Mansoura Engineering Journal Vol. 19, No.1, March, 1994

Natural Flow Convection In A Vertical Multilayered Porous Media With Varying Permeabilities (Part 2: Heat Transfer)

سريان الحمل الطبيعي في وسط مسامي رأسي متعدد الطبقات ذو درجات نفاذيه مختلفه (الجزء الثاني: انتقال الحراره)

M. S. EL KADY

Mechanical Engineering Department
Mansoura University, Egypt

خلاصـــــــــــه:

يعرض هذا البحث دراسه عدديه لخاصيه إنتقال الحراره بالحمل الطبيعي وذلك في وسط مسامي ثنائي البعد متعدد الطبقات الرأسيه وذو درجات نفاذيه مختلفه ومعرض لفرق في درجات الحراره بين الجدارين الرأسيين. وقد أهتم هذا الجزء بدراسه تأثير عدم تساوى درجات النفاذيه للطبقات الرأسيه ونسبه البعد للطبقه الوسطى لوسط ثلاثي الطبقات على الانماط المختلفه لانتقال الحراره بالحمل. الطبقه الاولى والثالثه لهما نفس العرض ودرجه النفاذيه، بينما يتراوح مدى التغير في نسبه درجه النفاذيه للطبقه الوسطى الى الطبقات الاخرى $K_r<10$ $0,1<$ ونسبه بعدها الى العرض الكلى $0 < W_r < 1$. وذلك لنسبه ألابعاد (ارتفاع/عرض) للوسط المسامي $A=3$. وقد أجريت مقارنات مع النتائج التي حصل عليها لوريات وبراساد [13] أظهرت تطابقا جيدا وأثبتت صحه هذا النموذج. وقد أظهرت النتائج أن خواص لإنتقال الحراره تعتمد على كل من W_r و K_r وذلك بالاضافه الى الاعتماد المعروف مسبقا على النسبه الباعيه للاطوال وعدد رايلي. وقد اظهر التغير في عدد رايلي ثلاثه أنماط مختلفه لانتقال الحراره وهي أنتقال الحراره بالتوصيل، نمط أنتقال الحراره خلال الطبقه الحديه وكذلك النمط الانتقالي بينهما. وقد استنتجت علاقه عدديه تعبر عن تأثير كل من نسبه درجه النفاذيه K_r ونسبه البعد للطبقه الوسطى W_r على عدد نوسيلت Nu وذلك في نمط أنتقال الحراره بالحمل من خلال الطبقه الحديه في الحاله المدروسه كما يلي:

$$Nu = C.(\pm 0.3836 W_r + 1).(0.0454 K_r + 0.9546) Ra^{0.5}$$

العلامه الموجبه عندما تكون $K_r > 1$ والعلامه السالبه في حاله $K_r < 1$ والثابت C يعتمد على نسبه الابعاد للوسط المسامي.

ABSTRACT

This paper outlined numerically a solution for the phenomenon of heat flow by natural convection in a two dimensional multilayered vertical porous *medium heated from one side with vertical isothermal walls and insulated horizontal walls. The study is concentrated on the effect of changing both the permeability ratio K_r and width ratio W_r of an inner sublayer in a three layered porous medium on the different regimes of the heat flow. Numerical results are reported for the range of $0<W_r<1$, $0.1<K_r<10$ and $0<Ra<6000$ and for an Aspect ratio $A=3$. It is found that the heat transfer characteristics depend on both K_r and W_r besides the known dependence on the Ra and the aspect ratio. Three regimes of heat flow is obtained by increasing Rayleigh number, conduction, transient and boundary layer regime. A numerical correlation expresses the effect of both W_r and K_r on Nu is derived for the boundary layer regime for this paricular case as follows:*

$$Nu = C\,.(\pm 0.3836\, W_r + 1).(0.0454\, K_r + 0.9546)\, Ra^{0.5}$$

where the +ve is for the case of $K_r>1$, the -ve is for the case of $K_r<1$ and C is constant depends on the Aspect ratio.

1. INTRODUCTION

Recently considerable attention has been focused upon problems involving heat transfer in porous media. In many of these systems, such as building insulation, geothermal sources, and pebble bed thermal storage units, the mode of heat transfer is natural convection. The majority of existing studies were concerned with the natural convection in a homogeneous porous medium, which may not always be a good model for the physical configuration. A better model is to consider the non-homogenuous case. To handle the non-homogenous porous medium case, a model is assumed in the present work, in which the porous medium s composed of three vertical porous layers of different permeabilities, with vertical isothermal walls at different temperatures and adiabatic top and bottom walls. Considerable work has been performed on natural convection in an isotropic porous layer. An overview of this research is provided in the first part of this study [1]. The boundary layer regime for convection in the porous media was attractive for some researchers. One of the earlier theoretical studies for the boundary layer regime for convection in a vertical porous ayer was done by Weber [2]. He showed that when the temperature difference between the walls, or equivalently, the Rayleigh number, is sufficiently increased, the basic flow exhibits boundary layer character. Tong and Subramanian [3] have solved the boundary layer equaion derived from the Brinkman's extended model using the Weber's approach.

The purpose of this study is to analyse and understand the phenomenon of the convective heat transfer in a vertical layered porous media with vertical isothermal walls at different temperatures, under the effect of the inhomogenity of the porous medium, and determine the different regimes of convective heat transfer, specially the boundary layer regime.

2. MATHEMATICAL MODEL AND SOLUTION PROCEDURE

Consider a two-dimensional rectangular vertical cavity (shown in Fig. 1) of width W and height H, filled with multilayered porous medium. Each layer is homogenuous, isotropioc and has constant permeability K. The porous medium is saturated with a single phase fluid of density ρ and viscosity μ. In Fig.1 T_H and T_C represent the hot and cold vertical walls of the cavity respectively while the horizontal top and bottom walls are insulated. The effect of both the drag and inertia are neglected and the flow will obey Darcy's law. The fluid properties are assumed constant except for the density change with temperature which gives rise to the buoyancy force. This is treated by invoking the Boussinesq approximation. While the permeability values K of the porous layers are different, the values of the thermal diffusivity in the layers is the same. This assumption is done to study the effects of the change of the permeability alone.

A nondimensional form of the mass, momentum and energy equations, using the Darcy's law and the Boussinesq approximation are used in a model solved by the finite difference technique. Nondimensional variables X, Y, Ψ, θ, U and V are used for the distances x, y, stream function φ, temperature T and horizontal and vertical velocities respectively. Their values are defined by:

$$X = x/W, \quad Y = y/H, \quad \Psi = \varphi H/ \alpha W, \quad \text{and} \quad \theta = (T-T_C)/(T_H- T_C)$$

The governing parameters for the present problem are the aspect ratio A and the Darcy-Rayleigh number based on the height of the cavity Ra ,

$$A= W/H \qquad \text{and} \qquad Ra = K g \beta (T_H -T_C) H/ \alpha \upsilon$$

where α, β, g ,υ and K are the thermal diffusivity, coefficient of thermal expansion, acceleration due to gravity, dynamic viscosity and Permeability of the porous medium, respectively.

The boundary conditions for the non-dimensional stream function and temperature are as follows:

$$\Psi = 0 \text{ on all walls}, \quad \partial\theta/ \partial Y = 0 \quad \text{for } Y=0,1 \text{ and } 0<X<1,$$

$$\theta = 1 \text{ for } X=0 \text{ and } 0 <Y<1, \quad \theta = 0 \text{ for } X=1 \text{ and } 0<Y<1$$

The effect of the fluid motion on the heat transfer across the layers can be expressed by the Nusselt number, which is defined for the vertical hot wall in the nondimensional form as:

$$Nu = \int_0^1 Nu_Y \Big|_{X=0} \cdot dY \qquad (1)$$

$$\text{and} \quad Nu_Y = - \partial\theta / \partial X$$

where Nu is the average (mean) Nusselt number at the wall ,
Nu_Y is the local Nusselt number

Because the vertical layers have different permeabilities K, the Darcy-Rayleigh number Ra will be different for each layer. It will be:

$$Ra = K_r \,.\, Ra_H$$

where:
Ra_H is the Darcy-Rayleigh number in the layer in contact with the hot wall,
K_r is the permeability ratio of each sublayer = K/K_H , and
K_H is taken as the permeability value of the porous medium layer in contact with the hot wall.

After the numerical solution of the continuity, momentum and energy equations and obtaining the stream function and the temperature fields as well as the velocities in both the x and y directions, equation (1) was integrated numerically to get the Nu. Detailed information for the numerical procedure is found in [1].

3. RESULTS AND DISCUSSION

The two dimensional natural convection in a multilayered with different permeability fluid saturated porous medium has been analyzed for vertical isothermal walls at different temperatures and with adiabatic top and bottom walls. The study was done for 3 layered porous medium in which the first and third layers have equal thicknesses and permeabilities. A wide range of the effective parameters are considered such as:

1. the permeability ratio of the core region varies from 0.1 to 10
2. the ratio of the width of the inner sublayers to the total width of the cavity varies form 0.0 to 1.0.

for a wide range of the Darcy Rayleigh number based on the height of the cavity up to 6000.

3.1 The validity of the model

A relation similar to equation (1) for the heat flow along the cold wall x=W, X=1 was obtained and intergrated numerically. The obtained Nu was compared with this calulated from equation (1). The results were found to be nearly identical, the discrepancy between the results of the two approaches was less than 0.75%.

A comparison was done in the first part of this work [1] with the work of Lauriat and Prasad [4] for the values of the vertical velocity V/Ra and the temperature θ at the midheight section where Y=0.5 and the horizontal velocity U/Ra at the middle vertial plane X=0.5 for the case of a single layer porous media with $W_r = 0.0$. The comparison shows good agreement.

Another comparison is done with the corresponding values of Lauriat and Prasad [4] for the local Nu_Y at an aspect ratio A=5 and Ra=500 and for the average Nu at the hot wall for an aspect ratio A=5 and 0<Ra<6000 for the case of the single layer, i.e. $W_r = 1$ and $K_r = 1$. The comparison is shown in Figs. 2 and 3. The Darcy Rayleigh number Ra is based on the width of the cavity by Lauriat and Prasad [4], while it is based on the height of the cavity in the present research, so the values of Ra by Lauriat and Prasad [4] must be changed by the multiplication of the aspect ratio to suit the values of the present definition. The comparison shows also a good agreement and proves the validity of the model.

3.2 Effect of permeability ratio

To show the effect of the permeability ratio, a case of three layers porous media with equal widths are studied. In which the aspect ratio A = 3, the permeability ratios for the inner to the outer sublayers K_r differ from 10 to 0.1 and Ra=250.

The behavior of the local rate of heat transfer Nu_Y along the hot wall is shown in Fig. 4. It takes the typical trend as obtained in [4,5], in which the higher values of the rate of heat transfer exist at the bottom of the wall and then it drops to the lower values at the top of the wall. Fig. 4 shows the increase of the rate of local heat transfer with the increase of the inner sublayer permeability ratio.

Fig. 5 shows the effect of the permeability ratio of the inner sublayer on the average rate of heat transfer expressed by Nu for different Ra. Fig. 6 shows the variation of Nu/Nu_1 with Ra for different values of K_r. Where, Nu_1 is the value of Nu for the case where the whole cavity is filled with the orous material of the layer adjacent to the hot wall, i.e. $K_r=1$ or $W_r=0$ for the inner sublayer. It is shown that Nu/Nu_1 is greater than 1 for $K_r>1$ and less than 1 for $K_r<1$, i.e. Nu is higher than its value for a homogenous filled cavity by $K_r>1$ and less than it by $K_r<1$.

Both the Nu/Nu_1-Ra and Nu-Ra curves take nearly the same form for the different permeability ratio K_r. In all cases where Ra = 0 at the hot wall, Nu/Nu_1 and Nu take the unity value and the heat is transferred by pure conduction. By the increase of Ra, Nu/Nu_1 increases in a transient region untill it takes the parallel straight line form by Ra>1000, where Nu/Nu_1 depends only on K_r.

The behavior of Nu/Nu_1 is shown in Fig. 7. It gives a linear relation as follows:

$$Nu/Nu_1 = 0.0454\ K_r + 0.9546 \qquad (2)$$

Kim and Vafia [6] studied the natural convection about a vertical plate embedded in a homogenous porous media. They concluded that the Nusselt number depends only on the Rayleigh number Ra in the thermal boundary layer, where the heating effect of the wall is felt. This conclusion is also concluded by Weber [2] by studying the boundary layer regime for convection in a vertical homogenuous porous layer. The mean value of Nu for the boundary layer heat flow in the homogenuous porous media can be given according to [2,6] in our notations by the following relation:

$$Nu = C\ Ra_H^{0.5} \qquad (3)$$

Where C is a constant and is dependent on the aspect ratio A. By Weber [2] this constant is obtained as $1/(\sqrt{3}.A)$ for the boundary layer flow. In this case, the heat conducted from the wall into the fluid is carried upwards by the convective movement of the fluid in the steady state, and the fluid is driven upwards by buoyancy and restricted by bulk friction. This means that outside this layer, where the fluid is isothermal and the buoyancy effect is absent the fluid is nearly motionless.

In our case, for the Nu/Nu_1 straight line zone, where Ra >1000, and Nu/Nu_1 is mainly function of K_r, it can be said that the heat is trasfereed in a boundary layer flow. This flow consists of upward boundary layer flow on the hot wall, downward boundary layer flow on the cold wall and motionless flow in the core, thus equation (3) can be considered. The constant C in equation (3) must be dependent on both the permeability ratio K_r and the Width ratio W_r of the inner sublayer besides the aspect ratio A. Because the aspect ratio A and the width ratio W_r are constants and equation gives that $Nu/Nu_1 = f(kr)$, it can be said that the effect of both K_r and W_r on C can be separated, and C can be written as:

$$C = f(W_r).\ f(K_r).\ f(A) \qquad (4)$$

3. 3 Effect of sublayers width

To show the effect of the sublayers width ratio W_r, constant values for the permeability ratios of the sublayers will be considered. Figs. 8-14 show the effect of the width ratio of the inner sublayer W_r on the Local and mean Nu at the vertical hot wall. The width ratio W_r takes the values 0.0, 0.2, 0.4, 0.6, 0.8, 1.0. for an aspect ratio A=3. Two cases are considered. In the first case, the permeability of the inner sublayer is taken as five times greater than the outer layers K_r =5 and Ra=150. In the second case, the permeability of the inner sublayer is taken as five times less than it in the outer layers K_r =0.2 and Ra=400.

Figs. 8 and 9 show the distribution of the local rate of heat transfer Nu_Y along the hot wall. The Figures show that the rate of local heat transfer inceases with the increase of the inner sublayer width ratio for K_r>1, and decreases with the increase of the sublayer width for K_r<1.

Figures 10 and 11 show the variation of the Nu for different values of the width ratio for the two cases of study. The behaviour of the Nu-Ra curve is nearly the same as that expressed in Fig. 5. It consists also from the three zones. The zero Ra at the hot wall where Nu=1 and the heat transferred by pure conduction, the transient zone and the boundary layer flow zone. It is shown that the Nu increases with the increase of the width ratio W_r by the case of K_r=5 and Nu decreases with the increase of the width ratio W_r by the case of K_r= 0.2.

Figures 12 and 13 show the behavior of the function Nu/Nu_1 for the two expressed cases. It is shown that the value of Nu is higher than its value for a homogenous filled cavity by K_r =5 and less than it by K_r = 0.2. It is also shown that in the boundary layer flow zone, where $Ra > 1000$, the value of Nu/Nu_1 is nearly constant and depends on the width ratio W_r only. This relation is shown in Fig. 14. It gives a linear relation as follows :

$$Nu/Nu_1 = \pm 0.3836\, W_r + 1 \qquad (5)$$

the +ve is for the case of $K_r > 1$ and the -ve for $K_r < 1$.

For the general case, equations 2 and 5 can be combined together to give the effect of both the permeability and width ratios in the boundary layer flow regime where Ra is nearly equal to 1000. It can be expressed as follows:

$$Nu/Nu_1 = (\pm 0.3836\, W_r + 1).(0.0454\, K_r + 0.9546) \qquad (6)$$

where the +ve is for the case of $K_r > 1$ and the -ve for $K_r < 1$.

Equation (3) which expresses the heat transfer in the boundary layer flow regime can be developed to take into consideraion the effects of both the permeability ratio K_r and width ratio W_r for the three layered porous media and written as:

$$Nu = C\,.(\pm 0.3836\, W_r + 1).(0.0454\, K_r + 0.9546)\, Ra^{0.5}$$

where the +ve is for the case of $K_r > 1$, the -ve for the case of $K_r < 1$ and the constant C depends on the aspect ratio A.

4. CONCLUSIONS

The significance of the effect of the non uniform permeability of the sublayers on natural convection heat transfer in a two dimensional vertical multilayered porous medium heated from one side has been thoroughly investigated numerically. Considerations were given to 3 layered porous medium in which the first and third layers have equal thicknesses and permeabilities. The variation was made for the permeability and thickness of the inner (second) sublayer.

For a constant width ratio, both the rate of local heat transfer and the mean heat transfer at the hot wall increase with the increase of the inner sublayer permeability ratio and are higher than its values for a homogenous filled cavity by $K_r > 1$ and less than it by $K_r < 1$.

For a constant permeability ratio both the rate of local heat transfer at the hot wall and the mean heat transfer incease with the increase of the inner sublayer width ratio for $K_r > 1$, and decreases with the increase of the sublayer width for $K_r < 1$.

The behavior of Nu at the hot wall with the increase of Ra_H takes 3 stages:

- Nu =1 and the heat transferred by pure conduction when $Ra_H = 0$,
- transient heat flow by 0< Ra <1000
- boundary layer flow by Ra >1000, in which the mean rate of heat transfer depends on both the width ratio, the permeability ratio in addition to the dependence on both the Rayleigh number and the aspect ratio. A correlation is derived numerically for this relationship as follows:

$$Nu = C .(\pm 0.3836 W_r + 1).(0.0454 K_r + 0.9546) Ra^{0.5}$$

where C is a constant depends of the aspect ratio A, the +ve is for the case of $K_r > 1$ and the -ve for the case of $K_r < 1$.

5. NOMENCLATURE

A	Aspect ratio = H/W
g	Acceleration due to gravity, m^2/s
H	Height of the porous material, m
K	Permeability of the porous layer, m^2
K_H	Permeability of the porous layer adjacent to the hot wall, m^2
K_r	Ratio of the permeability of the porous layer to the permeability of the porous layer adjacent to the hot wall $= K/K_H$
Nu	mean Nusselt number ,
Nu_1	Nu for the case where the whole cavity is filled with the porous material of the layer adjacent to the hot wall, i.e. K_r for the inner sublayer = 1 or $W_r = 0$.
Nu_Y	the local Nu at the vertical hot wall
p	Pressure, Pa
Ra	Darcy-Rayleih number= $g \ \beta K H (T_H - T_C)/ \alpha \upsilon$
Ra_H	Darcy-Rayleih number for the layer adjacent to the hot wall
T	Temperature, K
T_H, T_C	Temperature of the hot and cold isothermal surfaces, K
u,v	Field velocities in the x and y directions, m/s

U,V	Non-dimensional field velocities in the X and Y directions respectively
x,y	Spatial coordinates
X,Y	Dimensionless distances in the x and y axis respectively
W	Width of the porous material, m
W_r	Width ratio
α	Thermal diffusivity of the porous layers, m^2/s
β	Coefficient of volumetric thermal expansion, 1/K
μ	Dynamic viscosity of the fluid
υ	Kinematic viscoisity of the fluid, m^2/s
ρ	Fluid density
φ	Stream function
Ψ	Dimensionless stream function
θ	Non-dimensional temperature = $(T-T_C)/(T_H-T_C)$

6. REFERENCES

1. El Kady, M.S.,"Natural flow convection in a vertical multi-layered porous media with varying permeabilities (Part 1: Temperature and flow fields) to be published in Mansoura Engineering Journal.
2. Weber, J. E.,"The boundry layer regime for convection in a vertical porous layer." Int. Journal of Heat and Mass Transfer, Vol. 18, pp 569-573, 1975.
3. Tong, T, and Subramanian, E., "A boundary layer analysis for natural convection in a porous enclosure-use of the Brinkman extended Darcy model". Int. Journal of Mass and Heat Transfer, Vol.28, pp 563-571, 1985.
4. Lauriat, G. and Prasad, V., "Natural convection in a vertical porous cavity: a numerical study for Brinkman-extended Darcy formulation", Trans. of the ASME, Vol 109, pp 688-696, 1987.
5. El Kady, M.S.,"Numerical study of natural convection in a rectangular porous medium with vertical temperature gradient". Mansoura Eng. Journal (MEJ), vol. 15, No. 1, pp M72-M87, 1990.
6. Kim, S. J., and Vafia, K, "Analysis of natural convection about a vertical plate embedded in a porous medium" Int. J.of Heat and Mass Transfer, Vol. 32, No. 4, pp 665-677, 1989

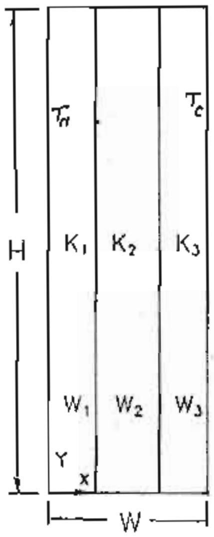

Fig. 1 Schematic diagram of the rectangular multilayered porous cavity

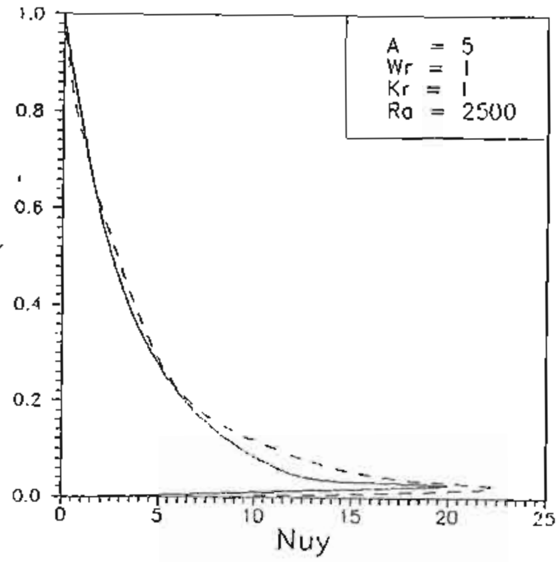

Fig. 2 The local Nu along the hot wall
- - - - - Lauriat and Prasad [4]
——— Present work ,

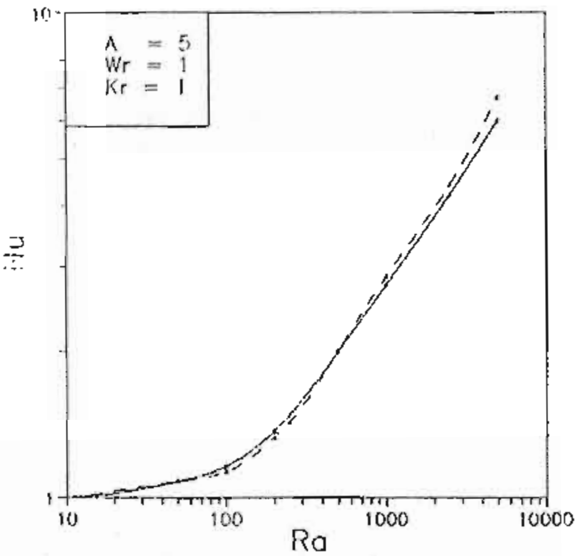

Fig. 3 The average Nu with Ra along the hot wall
- - - - - Lauriat and Prasad [4]
——— Present work ,

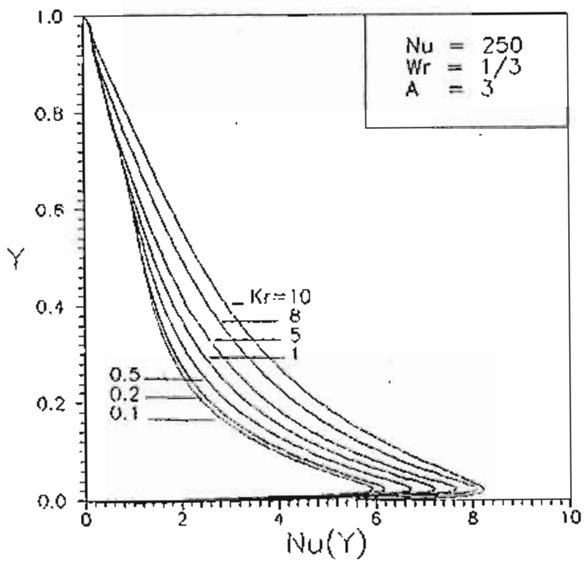

Fig. 4 Effect of the permeability ratio of the inner sublayer Kr on the local Nu at the hot wall

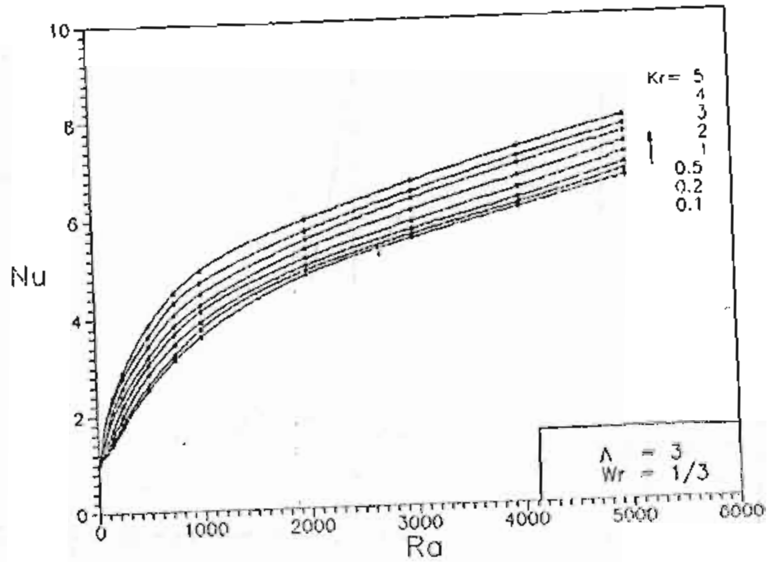

Fig. 5 Effect of the permeability ratio of the inner sublayer Kr on the average Nu at the hot wall

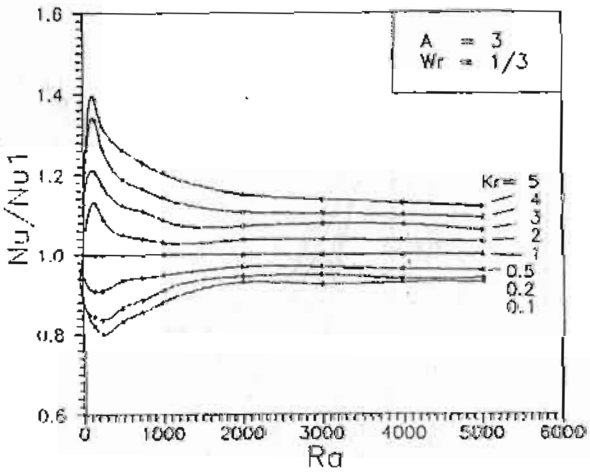

Fig. 6 Effect of the permeability ratio of the inner sublayer on the behavior of Nu/Nu1 at the hot wall

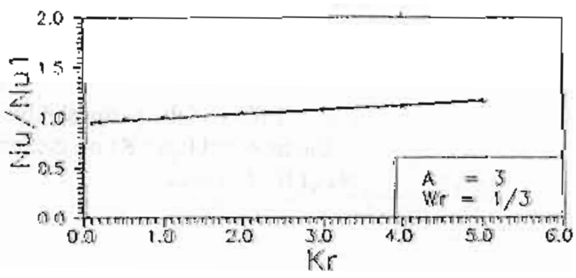

Fig. 7 The behavior of Nu/Nu at the hot wall with the permeability ratio of the inner sublayer in the boundary layer regime

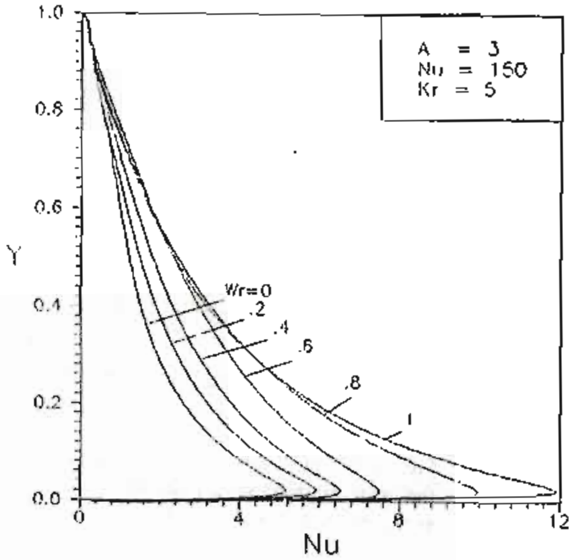

Fig. 8 The local Nu at the hot wall for Kr=5 and Ra= 150

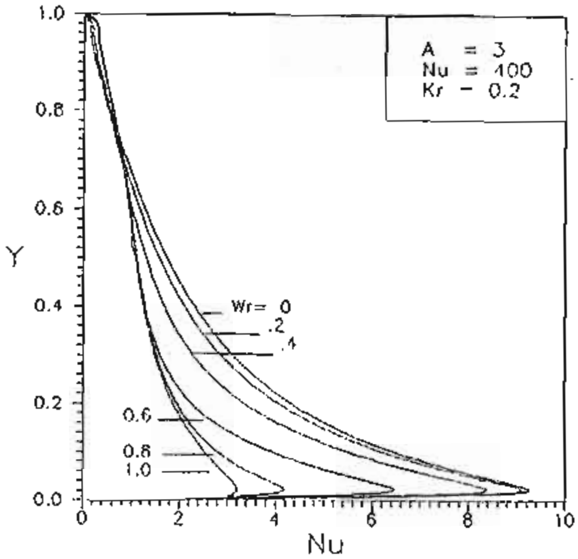

Fig. 9 The local Nu at the hot wall for Kr=0.2 and Ra= 150

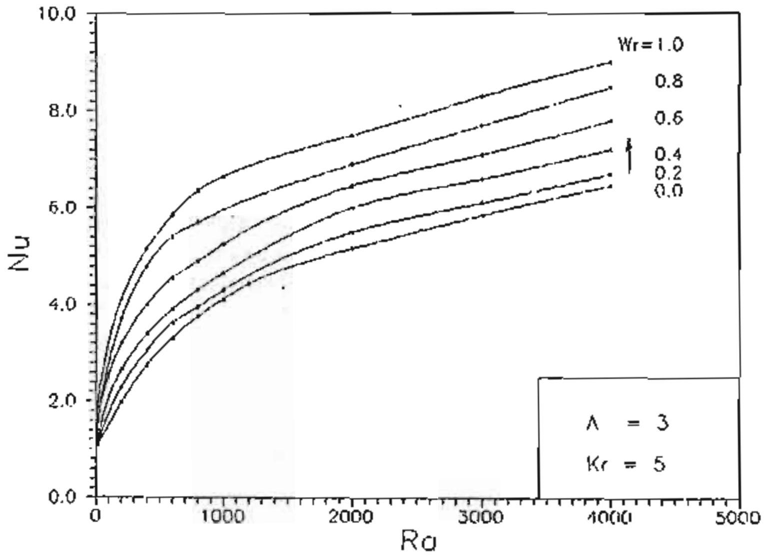

Fig. 10 Effect of the Width ratio of the inner sublayer Wr on the average Nu at the hot wall for Kr=5 ,

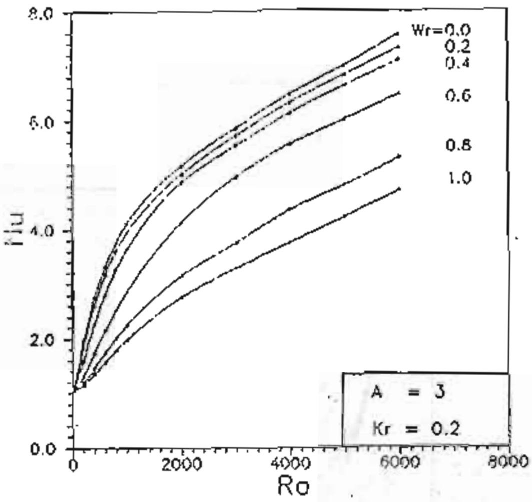

Fig. 11 Effect of the Width ratio of the inner sublayer Wr on the average Nu at the hot wall for Kr=0.2

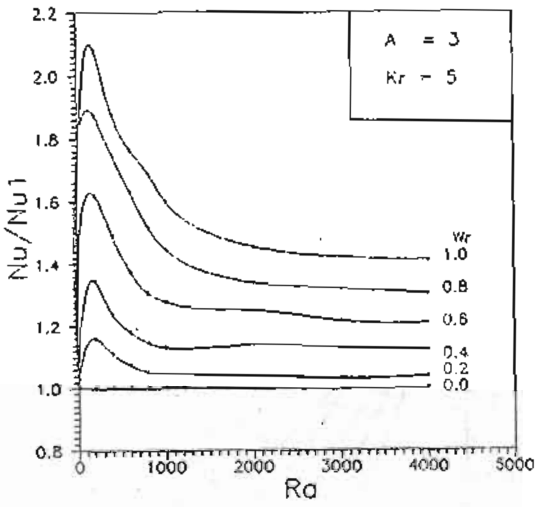

Fig. 12 Effect of the Width ratio of the inner sublayer Wr on Nu/Nu at the hot wall for Kr = 5

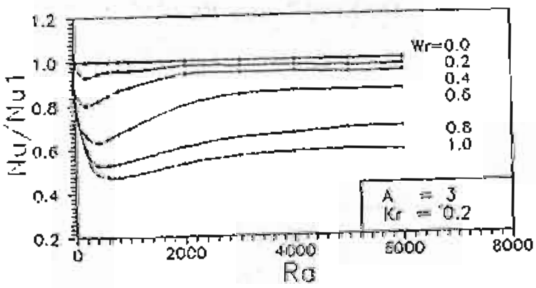

Fig. 13 Effect of the Width ratio of theinner sublayer Wr on Nu/Nu at the hot wall for Kr = 0.2

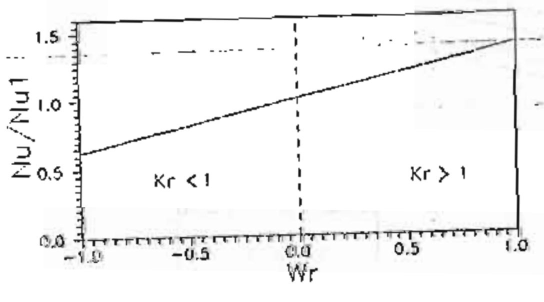

Fig. 14 The behavior of Nu/Nu at the hot wall with the Width ratio of the inner-sublayer in the boundary layer regime