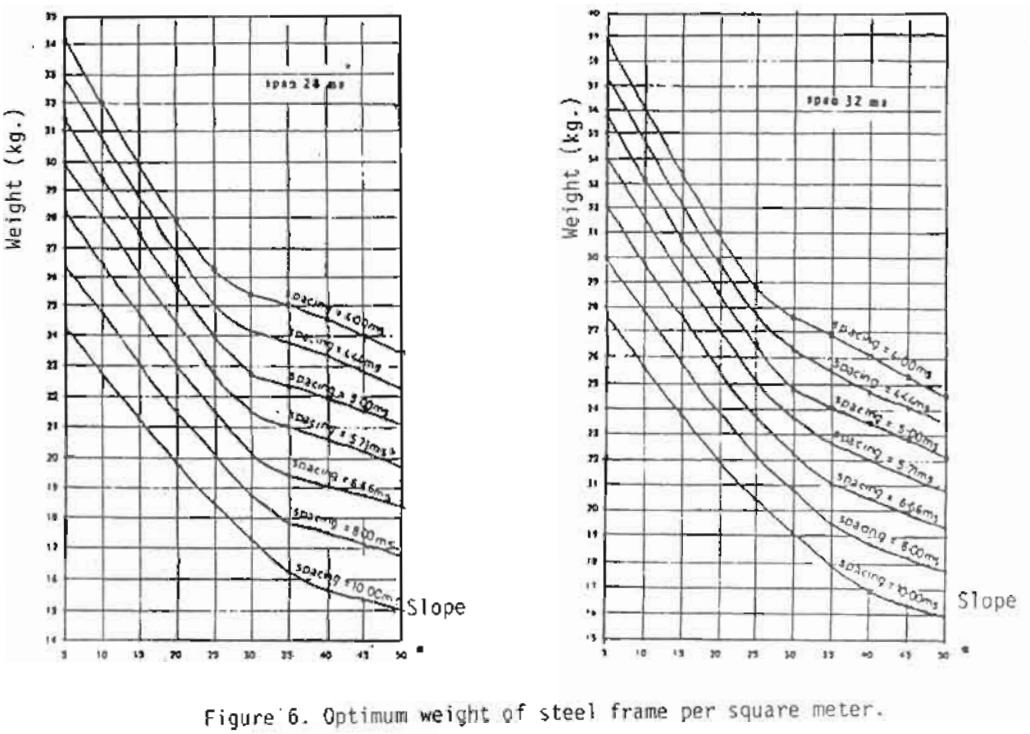

Figure 6. Optimum weight of steel frame per square meter.

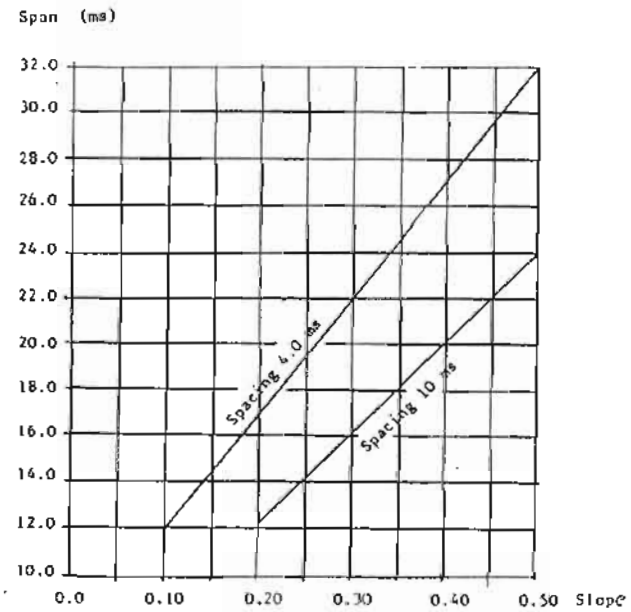

Figure 7. Relation between optimum slopes and spans.