Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year: 2017-2018 Date: 3/1/2018	Subject: Mathematical Physics Code: BES 522 Time Allowed: 3 hours Year: Master Total Marks: 100 Marks

Allowed Tables and Charts: None

Answer all the following questions: [100 Marks]

Q.1 [25]

a) Deduce, classify and check the partial differential equation which has the general solution $u = x\,F(y) + y\,G(x)$, where $F(y)$ and $G(x)$ are arbitrary functions.

b) Classify, solve and check the equation $u_{xy} = u_x + 2$ with the boundary conditions: $u(0.y) = 0,\ u_x\,(x.0) = x^2$

Q.2 [25]

a) Derive the heat conduction equation $u_t = k\,\nabla^2 u$, where $u(x.y.z.t)$ is the temperature at position $(x.y.z)$ in a solid body at time t. The constant k is the diffusivity, $k = \frac{\alpha}{\sigma\rho}$, the specific heat σ and the density (mass per unit volume) ρ are assumed to be constant, α is the thermal conductivity.

b) Classify, solve and check the heat equation $u_t = u_{xx}$ with the conditions:

$u(0.t) = 0,\ u(\pi.t) = 0, \quad t > 0$

$u(x.0) = 2\sin x + 4\sin 2x + 6\sin 3x, \quad 0 < x < \pi.$

Q.3 [25]

a) Proof that:

(i) $\beta(m.n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\ \cos^{2n-1}\theta\ d\theta$

(ii) $\Gamma(n+1) = (n)!$

(iii) $\Gamma(n+1) = n\,\Gamma(n),\ n > 1$

b) Evaluate the following integrals:

(i) $\int_0^{\infty} \sqrt{y}\, e^{-y^3}\, dy$

(ii) $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$

(iii) $\int_0^{\infty} x^6\, e^{-2x}\, dy$

(iv) $\int_0^1 e^{-x^3}\, dx$

(v) $\int_0^2 \frac{x^2}{\sqrt{2-x}}\, dx$

(vi) $\int_0^1 x^4\,(1-x)^3\, dx$

(vii) $\int_0^2 \sin^6\theta\ dx$

(viii) $\int_0^{\frac{\pi}{2}} \tan^6\theta\ dx$

Q.4 **a)** Find the Fourier series of the function defined by: [25]

$$f(x) = x, \qquad 0 < x < 2\pi$$

b) Find the Fourier series of the function defined by:

$$f(x) = \begin{cases} a & 0 < x < L \\ -a & -\pi < x < 0 \end{cases}$$

(A) Find the Fourier sine series of the function defined by:

$$f(x) = x^2,\ 0 < x < \pi$$

Good Luck

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