EVALUATION OF SOME EMPERICAL CRITERIA FOR TURBULENT SEPARATION

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SUMMARY

This paper presents an evaluation of accuracy for five separation criteria to predict the separation point of two-dimensional turbulent boundary layers. Predicted separation points were compared with the experimentally measured positions. In addition the separation point, in which the skin friction approaches zero, obtained from solution of Prandtl's boundary layer equations for one case was compared with the predicted points. The evaluated criteria were Stratford's, stratford's with modification of it's constants by Cebeci et al., Townsend's, Sandborn-Liu and the shape-factor criterion. It was concluded that criteria of Townsend, Sandborn-Liu and of shape-factor predict separation points with enough accuracy needed for engineering purposes. The separation point obtained from boundary layer solution gives a point far upstream from the measured one. An emperical criterion to predict the separation point is conducted here for separated flow induced with forward-facing wall jet injected from base of a forward-facing step.

NOMENCLATURE:

A = dimensionless parameter, Eqn.(6)
C = constants, (eqns. 5 to 7).
C_f = skin friction coefficient, $\tau_w/(1/2)\rho U^2$
C_p = pressure coefficient, $(p-p_o)/(1/2)\rho U_o^2$
C_{px} = pressure gradient in x-direction, (dC_p/dx).
h = slot-height.
H = step-height.
H_{12} = shape-factor, δ^*/θ
I = momentum ratio, $\lambda^2(h/\delta 1)$
n = exponent, (eqn.7)
p = pressure
R_e = Reynold's number, $(U.\delta 1/\nu)$.
u,v = X and y components of velocity, respectively.
U = velocity outside of boundary layer.
V = jet velocity.
x = streamwise distance.
y = distance normal to the surface of the body.
$\bar{X}_s$, = normalized distance, (eqn.9).
$\delta 1$ = displacement thickness of the main flow at position of step-front before fitting it.
θ = momentum thickness, $\int_0^\infty u/U \, (1-u/U)\, dy$.

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δ^* = displacement thickness, $\int_0^\infty (1-u/U)dy$
ν = kinematic viscosity
ρ = density.
τ = shear stress.
ϑ = deviding streamline engle, (eqn.4).
λ = velocity ratio, (V/U_∞)

SUBSCRIPTS

o = minimum pressure point.
ps = past separation
s = separation point
w = wall
∞ = free stream condition before fitting the step.

INTRODUCTION

The boundary layer equation for steady, incompressible two-dimensional flow without body force which are known as Prandtl's boundary layer equations are [9] :

continuity eqn. : $(\partial u/\partial x) + (\partial v/\partial y) = 0$ (1)

x-momentum eqn. : $u(\partial u/\partial x) + v(\partial u/\partial y) = -(1/\rho)\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$ (2)

y-momentum eqn. : $\frac{\partial p}{\partial y} = 0$ (3)

Here equation (3) indicates that the pressure is transmitted without change through the boundary laver to the surface.

Oswatitsch [8] has shown that the Navier-Stokes equations requires the condition

$$\left(\frac{\partial p}{\partial y}\right)_w = -\left(\frac{\partial \tau_w}{\partial x}\right) = \frac{1}{3}\tan\vartheta\left(\frac{\partial p}{\partial x}\right) \quad (4)$$

for the separation.
Here ϑ is the angle at which the deviding streamline leaves the wall, τ_w is the wall shear stress and the suffix w denotes the wall conditions (see Fig.1).

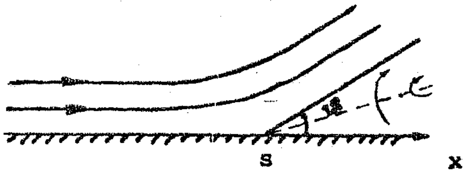


Fig. (1)- Separation point on flat plate

The relation given in eqn. (4) shows that the pressure changes in y-direction in the vicinity of separation, and therefore the boundary layer equations are not more valied. This means that in regions up to the vicinity of separation in boundary layer flows over solid boundaries with adverse pressure gradients the boundary layer equations are valid. However, in the separation region the solution methods of the boundary layer equations do not apply. This is evident because the pressure distribution of a separated flow is usually different from that of an unseparated flow about the same body as shown by the result given by Oswatitsch. Furthermore, all the assumptions made in the derivation of the boundary layer equations break down in the separation region. It is clear that the Navier-Stokes equations must be used to calculate the fluid motion in separated region; but no satisfactory theory for obtaining this has been found and experimental solution is the only resort at present time.

Recently, several investigators (refs 1 to 5) have developed new approximate techniques for separation point predictions depending only on emperical determined coefficients. By these criteria, the separation point can be predicted, for engineering purposes, with great easy and modirate accuracy. These simplified expressions for predicting the separation locations of two-dimensional, incompressible turbulent boundary layers are presented shortly in this work. Using the experimental measurements of separated turbulent boundary layers conducted in [6], an evaluation of the accuracy for the prediction criteria given in [1-5] by comparing the measured and predicted separation points is carried out in this paper. The used measurements in [6] were carried out in separated turbulent boundary layer using forward-facing steps with and without forward-facing wall jets. The pressure distributions, separation points, skin friction, some velocity profiles and the initial conditions were measured with good accuarcy.

CRITERIA FOR PREDICTING TURBULENT BOUNDARY-LAYER SEPARATION

Stratford's Criterion [1]

Stratford derived a simple criterion based on the two layer concept of the turbulent boundary layer to predict the separation positions. He assumed that the outer part of the boundary layer is affected only by the downstream pressure gradient and the initial velocity profile and the inner part is locally in equilibrium and is independent of upstream conditions. The theory results in following formula to be satisfied at the turbulent separation point at Reynold's number of order 10^6 :

$$C_p \left(x \frac{dC_p}{dx}\right)^{0.5} (10^{-6} Re_o)^{-0.1} = C \qquad (5)$$

This expression is valid for fully turbulent flow with an adverse pressure gradient following a minimum pressure region where the pressure gradient is zero as shown in Fig.(2). When there is a region of laminar flow, or a region of turbulent flow with a favorable pressure gradient,* Stratford assumed that the velocity profile at the

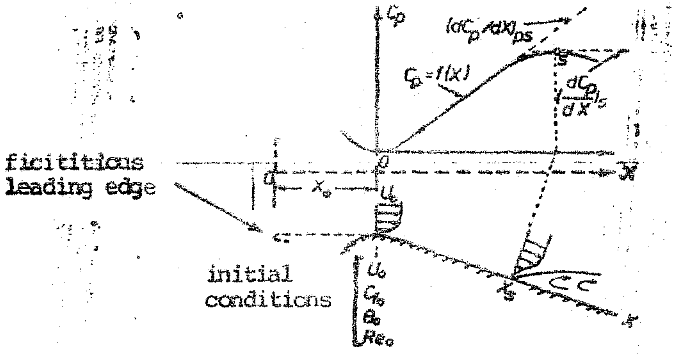


Fig.(2) Adverse pressure region with separation.

minimum pressure point (x_o) is approximatly that of a flat-plate turbulent boundary layer without pressure gradient starting from a ficititious leading edge. The distance x in eqn. (5) is measured from this edge. The pressure coefficient C_p is based on conditions of the location of minimum pressure and given by $C_p = (p-p_o)/(\varrho/2)U_o^2$; this means that C_p is equal to zero at the point of minimum pressure. The Reynold's number Re_o is based on conditions at the minimum pressure location and the equivalent distance x_o ; i.e; $Re_o = (U_o x_o / \nu)$. The right-hand side of Stratford's criterion C is an emperical constant which was subjected to change from 0.39 to 0.35 in the course of the development of the criterion according to the following :

- if C is greater than 0.39, than separation is predicted when C=0.39.
- if C has a value between 0.35 and 0.39, then separation occurs at the maximum value of C, and
- if C is less than 0.35, then no separation occurs.

According to the study made by Cebeci et al. [3] using the mixing length theory, the values 0.35 and 0.39 which are suggested by Stratford for C were modified by 0.3 and 0.5 respectively.

Stratford's method uses only the pressure distribution and the initial conditions to predict the values of C_p and x at the separation point. One must, therefore, calculate the left-hand side of eqn. (5) in maching fashion toward separation, using local C_p and dC_p/dx corresponding to each x-position. The values of C_p and x at the separation point are determined, when the calculated values reaches the constant C for separation in the right-hand side. Stratford's criterion is independent upon the pressure history upstream of the separation point.

Townsend's Criterion [2]

This theory can be regarded as a development of Stratford's method. It results in a formula for the pressure coefficient at separation which involves the skin friction coefficient at the beginning of the adverse pressure gradient region; i.e, the separation pressure depends upon the Reynold's number at the initial position. Then, according to this theory, the separation positions are very sensitive to the skin friction value and therefore, an accurate determination of the skin friction is required. Townsend replaced the distance x in Stratford's criterion with the local pressure and noted that the pressure gradient at separation is an integral part of the separation process and that a criterion should be based on the pressure gradient upstream of separation $(dC_p/dx)_{ps}$. The criterion is given in the following expression :

$$\log\left[\frac{\nu}{U_o}\cdot\left(\frac{dC_p}{dx}\right)_{ps}\cdot\frac{1}{C_{fo}^{1.5}}\right] + A + \log(A-1) = C \qquad (6)$$

where ,

$A = (0.3362\,\frac{C_p}{C_{fo}} + 2.6896)^{0.5}$; $C_p = (p-p_o)/(\varrho/2)U_o^2$ and C is an emperical constant which is suggested by Townsend by the value 3.4634. The other nomenclatures in eqn. (6) are defined in figure (1).

Using the local C_p , (dC_p/dx) and the initial condition values the left-hand side of Townsend's criterion given by eqn. (6) can be calculated in marching fashion toward separation. When the calculated values reaches the value of C, then the separation pressure coefficient is determined.

This means that Townsend's theory predicts only the separation pressure, not the separation location.

Sandborn-Liu Criterion [4]

Sandborn and Liu developed their criterion based on the same concept of Stratford's for two-dimensional turbulent boundary layer with positive pressure gradients when $C_p \leqslant 0.5$. This criterion is given in the following form:

$$C_p^{(2n-1)/4} \cdot \left(x \frac{dC_p}{dx}\right)^{0.5} = C \qquad (7)$$

where n is the power-law exponent of the turbulent velocity profile and has a value between 6 and 8. The emperical constant C takes the following values :

for n=6 C =0.08294
and for n=8 C =0.0528

The coordinate x is to determine here also according to the same way as given in Stratford's criterion.

When the calculated value of the left-hand side in eqn. (7), using C_p, (dC_p/dx) and n at every position x in marching fashion toward separation, reaches the corresponding value of C in the right-hand side of equation, then the separation location and separation pressure are determined. The two limits of C in Sandborn-Liu criterion give a range in which separation occurs. The value of C = 0.0528, which is based on n=8, gives the highest possible separation pressure.

Shape Factor Criterion

This criterion is given firstly at 1931 by Gruschwitz [5] through comparison between measurements made on separated flow around an airfoil and solution of the momentum integral equation for two-dimensional turbulent boundary layer. He found that the separation exists at the position in which the shape factor H_{12} is greater or equal to 1.8, i.e;

$$H_{12} = (\delta^* / \theta) \geq 1.8 \qquad (8)$$

where δ^* is the displacement thickness and θ is the momentum thickness of the boundary layer. In most integral solution methods of the boundary layer equation, the shape factor H_{12}, as given in the relation before, is taken as criterion of seperation. When H_{12} takes a value between 1.8 and 2.4, separation is assumed to exist. The difference between these two limits of H_{12} makes very little difference in the locating of separation point, where close to separation the shape factor increases quickly. This fact is proved in the measurements given in [6] and the other similar measurements.

EVALUATION OF CRITERIA WITH EXPERIMENTAL DATA:

The experimental data to be used for evaluation of the separation criteria described above have been taken from measurements carried out by Hewedy [6] . He conducted two series of experiments through measurements of separated flows. The first one (called series I) is separated flow induced upstream of a forward-facing step. In second series (II) the separation exists upstream of a forward-facing step in addition to a thin forward-facing wall jet injected from the base of the step. In series (I the initial conditions have been taken from the upstream region, which is rather not influenced by the step. The separation locations are clearly defined by the experimental data. For the experimental results conducted from the second series (II) the initial conditions have been taken from the measurements at the position of minimum pressure. The jet-flow in the wall side is considered here as developed turbulent boundary layer. The separation points of the jet-flow from the wall are given also by the experimental data. The measurements used here include details of boundary layer profiles at the initial conditions, which are required for the evaluation. For each set of data, the evaluation of the criteria described before will described.

a) Measurements of Series (I)

In this set of experiments the separation was induced in the flow over a flat plate at zero incidence upstream of a forward-facing step. The step height H had the values 33 and 60 mm; $8.1 \leq H/\delta 1 \leq 17.8$ and $1300 \leq (Re_{\delta 1} = U_{\infty}\delta 1/\nu) \leq 3520$. Here $\delta 1$ is the reference displacement thickness of the main flow at the step front position before fitting it and U_{∞} is the free stream velocity of main flow before fitting the step. The initial conditions were taken at a distance of about 800 mm upstream of the step and the measured wall pressures were normalized with the pressure p_o and the dynamic pressure at the initial condition $(\varrho/2)U_o^2$ in the form $C_p = (p-p_o)/(\varrho/2)U_o^2$.

Fig.(3) shows four different pressure distributions induced with two different steps (step heights H=33 and 60 mm) and at two different Reynold's number ($Re_{\delta 1}$ = 1300 and 3520). These distributions show that the pressure gradient upstream of separation is nearly constent, but varies rapidly near sparation . The measured - as well as the predicted separation points for each pressure distribution according to the separation criteria discribed above are shown. As Known, Stratford's criterion require the local pressure gradient to determine the separation pressure and position. However, it is very difficult to determine the local pressure gradient near separation. For this present purpose, the pressure gradient near separation has been replaced with that determined by the extrapolated pressure distribution from the attached flow upstream of separation $(C_{px})_{ps} = (dC_p/dx)_{ps}$. Values of $(C_{px})_{ps}$ are given in (1/m) for each distribution of pressure in the figure.

It can be shown that Stratford's criterion predicts earlier separation than is measured experimentally for all measurements in Fig.(3). Using the modification of the constants after Cebeci et al. [3] instead of the suggested by Stratford's the predicted separation point approaches to the measured but still before it.

The criterion of Sandborn and Liu gives a predicted separation region in which the separation occurs. The region exists still before the measured separation point. The upper limit of n (i.e. n=8 and C=0.0528) in the criterion gives a separation point close to the measured for all four cases given in Fig.(3).

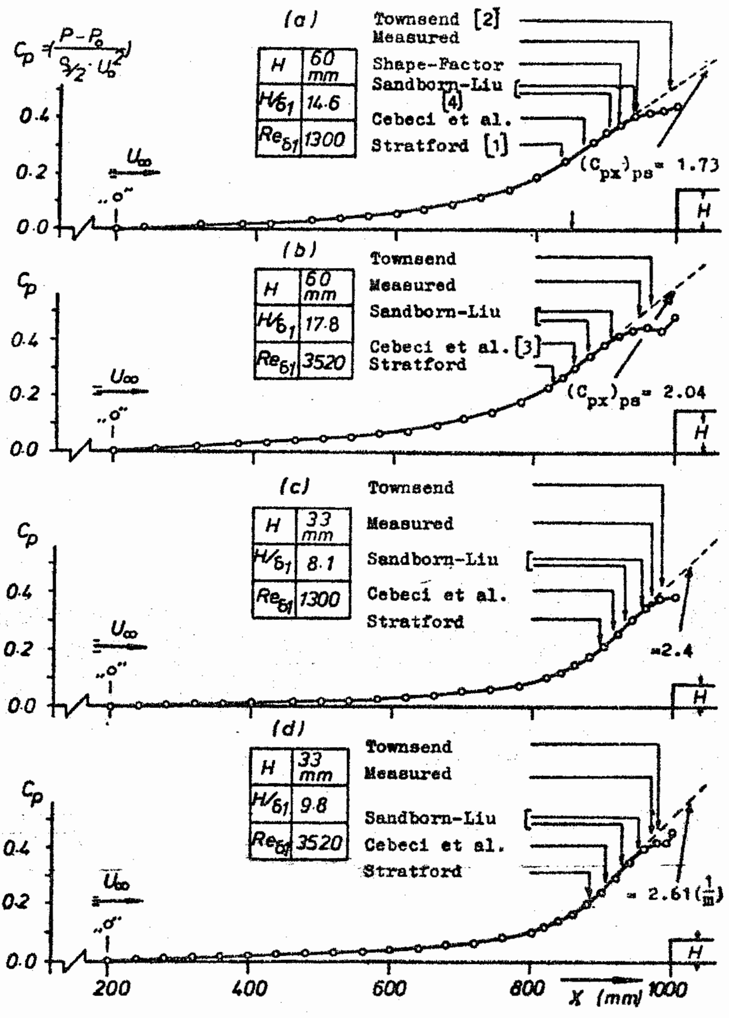


Fig. (3) Measured and predicted separation points upstream of forward-facing steps.

The separation pressures predicted by Townsend's criterion are higher than the experimental values for all four cases, as shown in the figure. It should be noted again that Townsend's criterion does not require x as an input; therefore, the predicted C_p is not necessarily on the extrapolated pressure line. The separation locations obtained by this method are in general close to the experimental values for cases (b), (c) and (d) in Fig. (3).

The predicted separation point resulted from the shape-factor criterion taking the value of $H_{12}=1.8$ as separation point is shown also in Fig. (3-a) in which only the velocity profiles were measured in [6]. Taking a shape factor of about 2.1, the separation location lies on the measured position.

The arrow over the x-axis of measurements plotted in Fig. (3-a) shows the predicted separation point obtained from solution of Prandtl's boundary layer equations for attachen flows. As known, the boundary layer equations are not valid near separation, where the pressure gradient normal to the wall is not more equal to zero. Therefore, the predicted separation point is given, as shown, so far upstream of the measured point.

b) Measurements of Series (II)

In this series the separation was induced also in the flow over a flat plate at zero incidence upstream of a forward-facing step with a height H of 60 mm and a forward-facing wall jet injected from the base of the step. The slot height of the jet h was 2.8 mm; $Re_{\delta 1} = (U_\infty \delta 1/\nu)=1300$ and the ratio $H/\delta 1=14.6$.

A normalized pressure distributions using the pressure and the maximum dynamic pressure in the jet profile at the minimum pressure location as reference values are plotted in Fig. (4) The main stream flows from left- to right-hand side with the velocity U_∞. The step was fitted at about one meter from the rounded leading edge of the plate and at the base of the step the slot was formed. The forward-facing wall jet was blowed from the slot with velocity V. The jet separation point from the wall was measured for the velocity ratios $\lambda = (V/U_\infty) = 2.84$, 3.45, 4.24 and 4.99. Fig. (4) shows the normalized pressure distributions for these four cases. The predicted separation point according to the theories of Stratford, Stratford with the modified constants suggested by Cebeci et al. and Sandborn-Liu are shown in the figures compared with the measured locations.

As in series (I), the Stratford's theory predicts also earlier separation points than all the others. With the modified constants of Stratford's criterion the predicted separation point lies close to the measured. The region in which the separation position lies according to Sandborn-Liu criterion exists before the measured position for cases of small pressure gradient in the attached flow upstream of separation $(C_{px})_{ps}$ as in cases (c) and (d) in Fig. (4) where the measured separation points exist inside the region for higher pressure gradients (cases a and b). It seems that the predicted separation point according to the criterion of Sandborn and Liu is dependent upon the pressure gradient before the separation point.

The Townsend's criterion could not be applied here where the skin -

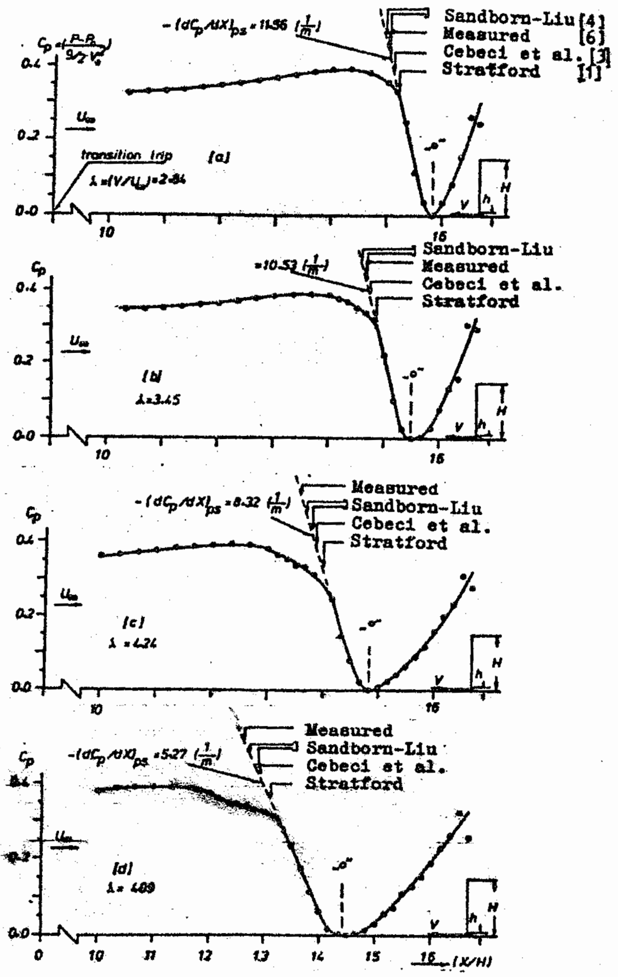


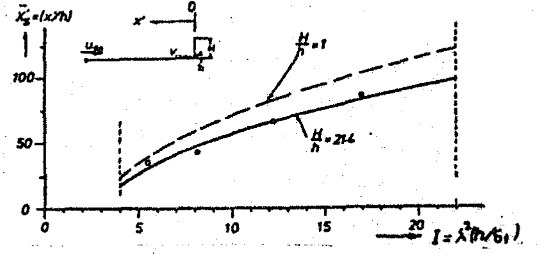
Fig. (4) Measured and predicted separation points upstream of forward-facing step with a forward-facing wall jet. (H=60 mm; H/h=21.4; $2.84 \leq \lambda \leq 4.99$ and $Re_{\delta 1}=1300$)

friction coefficient in the initial position was not available in the measurements. Also, the form-factor criterion needs measurements of velocity profiles of the attached flow upstream of jet separation which were not carried out also; therefore, the criterion could not be applied also.

Similar measurements for series (II) were carried out in [7] but for a forward-facing wall jet without step. An emperical separation criterion is given in that work for H/h =1. From the measurements conducted in [6] which are used in the present work for evaluation of the criteria, the influence of the step height to the slot height ratio (H/h) can be included in the emperical criterion given in [7] for the range of the measurements in the following form for $0.35 \leq (h/\delta_1) \leq 0.91$; $1 \leq (H/h) \leq 22$; $1300 \leq Re_{\delta 1} \leq 2900$ and $4 \leq I \leq 22$:

$$\bar{X}'_s = 25\ (I-3.21)\,(h/H)^{0.07} \tag{9}$$

where, $I = (V/U_{\infty})^2\ (h/\delta_1) = \lambda^2.(h/\delta_1)$ and $\bar{X}'_s$ is the distance between the jet separation point and the step front measured in the upstream direction of the main flow (or downstream direction of the jet flow) and normalized with the slot height h.



$1300 \leq Re_{\delta 1} \leq 2900$
$4 \leq I \leq 22$
$1 \leq H/h \leq 22$
and $h/\delta 1 = 0.91$

Fig.(5) Correlation of jet-separation point.

Eqn.(9) is plotted in Fig.(5) for the given range of measurements after the results conducted in [6] and [7]. The curve for H/h=1 represents the equation given in [7] and the curve for H/h=21.4 is plotted as well as the measurements carried out in [6] for that case.

CONCLUSIONS :

The solution of differential boundary layer equations predicts a separation point which lies so far upstream of the measured point. This is due to the break down of the assumptions on which the equations are based. Stratford's criterion predicts also an separation position which lies far upstream from the measured. Using the modified constants suggested by Cebeci et al. instead of that given by Stratford, the predicted separation point lies more close to the measured than the original one but still upstream of it. Townsend's criterion gives separation point downstream of the measured in the direction of the adverse pressure gradient. The distance between the predicted and the measured separation points seems to depend upon the pressure gradient upstream of separation $(C_{px})_{ps} = (dC_p/dx)_{ps}$. The criterion predicts satisfactorily the separation pressure. The Sandborn-Liu criterion predicts a separation point using the higher value of n close to the measured location.

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