Transient and Steady State Analysis of High Efficiency Single-Phase Induction Motor

دراسة تحليلية للحالة الديناميكية والمستقرة لمحرك حثى عالى الكفاءة مغذى من مصدر كهربى احادى الوجه

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ملخص:- يدرس هذا البحث عملية تشغيل محرك ثلاثى الأوجه من خلال مصدر كهربى احادى الوجه بإستخدام ثلاثة مكثفات كهربية. بضبط قيم المكثفات لكل سرعة دوران يمكن تشغيل المحرك فى الحالة المتزنة للأوجه الثلاثة. تم خلال هذا البحث استنتاج نموذج للمحاكاة للمحرك قائم على المعادلات اللحظية لمركبات التيار والجهود المتماثلة وبأستخدام الحاسب الآلى تم عمل محاكاة للمحرك ودراسة الأداء الوظيفى للماكينة المستقرة وكذلك المتزنة والغير متزنة. وقد اثبتت فاعلية البرنامج بمقارنة النتائج النظرية مع نظيرها العملية.

Abstract:- Operating a three-phase induction motor from a single-phase supply via three capacitors is studied. The capacitor values may be adjusted so as to get a motor balanced mode of operation. The model of the machine is derived using instantaneous symmetrical components. Equations describing the machine dynamics have been derived then used to simulate and analyze the machine performance for different modes of operation. The machine transient and steady state performance is predicted using Matlab-Simulink software. Experimental results have been obtained and compared with the simulated waveforms, showing the effectiveness of the proposed machine model.

Keywords: Induction motor, Single-phase supply, Simulink, Symmetrical components

1. Nomenclature

I_A, I_B, I_C	RMS motor phase currents
I_1, I_2, I_3	RMS currents through capacitors
I_o, I_p, I_n	RMS zero, +ve and –ve phase currents
I_{sr}, I_{rr}	RMS of real part of stator and rotor currents
I_{sg}, I_{rg}	RMS of imaginary part of stator and rotor currents
i_A, i_B, i_C	Instantaneous phase eurrents
i_{sr}, i_{rr}	Real part of instantaneous stator and rotor currents
i_{sg}, i_{rg}	Imag. part of instantaneous stator and rotor currents
i_{ps}, i_{ns}	Instantaneous +ve and -ve stator currents
i_{pr}, i_{nr}	Instantaneous +ve and -ve referred rotor currents
V_A, V_B, V_C	RMS phase voltages
V_1, V_2, V_3	RMS voltages across capacitors
V_S, I_S, ϕ_S	RMS supply voltage, current and phase shift
V_o, V_p, V_n	RMS zero, +ve and -ve phase voltages
v_A, v_B, v_C	Instantaneous phase voltages
v_p, v_n	Instantaneous +ve and -ve stator voltages
v_S, i_S	Instantaneous supply voltage and current
R_s, R_r	Stator and referred rotor resistances
T_L, T_e	Load torque and motor developed torque
T_{av}, T_{OSC}	Average motor torque and double supply frequency oscillatory torque component
X_m, X_1, X_2	Magnetizing, stator and referred rotor reactance
C_1, C_2, C_3	Balancing capacitors
B_1, B_2, B_3	Balancing susceptances
J	Inertia of the rotating parts
ω, n, s	Electrical angular speed, per unit speed and slip
P	Number of poles
a	Unit complex operator ($exp[j2\pi/3]$)
Y_p, Y_n	Stator +ve and -ve admittance
y_p, ϕ_p	Magnitude and angle of stator +ve admittance
$\phi_{sr}, \phi_{rr}, \phi_{sg}, \phi_{rg}$	Angle of I_{sr}, I_{rr}, I_{sg} and I_{rg}

2. Introduction

The 3-ph induction motor (IM) can be operated from a 1-ph supply. This can be achieved by leaving one terminal unexcited and the machine is partially loaded [1]. Capaeitor banks have been used to generate the split phase [2]. Autotransformer with a capaeitor were also used to excite the motor third terminal for full load operation [3]. In [4], 3-ph IM is supplied by a 1-ph supply via three eapacitors as shown in figure 1. This phase balancing scheme is called high efficiency single-phase induction motor [4], or SEMIHEX [5]. The machine can work at full load with the same high efficiency, which would have had from a 3-ph supply. With appropriate choice of the terminal capacitors, it is possible for the induction motor to operate with balanced winding voltages and currents. This configuration has lower cost and higher efficiency than a comparable 1-ph motor [5].

3. Performance Analysis

This machine performance is sensitive to the phase connection for a given direction of rotor rotation [5]. Referring to figure 1, the machine sequence is ACB. Therefore the vector diagram of

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the machine can be depicted as shown in figure 2 [4]. Applying Kirchoffs' laws, the circuit equations at steady state can be written as follows:

$$V_S = V_A - V_C \quad (1)$$

$$I_P = I_1 + I_2 \quad (2)$$

$$I_A + I_C + I_1 + I_3 = 0 \quad (3)$$

$$I_1 = Y_1 V_1 = Y_1 (V_A - V_B) \quad (4)$$

$$I_2 = Y_2 V_2 = Y_2 (V_A - V_B - V_C) \quad (5)$$

$$I_3 = Y_3 V_A \quad (6)$$

$$I_S = I_2 - I_C \quad (7)$$

$$Y_1 = jB_1,\ Y_2 = jB_2 \text{ and } Y_3 = jB_3 \quad (8)$$

Where: $B_1 = \omega C_1$, $B_2 = \omega C_2$ and $B_3 = \omega C_3$

4. Balanced Operation

Under perfect phase balance condition, the sum of the machine phase currents must equal zero.

This requires the currents I_2 and I_3 to be equal (compare equations (2) and (3)), implying that C_3 must be equal to twice C_2, as follows:

$$I_2 = I_3 \quad (9)$$

Substitute by (5) and (6) into (9) to get:

$$Y_3 V_A = Y_2 (V_A - V_B - V_C) \quad (10)$$

And for balanced operation, the sum of phase voltages is zero and from (5):

$$V_2 = (V_A - V_B - V_C) = 2V_A \quad (11)$$

Substitute by (8) and (11) into (10) to get:

$$C_3 = 2C_2 \quad (12)$$

Also, substituting by (4) and (5) into (2) results in:

$$I_B = (Y_1 + 2Y_2)V_A - Y_1 V_B \quad (13)$$

Note that I_A, I_B and I_C are lagging behind V_A, V_B and V_C respectively by angle (ϕ_p), which is the machine load angle. Also I_1 and I_2 lead V_1 and V_2 respectively by 90°.The voltages V_1 and V_S are line voltages (i.e. ?3 phase voltage), where supply voltage leads V_A by 30° and V_1 lags behind V_A by 30°. With balanced currents flowing in the stator phases, a perfect rotating magnetic field is produced and the machine runs as if it is supplied from a balanced 3-ph supply. Calculating the capacitors can be carried out using symmetrical components method. The symmetrical component equations for a Y-connected system may be written as follows:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_0 \\ V_p \\ V_n \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_0 \\ I_p \\ I_n \end{bmatrix} \quad (15)$$

For balanced operation, the sum of phase voltages and currents are zeros, and therefore zero sequence current and voltage are absent. Since

$$I_p = Y_p V_p \ \& \ I_n = Y_n V_n \quad (16)$$

Substitute by (14), (15) and (16) into (1) and (13) yields:

$$V_p = \frac{\sqrt{3}V_S(2Y_2 - a^2 Y_n + (1-a^2)Y_1)}{a(1-a)(Y_p + 2Y_2 + 3Y_1 + Y_n)} \quad (17)$$

$$V_n = \frac{\sqrt{3}V_S(aY_p - 2Y_2 - (1-a)Y_1)}{a(1-a)(Y_p + 2Y_2 + 3Y_1 + Y_n)} \quad (18)$$

For balanced operation, the negative sequence voltage must be absent. It could be calculated from (18) that:

$$aY_p - 2Y_2 - (1-a)Y_1 = 0 \quad (19)$$

Substitute by (8) into real and imaginary parts of (19), and rearranging, the phase capacitors can be written as follows:

$$\left.\begin{aligned} B_1 &= \frac{2}{\sqrt{3}}(y_p)\sin(\phi_p - 30^\circ) \\ B_2 &= (y_p)\sin(60^\circ - \phi_p) \\ B_3 &= 2B_2 = (2y_p)\sin(60^\circ - \phi_p) \end{aligned}\right\} \quad (20)$$

The values of phase converter capacitors depend on (y_p) and (φ_p) which are both function of the rotor speed only. Therefore, each operating speed has certain susceptance values for the balanced mode. Table 1 shows the type of the balancing susceptances as the load angle (φ_p) varies. Normally, the load angle of the machine within the torque-speed stable range varies between π/6 to π/3. Therefore in this paper, the balancing susceptances are considered capacitive. A software program is written to predict the performance of the machine at balanced mode for each speed.

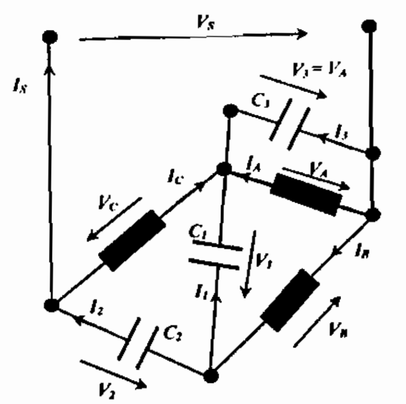


Fig. 1 Connection diagram of a 3-ph IM fed from a 1-ph supply using three capacitors

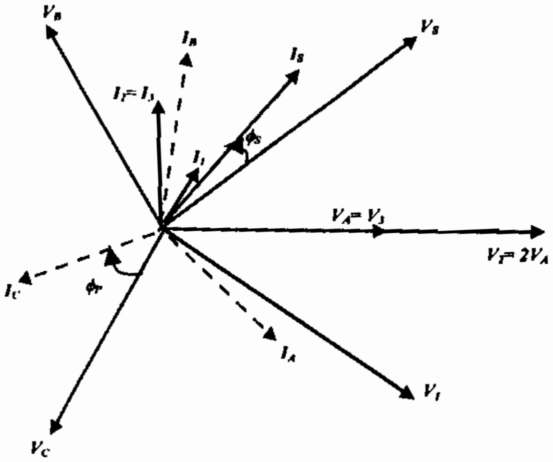


Fig. 2 Motor under perfect phase balance

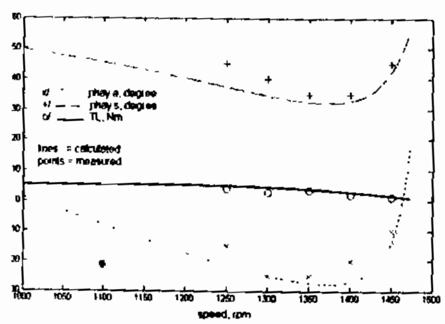
The values of (y_p) and (φ_p) are calculated from the machine equivalent circuit, then C_1, C_2 and C_3 are obtained from cquation (20).For a given phase voltage, relationships between speed versus torque, admittance angle, power factor, balancing capacitors, current magnitudes and current phase angles are evaluated as depicted in figure 3. Experimental results, between 1250 rpm and 1450 rpm, are obtained and plotted as points in the same graph as shown in figure 3. A sort of correlation between simulated and experimental results can be seen from figure 3.

Table 1: Type of balancing susceptances with the load angle

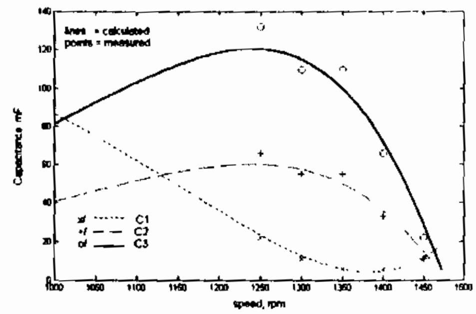
	B1	B2	B3
$\pi/6 > \phi_p$	inductor	capacitor	Capacitor
$\pi/6 < \phi_p < \pi/3$	capacitor	capacitor	Capacitor
$\phi_p > \pi/3$	capacitor	inductor	Inductor

5. Motor Dynamic Equations

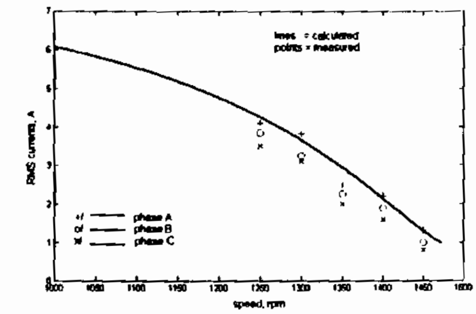
The equations describing motor transient performance can be derived using instantaneous symmetrical component (ISC) approaches [6-7].



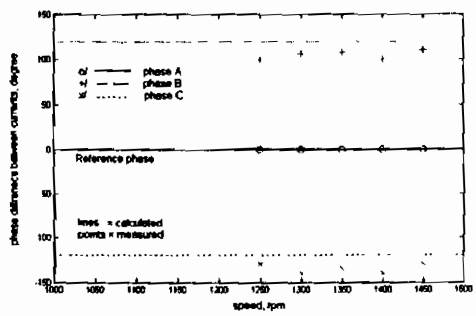
(a)



(b)



(c)



(d)

Fig. 3 Simulated (lines) and experimental (points) relationship at balanced operation between speed versus (a) torque, admittance angle and power factor (b) balancing capacitors (c) current magnitudes and (d) current phase angles

Equations (1) and (13) may be rewritten in an instantaneous form as:

$$v_s = v_a - v_C \quad (21)$$

$$i_b = B_1 p(v_a - v_b) + B_2 p(v_a - v_b - v_c) \quad (22)$$

Where: $p = \frac{1}{\omega}\frac{d}{dt}$

Equations (14) and (15) are also valid in the transient period, they represent the instantaneous symmetrical components.

Substitute by (14), (15) into (21) and (22) yields:

$$\sqrt{3} v_s = (1-a^2)v_p + (1-a)v_n \quad (23)$$

$$a i_{pt} + a^2 i_{ns} = \begin{array}{l}(B_1 p(1-a) - 2B_2 p)v_p + \\ (B_1 p(1-a^2) + 2B_2 p)v_n\end{array} \quad (24)$$

Voltage balance equations in terms of sequence current and impedance can be derived from the equivalent circuit shown in figure 4, where the rotational losses are neglected, as follows:

$$v_p = (R_s + X_s p)i_{ps} + X_m p i_{pr} \quad (25)$$

$$v_n = (R_s + X_s p)i_{ns} + X_m p i_{nr} \quad (26)$$

$$\left(\frac{R_r}{1 - j\frac{n}{p}} + X_r p\right)i_{pr} + X_m p i_{ps} = 0 \quad (27)$$

$$\left(\frac{R_r}{1 + j\frac{n}{p}} + X_r p\right)i_{nr} + X_m p i_{ns} = 0 \quad (28)$$

Where: $X_s = X_1 + X_m$ and $X_r = X_2 + X_m$

The ISC currents (***i_{ps}, i_{ns}, i_{pr}*** and ***i_{nr}***) can be written in terms of its real and imaginary parts as:

$$\left.\begin{aligned} i_{ps} &= i_{sr} + j i_{sg} \\ i_{ns} &= i_{sr} - j i_{sg} \\ i_{pr} &= i_{rr} + j i_{rg} \\ i_{nr} &= i_{rr} - j i_{rg} \end{aligned}\right\} \quad (29)$$

Equations (23) to (29) may be rearranged to conclude the following state space representation:

$$p[i]^{s,r} = [X]^{-1}.[v] \quad (30)$$

Where: $[i]^{s,r} = [i_{sr} \quad i_{sg} \quad i_{rr} \quad i_{rg} \quad q_{sr} \quad q_{sg}]^T$

And $q_{sg} = \frac{i_{sg}}{p}$ and $q_{sr} = \frac{i_{sr}}{p}$

The matrices [X] and [v] are given by:

$$[X] = \begin{bmatrix} \sqrt{3}X_s & -X_s & \sqrt{3}X_m & X_m & 0 & 0 \\ X_m & 0 & X_r & 0 & 0 & 0 \\ 0 & X_m & 0 & X_r & 0 & 0 \\ (3B_1 + 4B_2)X_s & \sqrt{3}B_1 X_s & (3B_1 + 4B_2)X_m & \sqrt{3}B_1 X_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

$$[v] = \begin{bmatrix} v_s - \sqrt{3}R_s i_{sr} + R_s i_{sg} \\ -nX_m i_{sg} - R_r i_{rr} - nX_r i_{rg} \\ nX_m i_{sr} + nX_r i_{rr} - R_r i_{rg} \\ -q_{sr} - \sqrt{3}q_{sg} - (3B_1 + 4B_2)R_s i_{sr} - \sqrt{3}B_1 R_s i_{sg} \\ i_{sr} \\ i_{sg} \end{bmatrix} \quad (32)$$

Moreover, the instantaneous electromagnetic torque can be expressed in terms of real and imaginary parts of motor currents as follows:

$$T_e = \frac{PX_m}{\omega}(i_{rr} i_{sg} - i_{sr} i_{rg}) \quad (33)$$

The mechanical equation of the machine is given by:

$$T_e - T_L = \frac{2}{P} J\omega \frac{dn}{dt} \quad (34)$$

6. Transient Analysis

Figure 5 shows a schematic representation of the machine model. A Matlab program is written to calculate the matrix $[X]^{-1}$ for a given operating point correspondent to certain values of (***B_1***) and (***B_2***) which are in turn functions of speed. Then a Simulink program simulates the model given in figure 5 whose input is sinusoidal waveform having the supply frequency and maximum voltage. Each operating point involving certain supply voltage, load torque and capacitors will result in balanced operation of the machine at certain speed. Having get the currents $[i]^{s,r}$, the positive and negative currents can also be obtained from (29).

6. Steady State Analysis

Equations (26) to (30) may also prediet the steady state analysis of the machine by substituting "*j*" for "*p*" and "***1-s***" for "***n***". This can be written as:

$$[V] = [Z][I] \quad (35)$$

Where:

$$[I] = [I_{sr} \quad I_{sg} \quad I_{rr} \quad I_{rg}]^T, \quad [V] = [V_{s(RMS)} \quad 0 \quad 0 \quad 0]^T,$$

$$[Z] = \begin{bmatrix} \sqrt{3}Z_s & -Z_s & \sqrt{3}jX_m & -jX_m \\ jX_m & (1-s)X_m & Z_r & (1-s)X_r \\ -(1-s)X_m & jX_m & -(1-s)X_r & Z_r \\ 1 + Z_s(3Y_1 + 4Y_2) & \sqrt{3}(1 + Z_s Y_1) & -(3B_1 + 4B_2)X_m & -\sqrt{3}B_1 X_m \end{bmatrix},$$

$Z_s = R_s + jX_s$ and $Z_r = R_r + jX_r$

Solving (35) results in the vector of state currents [I] which when transferred to the time domain yields:

$$i_{(sr,sg,rr,rg)} = \sqrt{2} I_{(sr,sg,rr,rg)} \sin(\omega t - \phi_{(sr,sg,rr,rg)}) \quad (36)$$

Substitute by (36) into (33), the instantaneous torque (***T_e***) at steady state then can be expressed as:

$$T_e = T_{av} + T_{osc} \quad (37)$$

where:

$$T_{av} = \frac{X_m p}{\sqrt{2}\omega}(I_{rr} I_{sg} \cos(\phi_{sg} - \phi_{rr}) - I_{sr} I_{rg} \cos(\phi_{rg} - \phi_{rr}))$$

$$T_{osc} = \frac{X_m p}{\sqrt{2}\omega}(I_{rr} I_{sg} \cos(2\omega t - \phi_{sr} - \phi_{rg}) - I_{rr} I_{sg} \cos(2\omega t - \phi_{sg} - \phi_{rr}))$$

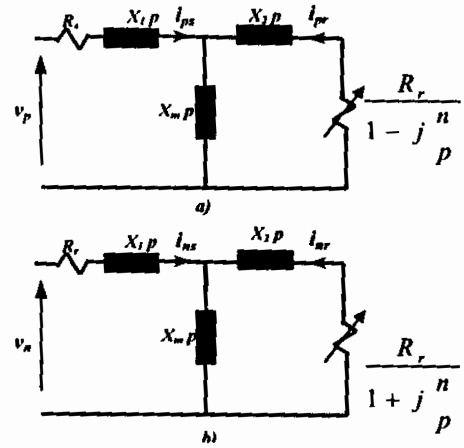


Fig. 4 Motor equivalent circuit (a) positive sequence (b) negative sequence

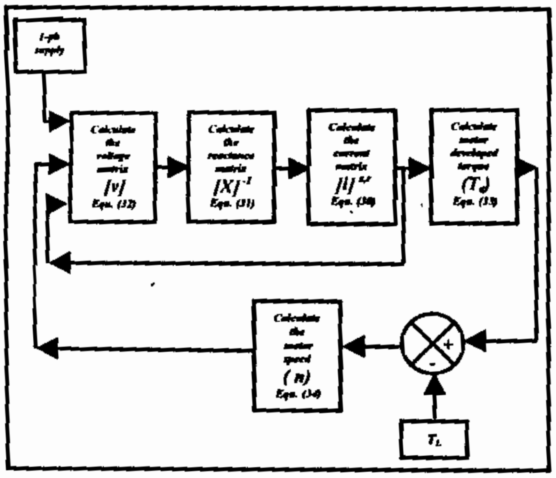


Fig. 5 SIMULINK model of the machine

It should be noted that the researeh of this paper is concentrated at steady state with balanced mode of operation. Also the steady state results for the unbalanced mode can be obtained using (35) as far as B3 = 2B2, Thus, phase currents, voltages and torque can be determined using (14), (15) and (29) for any values of C1, C2 , speed and supply voltage.

7. Results

The simulation is carried out on a 3-ph 1.5kW 220V 1500rpm Y-conneeted induction motor whose data is given by: R_1=2.08Ω, R_2=2.64Ω, X_1=5.28Ω X_2=5.28Ω, X_m=104Ω, P= 4-pole and J = 0.01kg-m^2. Figure 6a depicts the experimental waveform of the motor speed, scored by digital scope, when the 3-ph IM is fed from 1-ph supply and three capacitors are connected as shown in figure 1. Figure 6b depicts the corresponding simulated speed waveform using Simulink model based on figure 5. Figures 6c to 6h illustrate different simulated current, torque and voltage waveforms. Figure 6 can be divided into the following four periods:-

Period 1 (t_0 - t_1) : The machine starts with a ramp increasing voltage from standstill up to the operating speed, during which the machine is expected to run at unbalanced mode, and reaches the balanced mode at the operating speed. The machine will start if the mean value of the torque at starting is higher than the load torque. Therefore, conventional methods to start the induction motor may be used or the machine is started at no-load then the load is introduced when the machine runs as shown in figures 6e and 6f.

Period 2 (t_1 - t_2) : At t_1 the machine reaches the operating speed, whose capacitors are determined as given in (20), henee operates at a balanced mode. At steady state, the machine runs as if it is supplied from a 3-ph balanced supply whose phase voltage equals to (1/√3) of the 1-ph supply voltage.

Period 3 (t_2 - t_3) : While the machine runs at the balanced mode for a certain speed and load torque, the load torque is changed suddenly as shown in figure 6e. Since the value of the capacitor results in a balanced mode of operation depends on the operating speed, and if the load is changed, the speed will change causing the machine to run in an unbalanced mode .

Period 4 (t_3 – t_4) : To bring the motor back to the balaneed mode of operation, two methods may be used:

a) The capacitors are changed to new values correspondent to the new speed. A smooth variation of capacitor can be obtained using a static exciter employing a fixed capacitor-thyristor controlled reactor (FC-TCR) [9].

b) The voltage is changed to run the machine at the previous speed and the new load torque. The machine can resume the balanced mode by controlling the supply voltage, as shown in figure 6f, to keep the speed constant regardless the variation of the load torque.

From figures 6d and 6e it can be seen that during the balanced operation (t_1 - t_2 or t_3 - t_4), the negative sequence current I_n and the oscillatory torque T_{OSC} are zero while during the unbalanced operation (t_0 - t_1 or t_2 - t_3) I_n and T_{OSC} are not zero. Also during the balanced operation the 3-ph currents are equal (see figures 6c and 6g), while these currents are not equal through the unbalanced operation (see figures 6c and 6h). It should be noted that the induction motor is seen by the supply as a lagging power factor load.

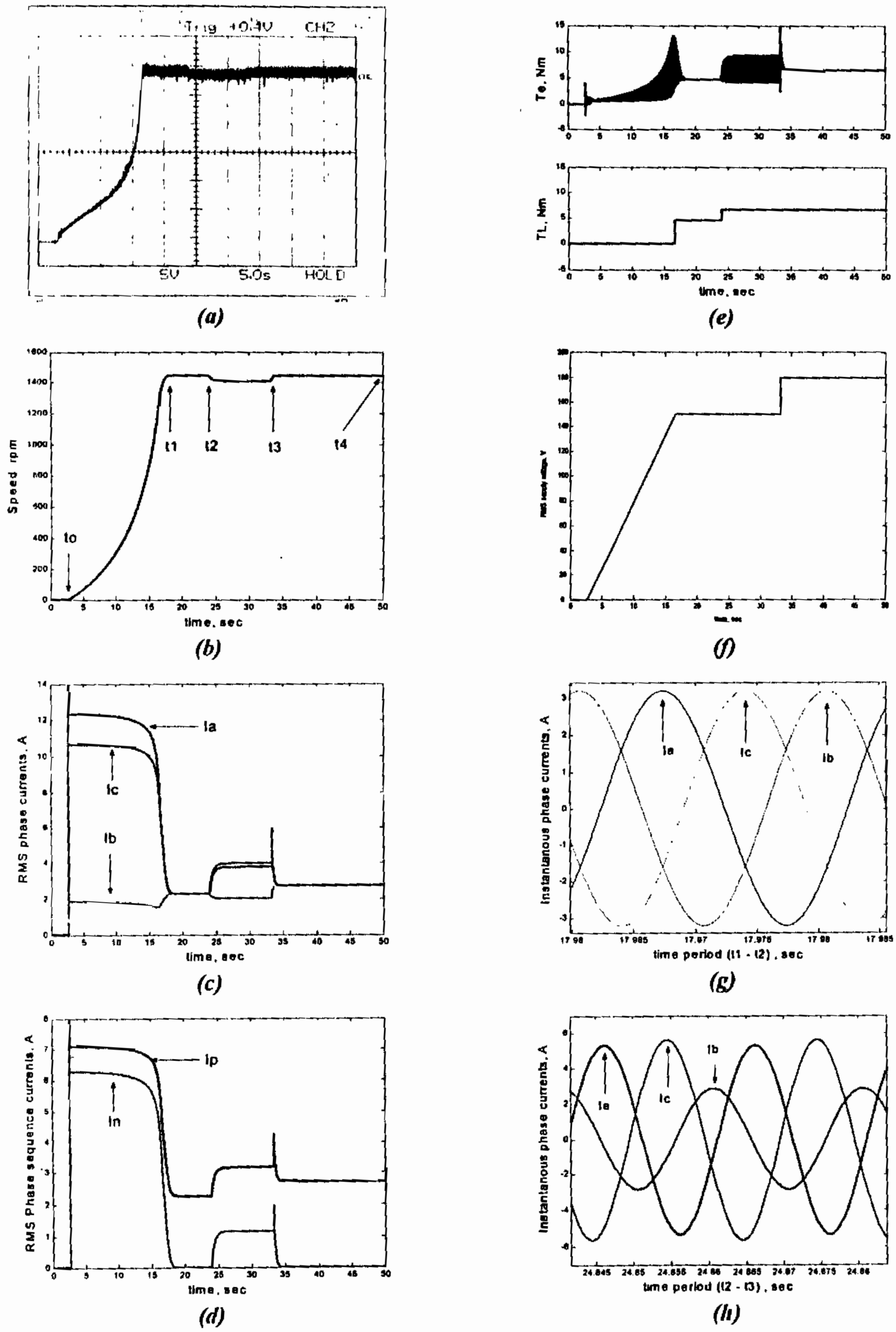


Fig. 6 Relationship between time versus (a) experimental speed, (b) simulated speed, (c) 3ph RMS phase currents, (d) RMS +ve and –ve phase sequence currents, (e) motor and load torques, (f) RMS of the 1-ph supply voltage, (g) closer view for the instantaneous 3-ph current during balanced operation [t1 - t2] and (h) closer view for the instantaneous 3-ph current during unbalanced operation [t2 - t3]

This configuration may operate at leading power factor due to the connection of the capacitors as seen from figure 2.

8. Conclusions

The three-phase induction motor can be supplied from a single-phase supply in a balanced mode of operation if three capacitors are used whose values are function of operating motor speed. The machine can work at full load with the same high efficiency that it would have had on a 3-ph supply. The machine performance is sensitive to the phase rotation. The steady state transient response of the machine is predicted using instantaneous symmetrical components. The Simulink model has been implemented and used to analyze the machine performance for different operating conditions. The simulated speed is verified and matched with the corresponding experimental waveform. If the load is changed, the speed will correspondingly change and the machine will continue to run but at unbalanced mode. The supply voltage can be then controlled to bring the machine to the balanced mode.

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