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EFFECT OF HIGHER ORDER APPROXIMATION OF THE DEAD-TIME IN CONTROL ALGORITHM BASED ON LYAPUNOV'S DIRECT METHOD

*Dr. S. M. M. BADRAH**

ABSTRACT

Lyapunov's Direct Method has been applied by Badrah [1] to systems containing long delay time after approximating the exponential term representing this delay time to a second-order Padé-approximant and therefore, the whole control system, under investigation, has become of fourth order.

In present work, the exponential term is expanded to fourth-order Padé-approximant to get a whole system of sixth-order, in order to study the effect of increasing the order of the system on applicability of the proposed control algorithm.

A sinter plant model, given in Fig.1., is used as a numerical practical example in which the dead-time approximation is utilized in the observer loop only to obtain adequate estimation of state-variables without any additional instrumentation.

Computer programs have been written in BASIC language to suit the home micro computers. A list of computer programs is included in the Appendix.

The adequate responses of the computer simulation of the sinter plant confirm the generality and powerfulness of the techniques suggested by the author.

* *Lecturer in Mechanical Engineering Department, Faculty of Engineering and Technology, Menoufia University.*

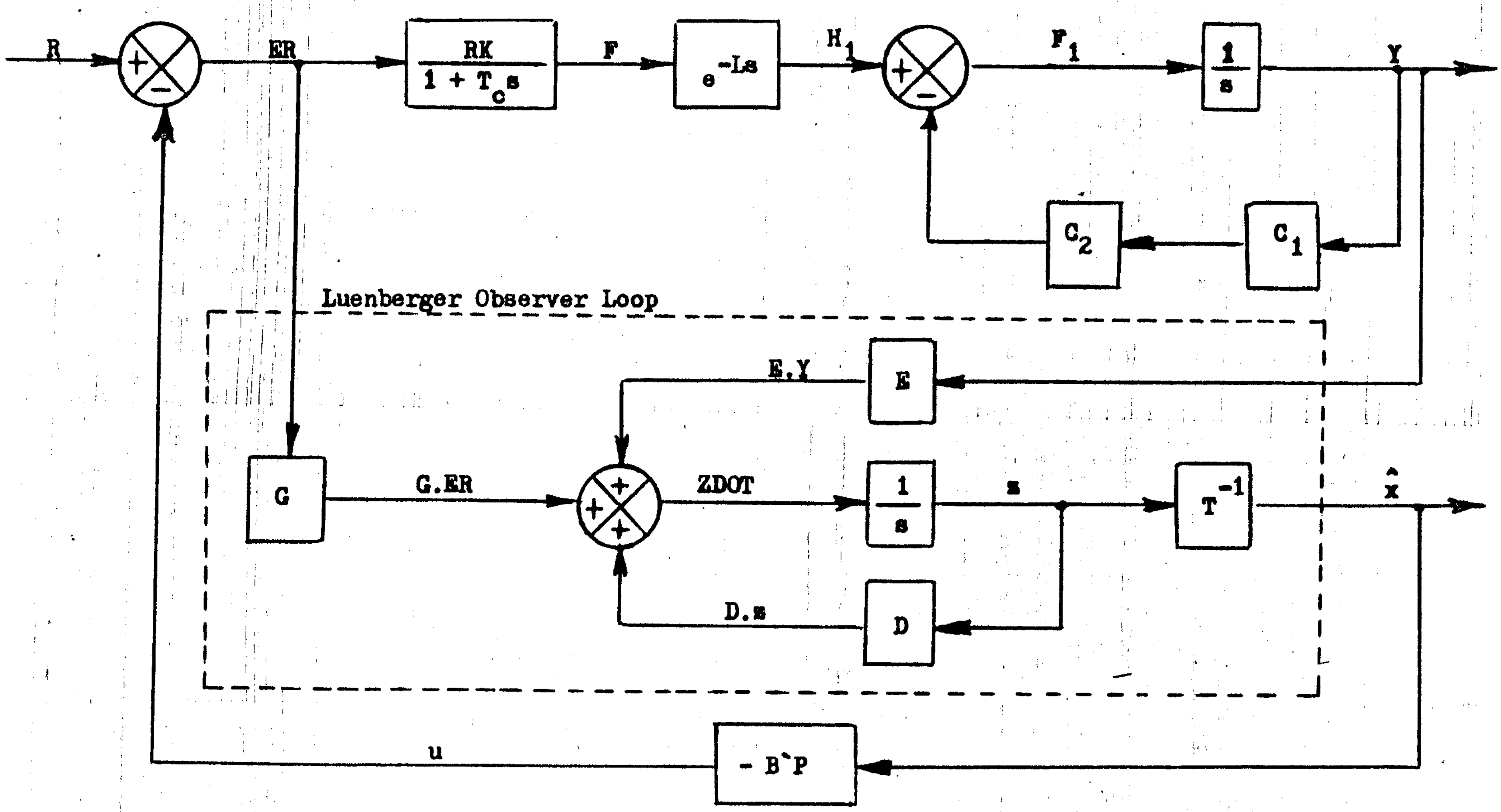

Figure 1 Block diagram for the sinter plant with the state estimator and Lyapunov controller

1. *INTRODUCTION*

In previous publications by the author [1-4], the exponential term representing the delay time in Laplace domain has been approximated using Padé-type approximant [5] to a second order rational polynomial before applying the suggested techniques based on either the Lyapunov's Direct Method or optimal regulator and poles positioning and in both cases,the sinter plant was used as an example to test the qualification of the proposed techniques.

The sintering process itself consists mainly of mixing iron-ore fines and other necessary materials (limestone and/or dolomite)with coke breeze as a solid fuel. Water is added to moisten the fines to help the agglomeration process and to impart permeability to the mix. Igniting the mix on the strand layer by layer leads in bonding the grains and a strong agglomerate is formed. The transportation of the materials from storage bins and mixing drums to the strand and combustion zone takes about 360 seconds; which is the dead-time under investigation.

Applying the approximation to the exponential term will accordingly increase the order of the control system. Hence, it is of interest to study the effect of increasing the order of the approximant on the stability of the proposed control algorithms by Badrah [1-4]. This study is presented in this paper.The exponential delay-time term is approximated to a fourth order rational polynomial according to Baker [5] to give

$$e^{-Ls} = \frac{840 - 360\, Ls + 60\, L^2 s^2 - 4\, L^3 s^2}{840 + 480\, Ls + 120\, L^2 s^2 + 16\, L^3 s^3 + L^4 s^4} \qquad \text{........(1)}$$

To obtain a comparable study, the simplified model of the sinter plant in [1] is adopted here and redrawn in Fig.1. The order of the whole system is therefore increased to sixth.

A micro computer, SINCLAIR ZX-SPECTRUM 48K, was used to simulate the plant model and the control algorithm associated with the state estimator, observer. The programs have been written in BASIC language and located in the Appendix.

2. *STATE-SPACE EQUIVALENCE OF THE SYSTEM*

The required representation of the system in state-space form can be obtained with the help of the state-variable diagram given in Fig.2 as follows

$$\dot{x}_1 = -C_1 C_2 x_1 + 840\, x_2 - 360\, L\, x_3 + 60\, L^2\, x_4 - 4L^3 x_5$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_5 = (-840\, x_2 - 480\, L\, x_3 - 120\, L^2 x_4 - 16\, L^3 x_5 + RKx_6)/L^4$$

$$\dot{x}_6 = (u - x_6)/T_c$$

which can be written in the general state-variable form:

$$x = A\, x + B\, u \quad , \quad y = H\, x \qquad (2)$$

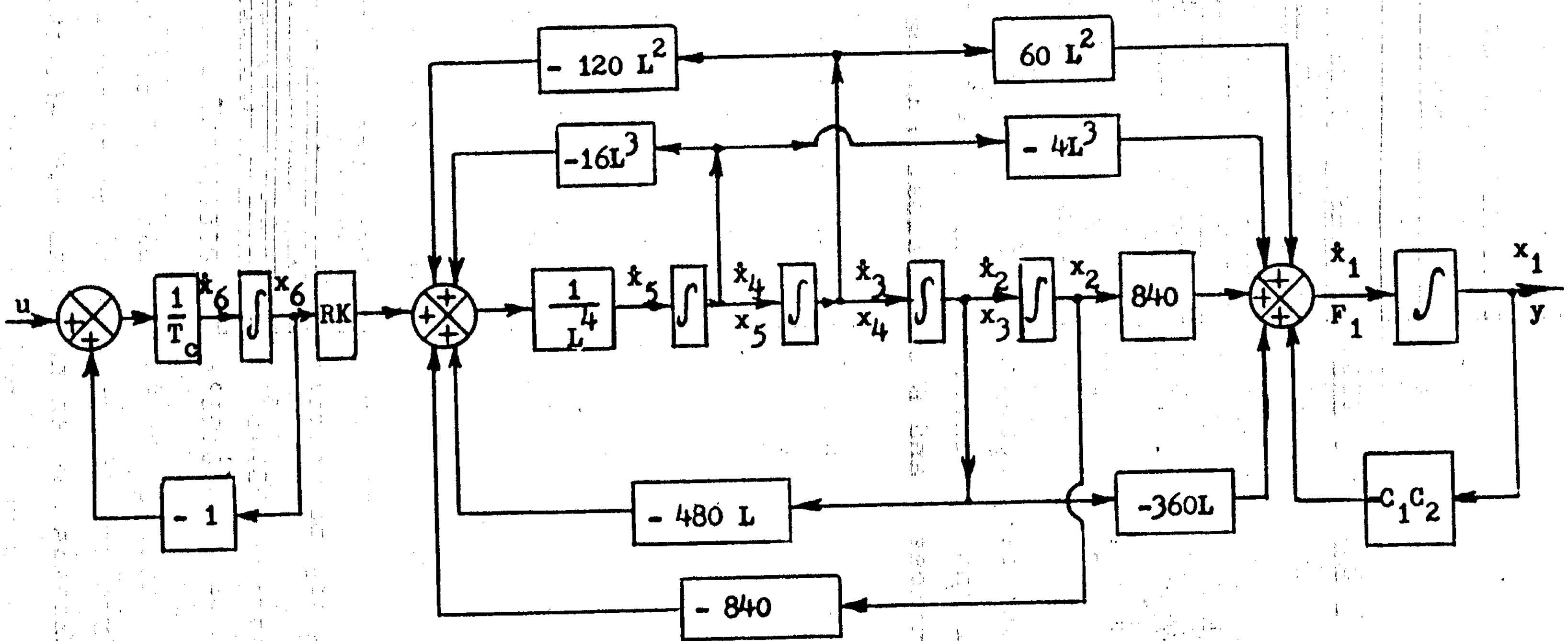

Figure 2 State-Variable diagram for the sinter plant

where

$$A = \begin{bmatrix} -C_1C_2 & 840 & -360\ L & 60\ L^2 & -4\ L^2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{840}{L^4} & -\frac{480}{L^3} & -\frac{120}{L^2} & -\frac{16}{L} & \frac{RK}{L^4} \\ 0 & 0 & 0 & 0 & 0 & -1/T_c \end{bmatrix}, \quad (3)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1/T_c \end{bmatrix}, \quad (4)$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Taking 360 seconds transportation time lag to be assigned for L and substitution for $C_1 = 0.0015$ and $C_2 = 10/3$, matrices A and B are evaluated to be

$$A = \begin{bmatrix} -0.005 & 840 & -129600 & 7776000 & -186624000 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -5.E-8 & -1.03E-5 & -9.26E-4 & -1/22.5 & 1.E-10 \\ 0 & 0 & 0 & 0 & 0 & -0.025 \end{bmatrix} \quad (6)$$

and the driving vector B will be :

$$B' = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.025] \quad (7)$$

The characteristic equation when calculated from the plant matrix A may be written as

$$(s + 0.005)(s + 0.025)[s^4 + 0.044444\ s^3 + 0.000926\ s^2 + 0.0000103\ s + 0.00000005] = 0 \quad (8)$$

The numerical values of the eigenvalues of this system are computed from eqn.(8) and are found to be: -0.005, -0.025, -0.00477345 ± j0.0058983, and -0.01744878 ± j0.0237479 to give a good indiction about the stability of this system.

3. *CONTROL ALGORITHM AND OBSERVER DESIGN*

As shown in Fig. 1, the control law is given by

$$u = - B' P \hat{x} \tag{9}$$

The Lyamunov's matrix equation

$$A'P + P A = - Q \tag{10}$$

where both P and Q are positive-definite-symmetric matrices, has been solved for the diagonal matrix Q with all elements on the diagonal of the same value as 0.1×10^{-4}, using the iteration method explained by the author in [3].

Luenberger observer is adopted to estimate the values of the state variables required for implementing the proposed technique. This will be of the form :

$$\dot{z} = D.z + E.y + G.ER \tag{11}$$

where

$$z = T . \hat{x}, \tag{12}$$

$$G = T . B, \tag{13}$$

and

$$T.A - D.T = E.H \tag{14}$$

The elements of vector H are the same as in eqn.(5), vector E has five elements and all are assigned to unity, and the fifth order D matrix is selected to be diagonal. The elements on the diagonal may be chosen, according to the given guide in [1-3], as follows :

$$D = \text{diag.}[-0.002, -0.0019, -0.0018, -0.0017, -0.0016] \quad .... \tag{15}$$

The matrix equation, given by eqn.(14), has been solved using the Kronecker product method illustrated by Badrah [1].

4. SIMULATION RESULTS AND DISCUSSION

A separate computer program was written by the author to compute the elements of the positive-definite-symmetric P matrix as a solution of the Lyapunov matrix equation, eqn.(10),using the iteration technique after eleven iterations only. Eqn.(16) gives the result of this iteration.

A second program was written to solve the matrix observer equation, eqn.(14), using Kronecker product method explained by the author in details in [1]. The result is expressed in eqn. (17).

Several attempts to apply some iteration techniques to solve eqn.(14) did not give reasonable results because both A and D matrices converge semultaneously to unit matrix leading to no solution. These attempts were tried to decrease the computer running time, since the Kronecker product method takes very long time and requires very big storage memory.

The values of matrix T and matrix P must be fed to the simulation computer program as a part of the input data, as shown in Appendix line no 332 through to line no 363. This program calculates first the inverse of matrix T after it is merged by the vector H and solves the differential equations included by applying Euler integration method.

A step input is applied in a form of 10% increase in the set point, R, after a running time of 600 seconds which is bigger than the dead-time itself. Fig.3, shows the response of the sinter plant to this disturbance with a steady state error of approximately -2%. The final value is exactly equal to 43.92 ton instead of 44 ton.

$$P = \begin{bmatrix} 1.00001243E-3 & 140.253956 & -3586.63551 & 323863.486 & 2775660.3 & 9.25240774E-3 \\ 140.253956 & 54366241.3 & 1.37022420E+9 & 1.40376566E11 & 2.35617786E12 & 7975.50582 \\ -3586.63551 & 1.3702242E+9 & 2.07973293E11 & 7.68638407E12 & 1.78165420E14 & 621163.451 \\ 323863.486 & 1.40376566E11 & 7.68638407E12 & 5.69282583E14 & 1.10200375E16 & 39208368.2 \\ 2775660.3 & 2.35617786E12 & 1.78165420E14 & 1.10200375E16 & 2.36297977E17 & 880010142. \\ 9.25240774E-3 & 7975.50582 & 621163.451 & 39208368.2 & 880010142. & 3.52024055 \end{bmatrix} \quad (16)$$

$$T = \begin{bmatrix} -333.333333 & -14786204 & -6.96854434E9 & -5.50933816E11 & -1.15144838E13 & -50062.973 \\ -322.580645 & -140238427 & -6.46142074E9 & -5.17486591E11 & -1.07484151E13 & -46529.9354 \\ -312.500000 & -133160689 & -5.99460790E9 & -4.86631579E11 & -1.00437848E13 & -43291.7590 \\ -303.030304 & -126577382 & -5.56418143E9 & -4.58117082E11 & -9.39454008E12 & -40319.9145 \\ -294.117647 & -120443118 & -5.16668661E9 & -4.31721524E11 & -8.79535623E12 & -37586.9925 \end{bmatrix} \quad (17)$$

Also, a start up condition has been tested by assigning a zero for all parameters for a period of 600 seconds, then a desired value of 40 ton is given to the set point. The response is shown in Fig.4 and has a maximum overshoot of 39.34 ton and a final steady state condition of 35.6 ton is obtained.

Comparing these results with those previously obtained, given in Fig.5, it can be seen that there is no significant effect of increasing the order of the approximation of the dead-time exponential term. Also, the proposed method via Lyapunov's Direct method is still simply applied with the higher order system, of sixth order here, which shows strength and generality of the methods suggested and previously investigated by the author.

[illegible]. CONCLUSION

The dead-time approximation to higher order approximant does not affect the applicability of the proposed techniques. The resulting increasing order of the whole control system shows the same simplicity, as before, in selecting the necessary and adequate parameters such as the D, E, and T matrices of the observer and in solving the Lyapunov matrix equation but longer computer running time is required. The system stability and the steady-state error are still within the acceptable engineering range.

The obtained results confirm the powerfulness and generality of the methods discussed and previously published by the author and perhaps encourage the attempts of applying many other modern techniques on similar systems containing long delay-time in future.

6. REFERENCES

1] Badrah, S.M.M., 1981, Ph.D. Thesis, Department of Mechanical Engineering, Leeds University, U.K.

2] Badrah, S.M.M., and Gill. K.F., 1981, Trans. Inst. M.C., Vol.3, No 3, July-Sept.

3] Badrah, S.M.M., and Gill., K.F. 1982, Proc. of 4th Int. Conference for Mechanical Power Engineering, V-32, Faculty of Engineering, Cairo University.

4] Badrah, S.M.M., 1983, Proc. of 2nd PEDAC Int. Conference, Alexandria University, Egypt.

5] Baker, G.A., Jr., 1975, Essentials of Padé Approximant, Academic Press, Inc., London.

7] APPENDIX

The simulation computer program has been written in BASIC language and the inverse of matrices can be computed using the following SUBROUTINE.

```
  2 REM **SYSTEM Progra simulat
ion of a SINTER PLANT ***
  3 DIM L(800): DIM G(6): DIM I
(6,6): DIM Q(6,6): DIM C(6)
  4 DIM A(6,6): DIM B(6): DIM D
(6,6): DIM H(6): DIM E(6)
  5 DIM Z(6): DIM P(6,6): DIM T
(6,6): DIM R(6): DIM J(6): DIM K
(6)
  6 DIM Y(6): DIM U(6): DIM F(6
): DIM N(6): DIM M(6): DIM X(6)
  7 DIM W(130)
  10 INPUT "ENTER THE ORDER OF T
HE SYSTEM: ";N: LET M=N-1: INPUT
 "Final Time: ";TF
  12 READ SAMP,DD,DL,C1,C2,R,R1,
YY
  14 READ TC,RK,IV
  16 FOR I=1 TO N
  17 READ C(I)
  18 NEXT I
  19 FOR I=1 TO M
  20 READ E(I)
  21 NEXT I
  22 FOR I=1 TO N
  23 FOR J=1 TO N
  24 READ A(I,J)
  25 NEXT J: NEXT I
  26 FOR I=1 TO N
  27 READ B(I)
  29 NEXT I
  30 FOR J=1 TO N
  31 READ H(J)
  32 NEXT J
  33 FOR I=1 TO M
  34 FOR J=1 TO M
  35 READ D(I,J)
  36 NEXT J: NEXT I
  37 LET FY=YY*C1*C2: LET UO=0
  38 FOR I=1 TO N
  39 LET G(I)=0: LET Z(I)=0: LET
 I(N,I)=H(I)
  41 IF I=N THEN GO TO 48
  42 FOR J=1 TO N
  43 READ T(I,J): LET I(I,J)=T(I
,J)
  44 LET G(I)=G(I)+T(I,J)*B(J)
  45 LET Z(I)=Z(I)+T(I,J)*C(J)
  47 NEXT J: NEXT I
  48 FOR I=1 TO N
  49 LET Q(1,I)=0
  50 FOR J=1 TO N
  51 READ P(I,J)
  52 LET Q(1,I)=Q(1,I)+B(J)*P(I,
J)
  53 NEXT J
  54 LET UO=UO+Q(1,I)*C(I)
  55 NEXT I
  57 LET R=R*C1*C2: LET R=R+UO:
LET RN=R*(1+R1): LET CN=C1*(1+R1
)
  58 BEEP 3,12: INPUT "ENTER 1 F
OR %change in R or 2 for %change
 in C1: ";ICR
  60 LET IT=1: LET TIM=0: LET IO
UT=1: LET IX=-1: LET SM=0
  64 LET LT=DD/DL
  65 FOR I=1 TO LT
  66 LET L(I)=FY
  67 NEXT I
  68 GO SUB 500
  69 LET F=FY
  73 LET VI=0
  74 REM ******* START SIMULATIO
N LOOP *****
  76 IF TIM<600 THEN GO TO 80
  77 IF ICR=2 THEN GO TO 79
  78 LET R=RN: GO TO 80
  79 LET C1=CN
  80 IF TIM<SM AND TIM>0 THEN GO
 TO 88
  82 LET U=0
  83 FOR K=1 TO N
  84 LET U=U+Q(1,K)*C(K)
  85 NEXT K
  86 LET U=-U*SGN (U): LET SM=TI
M+SAMP*DL: LET ER=R+U
  88 LET FDOT=(ER*RK-F)/TC
  89 LET F=F+DL*FDOT
  90 REM ** Represent Dead time
by shifting Technique ***
  91 LET H1=L(IT): LET L(IT)=F:
LET IT=IT+1
  92 IF IT>LT THEN LET IT=1
  93 LET SS=C1*YY: LET SH=SS*C2:
 LET F1=H1-SH
  94 LET YY=YY+DL*F1
  95 REM ** OBSERVER DYNAMIC CAL
CULATIONS **
  96 FOR I=1 TO M
  97 LET M(I)=0
  98 FOR K=1 TO M
  99 LET M(I)=M(I)+D(I,K)*Z(K)
 100 NEXT K
 101 LET F(I)=G(I)*ER: LET U(I)=
E(I)*YY
 102 LET N(I)=F(I)+U(I)+M(I)
 103 LET Z(I)=Z(I)+DL*N(I)
 104 NEXT I
 105 LET Z(N)=YY
 106 REM **CALCULATION OF XE=(T/
M) INVERSE *(Z/Y) ***
 107 FOR I=1 TO N
 108 LET C(I)=0
 109 FOR K=1 TO N
 110 LET C(I)=C(I)+I(I,K)*Z(K)
 111 NEXT K: NEXT I
 112 REM **CONTROLLING THE suita
ble time for PRINT OUT ***
 113 IF TIM<VI THEN GO TO 121
 114 LET VI=TIM+IV*DL
 115 BEEP 2,1: PRINT "****";TIM;
"......Y=";YY,"F=";F,"U=";U: LE
T W(IOUT)=YY
 116 FOR I=1 TO N: PRINT "XE(";I
;")=";C(I): NEXT I
 120 LET IOUT=IOUT+1
 121 LET TIM=TIM+DL
 122 IF TIM<=TF THEN GO TO 74
 124 BEEP 4,6: PRINT "Number of
output lines=";IOUT
 126 INPUT "ENTER 1 IF GRAPH IS
REQUIRED";I
 128 IF I=1 THEN GO SUB 400
 130 STOP
 300 DATA 10,360,.5,.0015,10/3,4
0,.1,40
 302 DATA 40,1,100
 304 DATA 40,4.231645E-7,-2.0637
76E-4,-3.18551E-6,3.68972E-7,-19
.258
 306 DATA 1,1,1,1,1
 308 DATA -.005,640,-129600,7776
000,-186624000,0
 310 DATA 0,0,1,0,0,0
 312 DATA 0,0,0,1,0,0
 314 DATA 0,0,0,0,1,0
 316 DATA 0,-5E-8,-1.03E-5,-9.26
E-4,-1/22.5,1E-10
 318 DATA 0,0,0,0,0,-.025
 320 DATA 0,0,0,0,0,.025
 322 DATA 1,0,0,0,0,0
 324 DATA -.2E-2,0,0,0,0
 326 DATA 0,-.19E-2,0,0,0
 328 DATA 0,0,-.18E-2,0,0
 329 DATA 0,0,0,-.17E-2,0
 330 DATA 0,0,0,0,-.16E-2
 332 DATA -333.333333,-14786204,
-6.96854434E9,-5.5093381E11
 333 DATA -1.15144838E13,-50062.
973
 335 DATA -322.580645,-140238427
,-6.48142074E9,-5.17488591E11
 336 DATA -1.07484161E13,-46529.
9354
 338 DATA -312.5,-133160689,-5.9
946079E9,-4.86631579E11
 339 DATA -1.00437848E13,-43291.
759
 341 DATA -303.030304,-126577382
,-5.56418143E9,-4.58117082E11
 342 DATA -9.39454008E12,-40319.
9145
 344 DATA -294.117647,-120443118
,-5.16688851E9,-4.31721524E11
 345 DATA -8.79535823E12,-37586.
9925
 347 DATA 1.00001243E-5,1.402539
56,-35.8663551,3238.63486
 348 DATA 27756.603,9.25240774E-
5
 350 DATA 1.40253956,543662.413,
1.3702242E7,1.40376566E9
 351 DATA 2.35617786E10,79.75505
82
 353 DATA -35.8663551,1.3702242E
7,2.07973293E9,7.68638407E10
 354 DATA 1.7818542E12,6211.6345
1
 356 DATA 3238.63486,1.40376566E
9,7.68638497E10,5.69282583E12
 357 DATA 1.10200375E14,392083.6
62
 359 DATA 27756.603,2.35617786E1
0,1.7816542E12,1.10200375E14
 360 DATA 2.36297977E15,8800101.
42
 362 DATA 9.25240774E-5,79.75505
82,6211.63451,392083.662
 363 DATA 8800101.42,.0352024055
```

```
500 REM ** SUBROUTINE "MATRIX"
INVERSE **
 504 LET MX=N
 508 IF IX>=0 THEN LET MX=N+1
 564 LET DT=1
 566 FOR K=1 TO N
 568 LET KM=K-1: LET PV=0
 572 FOR I=1 TO N
 576 FOR J=1 TO N
 577 REM Scan IROW and JCOL arra
ys for invalid pivot subscripts*
*
 578 IF k=1 THEN GO TO 590
 580 FOR F=1 TO KM
 582 FOR G=1 TO KM
 584 IF I=R(F) THEN GO TO 594
 586 IF J=J(G) THEN GO TO 594
 588 NEXT G: NEXT F
 590 IF ABS (I(I,J))<=ABS (PV) T
HEN GO TO 594
 592 LET PV=I(I,J): LET R(K)=I:
LET J(K)=J
 594 NEXT J: NEXT I
 595 REM **Insure that the selec
ted pivot is larger than zero**
 596 IF ABS (pv)>0 THEN GO TO 60
0
 598 PRINT "The given matrix is
sinular": RETURN
 599 REM **Update the determinan
t value***
 600 LET DT=DT*PU
 601 LET IK=R(K): LET JK=J(K)
 602 REM **Normalise the PIVOT r
ow elements **
 603 FOR J=1 TO MX
 604 LET I(IK,J)=I(IK,J)/PU
 605 NEXT J
 606 LET I(IK,JK)=1/PU: REM **Ca
rry out the eleaination and INVE
RSE
 607 FOR I=1 TO N
 608 LET AI=I(I,JK)
 609 IF I=IK THEN GO TO 614
 610 LET I(I,JK)=-AI/PU
 611 FOR J=1 TO MX
 612 IF J<>JK THEN LET I(I,J)=I(
I,J)-AI*I(IK,J)
 613 NEXT J
 614 NEXT I
 615 NEXT K
 617 REM **ORDER SOLUTION VALUES
 (IF ANY) AND CREATE K ARRAY**
 618 FOR I=1 TO N
 620 LET IU=R(I): LET JL=J(I): L
ET K(IU)=JL
 621 IF IX>=0 THEN LET X(JL)=I(I
U,MX)
 622 NEXT I
 623 REM **Adjust the sign of th
e determinant ****
 624 LET IH=0: LET N1=N-1
 626 FOR I=1 TO N1
 628 LET IP=I+1
 630 FOR J=IP TO N
 632 IF K(J)>=K(I) THEN GO TO 63
6
 634 LET JP=K(J): LET K(J)=K(I):
 LET K(I)=JP: LET IH=IH+1
 636 NEXT J: NEXT I
 638 IF (INT (IH/2)*2)<>IH THEN
LET DT=-DT
 639 REM *** IF IX is positive r
eturn the solution without the i
nverse***
 640 IF IX>0 THEN GO TO 670
 641 REM **IF IX is >=0;unscramb
le the inverse FIRST BY ROW**
 642 FOR J=1 TO N
 643 FOR I=1 TO N
 644 LET IU=R(I): LET JL=J(I): L
ET Y(JL)=I(IU,J)
 646 NEXT I
 648 FOR I=1 TO N
 650 LET I(I,J)=Y(I)
 652 NEXT I: NEXT J
 655 REM **Then BY COLUMN**
 656 FOR I=1 TO N
 658 FOR J=1 TO N
 660 LET IJ=R(J): LET JJ=J(J): L
ET Y(IJ)=I(I,JJ)
 662 NEXT J
 664 FOR J=1 TO N
 666 LET I(I,J)=Y(J)
 668 NEXT J: NEXT I
 670 PRINT "DETERMINANT of the g
iven MATRIX=";DT
 672 RETURN
```

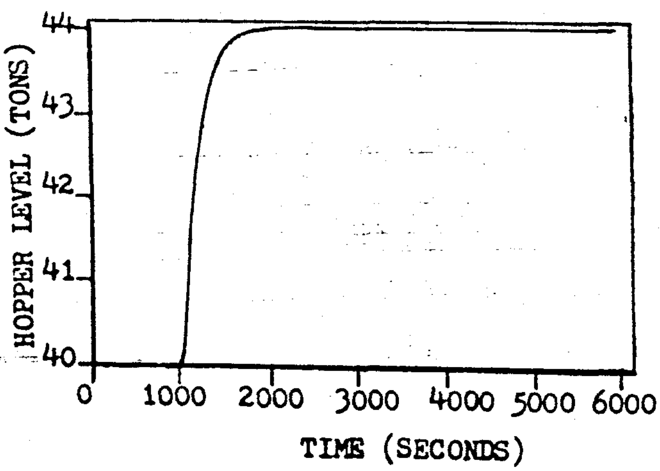

Figure 3 Set point is increased by 10%.

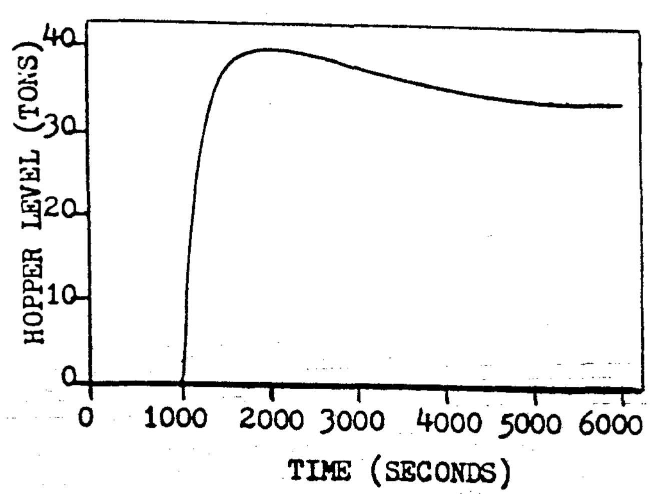

Figure 4 Set point is increased from zero to 40 tons.

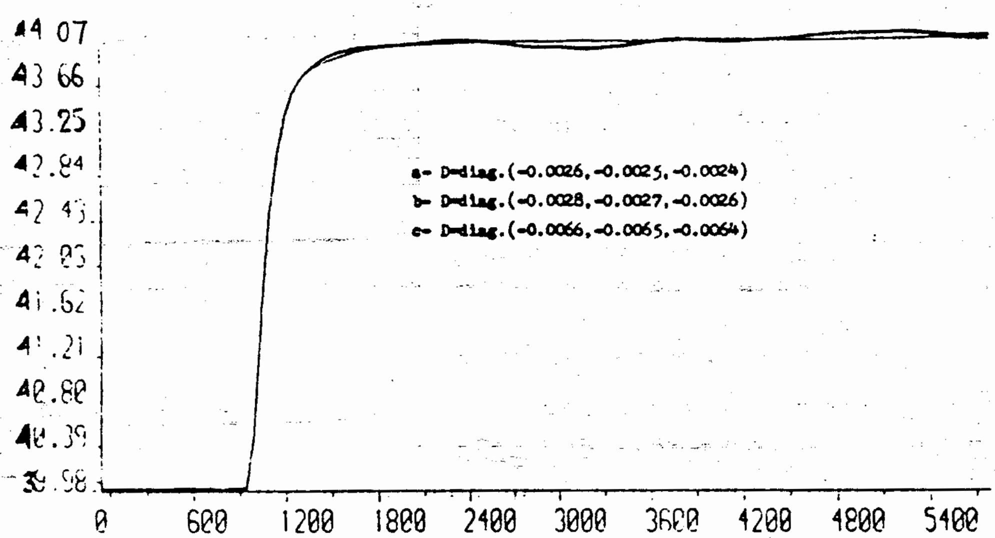

Figure 5 Responses to an increase of 10% in set point for different values of the observer D matrix with a second order approximation of the dead-time.