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A THEORETICAL STUDY ON THE IMPLEMENTATION OF ANALOG AND SAMPLED DATA ADAPTIVE FILTERS

Fayez W. Zaki and Yehia M. I. Enab

Faculty of Engineering, El-Mansoura University

دراسة تحليلية فى بناء المرشحات المتهايئة القياسية منها و الرقمية

ملخص

يصف هذا البحث التأثيرات الناتجة من عدم مثالية القطع الألكترونية على أداء المرشحات المتهايئة القياسية و الرقمية و يبين أن البناء القياسي يختلف بشكل كبير عن ذلك الموجود فى الحالات المثالية حيث أن تأثيرات إنحراف الجهد أو التيار و كذا عدم الخطية فى الخواص كلها تؤدى إلى زيادة فى متوسط مربع الخطأ بطريقة تتناسب عكسيا مع معامل التحكم فى الإستقرار و معدل التقارب للخوارزم المستخدم

يخلص البحث إلى تقديم بعض المقترحات لبناء دوائر التكامل وكذا دوائر الضرب القياسية بهدف التقليل من التأثيرات المشار إليها عن طريق توزيع كسب دوائر المضاعفات حول مسار الإشارة.

ABSTRACT

This paper explains the effects of nonideal circuit elements upon the performance of analog and sampled data adaptive filters. It is shown that the effects in case of analog implementations differ significantly from that encountered in ideal case. Moreover, it is shown that the effects of nonzero mean errors, such as offset voltages and nonlinearities of the input multipliers contribute an excess mean square error which is inversely proportional to the step size which controls the stability and the rate of convergence of the updating algorithm. To overcome most of these errors, solutions are presented and discussed.

I-INTRODUCTION

An adaptive filter is, in some sense, a self designing (really self optimising) filter [1]. In other words, it is a filter whose frequency response or transfer function is altered, or adapted, to pass without degradation the desired components of the signal and to attenuate the undesired or interfering signals, or to reduce any distortion on the input signal.

Although adaptive filtering techniques have been reported in literature for over two decades and have been implemented for some time as off line processors [2], it is only the recent advances in large scale integration (LSI) and very large scale integration (VLSI) circuit design techniques that have increased the interest in their hardware realisation. The interest in analog and sampled data adaptive filters was aided by the advances in charge coupled devices (CCD) and bucket brigade devices (BBD) [3] and [4]. However, the performance of these analog filters is ultimately limited by restrictions in dynamic range such as nonlinearities and noise effects, and this has fostered the development of hybrid or fully digital adaptive filters. Therefore, it was felt that by providing information on the sources of errors and limitations resulting from these errors as well as solutions, analog and sampled data adaptive filters might gain greater acceptance among researchers.

The first known work to consider component imperfections was performed in 1963 by Low [5]. He proved that systems constructed with imperfect components could be expected to converge to a solution if such a solution existed. Other previous work in adaptive filtering by Kaunitz [6] assumed an added random noise. Widrow et al [7] showed that the effect of an additive zero mean noise in the weight vector is an excess mean square error which is proportional to the step size μ. This noise was added to the weights to model the errors caused by the estimation of the actual gradient of the mean square error performance surface. Rosenberger [8] assumed that a zero mean band limited random process with a finite variance was added into the output of an adaptive noise canceller before it was fed back to the weights. His results showed that the maximum achievable echo suppression, for this case, was inversely proportional to μ, i.e., the smaller the μ the better the system worked. Thomas [9] made the same assumptions as Rosenberger, and showed that the choice of μ which allowed the most convergence is the smallest value of μ. Other papers [10,11,12] considering nonideal multipliers showed that the qualitative behaviour of adaptive filters, using nonideal multipliers is similar to that of ideal adaptive filters. These results agreed with the general belief that adaptive filters can adapt around their own internal errors. It will be shown in this paper that this is not the case when one considers the effects of internal nonzero mean errors.

II-BACKGROUND

The basic form of a sampled data adaptive filter is shown in Fig. 1. It consists of an adaptive linear combiner [1] with variable weights W_i, i=1,2,...M. These weights are controled by the least mean square (LMS) algorithm [1,2]. The algorithm attempts to minimise the mean square error (MSE) between the combiner output y(t) and a reference signal d(t). The way in which this reference signal is obtained depends on the application of the adaptive filter.

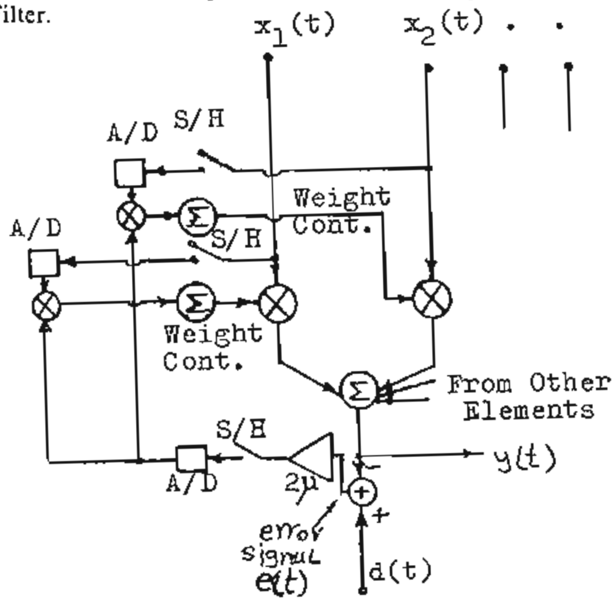

Fig. 1, Sampled Data Adaptive Filter Using LMS Algorithm

For an Mth order adaptive filter, the input signal vector may be defined as:

$$\underline{X}(n) = [\ x_1(n)\quad x_2(n)\quad \ldots\quad x_M(n)\ ]^T \tag{1}$$

The variable weight vector is defined as:

$$\underline{W}(n) = [\ w_1(n)\quad w_2(n)\quad \ldots\quad w_M(n)\ ]^T \tag{2}$$

The filter output is given by:

$$y(n) = \underline{W}^T(n)\underline{X}(n) = \underline{X}^T(n)\underline{W}(n) \tag{3}$$

therefore, the error signal $e(n)$ is expressed as:

$$e(n) = d(n) - y(n)$$

$$= d(n) - \underline{W}^T(n)\underline{X}(n) \tag{4-a}$$

or

$$e(n) = d(n) - \underline{X}^T(n)\underline{W}(n) \tag{4-b}$$

The squared value of $e(n)$ is therefore,

$$e^2(n) = d^2(n) - 2d(n)\underline{W}^T(n)\underline{X}(n) + \underline{W}^T(n)\underline{X}(n)\underline{X}^T(n)\underline{W}(n) \tag{5}$$

The mean square error (MSE) or cost function ξ is defined as:

$$\xi(n) = E[e^2(n)]$$

$$= E[d^2(n)] - 2E[d(n)\underline{X}^T(n)]\underline{W}(n) + \underline{W}^T(n)E[\underline{X}(n)\underline{X}^T(n)]\underline{W}(n) \tag{6}$$

where $E[.]$ denotes expectation operation. Define the cross correlation between the reference signal $d(n)$ and the vector $\underline{X}(n)$ as:

$$\underline{S}(n) = E[d(n)\underline{X}(n)] \tag{7}$$

and the input correlation matrix as:

$$\underline{R}(n) = E[\underline{X}(n)\underline{X}^T(n)] \tag{8}$$

Hence, the MSE $\xi(n)$ at time n can be expressed in terms of $\underline{S}(n)$ and $\underline{R}(n)$ as:

$$\xi(n) = E[d^2(n)] - 2\underline{S}^T(n)\underline{W}(n) + \underline{W}^T(n)\underline{R}(n)\underline{W}(n) \tag{9}$$

The MSE in Eq.(9) is a quadratic function of the weights $\underline{W}(n)$. Such a function has only one minimum. The object of the adaptive algorithm is to adjust the weights w_i, $i=1,2,\ldots M$ so that the minimum mean square point is reached. Gradient methods are commonly used for this purpose [1,2]. The gradient vector $\underline{G}(n)$ of the MSE can be obtained by differentiating Eq.(9) w.r.t $\underline{W}(n)$ as:

$$\underline{G}(n) = \begin{bmatrix} \partial\xi(n)/\partial w_1(n) \\ \partial\xi(n)/\partial w_2(n) \\ \ldots\ldots \\ \ldots\ldots \\ \partial\xi(n)/\partial w_M(n) \end{bmatrix} = -2\underline{S}(n) + 2\underline{R}(n)\underline{W}(n) \tag{10}$$

For stationary input process, the optimum weight vector $\underline{W}^*$ can be obtained by setting $\underline{G}(n)$ in Eq.(10) equal to zero, i.e.,

$$2\underline{S} - 2\underline{R}\underline{W}^* = 0 \tag{11}$$

and therefore,

$$\underline{W}^* = \underline{R}^{-1}\underline{S} \tag{12}$$

Note that $\underline{R}(n)$ and $\underline{S}(n)$ are replaced by $\underline{R}$ and $\underline{S}$ for stationary input process. Eq.(12) is referred to as Weiner-Hopf equation [12]

III-ADAPTIVE ALGORITHMS

The major task of the adaptive algorithm is to find a recursive solution to Eq.(12) avoiding matrix inversion. One way of doing this would be to use the steepest descent method [1]. In this method, the adaptation starts with an arbitrary set of initial values, $\underline{W}(0)$, for the weights. An iterated change in the weighting coefficients in the direction of the negative gradient of the MSE is performed until the minimum point is reached. Therefore, the steepest descent updating algorithm for the weights is expressed as:

$$\underline{W}(n+1) = \underline{W}(n) + \mu(-\underline{G}(n)) \tag{13}$$

where μ is a positive constant (step size) which controls stability and the rate of convergence.

The method of steepest descent described above requires the determination of the gradient vector at successive points on the MSE performance surface. In practice, the true values of the gradient are not available (the calculation of expectation is not feasible in practice). To overcome this difficulty, the least mean square (LMS) algorithm [1] determines a gradient estimate instead of the true gradient. This gradient estimate is obtained by considering the square value of the instantaneous error signal as an estimate of the MSE. Therefore, by differentiating Eq.(5) w.r.t $\underline{W}(n)$ yields

$$\hat{\underline{G}}(n) = \begin{bmatrix} \partial e^2(n)/\partial w_1(n) \\ \partial e^2(n)/\partial w_2(n) \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \partial e^2(n)/\partial w_M(n) \end{bmatrix} = -2e(n)\underline{X}(n) \tag{14}$$

Substituting Eq.(14) into Eq.(13) yields the LMS algorithm as:

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu e(n)\underline{X}(n) \tag{15}$$

Sampled data adaptive filters using LMS algorithm has been given already in Fig. 1. Although this was mainly intended to illustrate mathematical procedures and basically a block diagram representation of the equations, it is probably the most efficient implementation in terms of adaptation rate and quality of solution. It is also very expensive to implement (demanding real time digital data throughout) and for practical purposes it is better to adopt cheaper methods either analog or hybrid analog/digital schemes which have convergence rate penalty but give just as good as solution eventually. In many practical interference cases these slower implementation are entirely adequate.

Analog implementation of the LMS algorithm in Eq.(15), is done simply by setting [14]

$$d\underline{W}(t)/dt = -\mu\hat{\underline{G}}(t)$$

$$= 2\mu e(t)\underline{X}(t) \qquad (16)$$

and solving with a set of integrators, i.e.,

$$\underline{W}(t) = \underline{W}(0) + \mu\int_0^t \underline{G}(t)\,dt$$

$$= \underline{W}(0) + 2\mu\int_0^t e(t)\underline{X}(t)\,dt \qquad (17)$$

Continuous time (analog) implementation of the adaptive filter with the LMS algorithm in Eq.(17) is shown in Fig. 2.

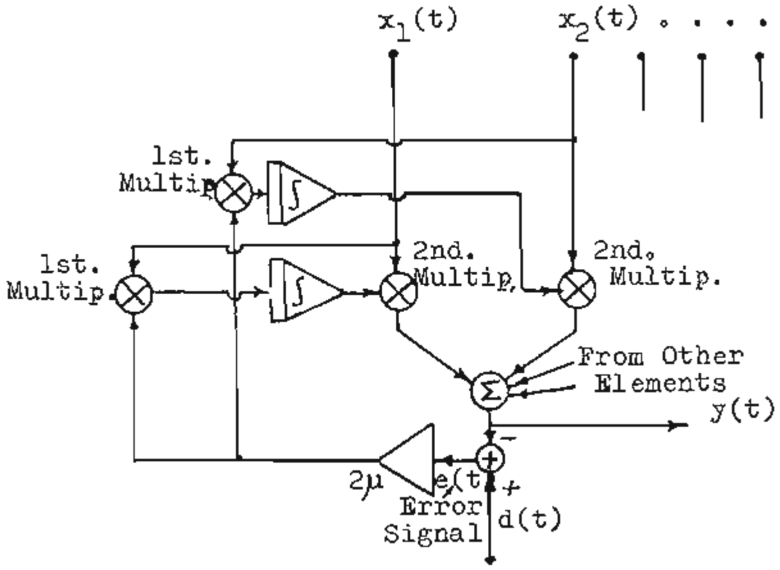

Fig. 2, Analog Adaptive Filter Using Continuous Time LMS Algorithm.

IV-EFFECTS OF COMPONENT IMPERFECTIONS

For a practical multiplier, the output will differ from the theoretical product of its inputs by an amount ϵ, as defined by

$$V_o = K_1 V_x V_y + \epsilon(V_x, V_y) \qquad (18)$$

Where V_o is the multiplier output voltage, V_x and V_y are the multiplier inputs, and $K_1 V_x V_y$ is the true multiplier product. The error term can be expanded into terms directly related to the error sources in the multiplier circuit [15]. In Fig. 2, each of the input multipliers has two inputs defined as $x_i(t)$ and $2\mu e(t)$, where i is the tap number. Therefore, the output voltage of the ith first multiplier is given by

$$V_{oi}^{1} = K_1[(x_i(t) + x_{os})(2\mu e(t) + y_{os}) + z_{os} + f(x_i(t), 2\mu e(t))] \qquad (19)$$

where the superscript 1 refers to first multiplier. The three sources of d.c. errors in an analog multiplier are: input offsets x_{os}, y_{os}, output offset z_{os}, and nonlinearity $f[x_i(t), 2\mu e(t)]$. Hence, Eq.(19) can be rewritten as:

$$V_{oi}^{1} = 2\mu K_1 e(t)x_i(t) + K_1[x_i(t)y_{os} + 2\mu e(t)x_{os} + z_{os} + k_2 x_i^2(t) + k_3(2\mu e(t))^2] \quad (20)$$

If the multiplier is followed by a gain amplifier such that the true product is $2\mu x_i(t)e(t)$, then one can assume that $K_1=1$. This yields

$$V_{oi}^{1} = 2\mu e(t)x_i(t) + [(x_i(t)y_{os} + 2\mu e(t)x_{os}) + z_{os} + k_2 x_i^2(t) + k_3(2\mu e(t))^2] \quad (21)$$

where $x_i(t)y_{os} + 2\mu e(t)x_{os}$ are feedthrough terms due to input offset voltages, z_{os} is an output offset voltage independent of $x_i(t)$ and $e(t)$, and $k_2 x_i^2(t)$ and $k_3(2\mu e(t))^2$ are nonlinear terms due to transistor mismatch. The feedthrough terms can be neglected in high frequency applications (since the output of the first multiplier goes into an integrator), but must be considered in low frequency applications. The term $k_2 x_i^2(t)$ and $k_3(2\mu e(t))^2$ result in a nonzero d.c. component being added to the weights. The output offset voltage z_{os} is added to the true product. The nonlinear terms are important because the value of their d.c. component is dependent on input signal power. Therefore, although one could adjust the d.c. balance of the multiplier so that $z_{os} + k_2 x_i^2(t) + k_3(2\mu e(t))^2$ is zero for particular values of $x_i(t)$ and $e(t)$, the balance will be destroyed when one of the signal power levels is changed.

The effects of component imperfections will be presented in four parts. First, the effects of input multiplier output offset voltages will be explained; second, the combined effects of input multiplier output offset voltages and nonlinearities will be presented; third, the effects of integrator offset voltages and bias currents will be shown; and fourth, the other nonlinearities will be considered.

a)The Effects of Input Multiplier Output Offset Voltages

Assuming that all of the circuit components used in the construction of a sampled data adaptive filter are ideal, except that an output offset voltage error occurs in the first multipliers, the output of the first multiplier in the ith coefficient (tap) can be expressed as:

$$V_{oi}^{1}(n) = 2\mu e(n)x_i(n) + z_{osi} \quad (22)$$

Define the vector

$$\underline{Z}_{os} = [z_{os1} \quad z_{os2} \quad \cdots \quad z_{osM}]^T \quad (23)$$

whose elements z_{osi} are the output offset voltages for the ith input multiplier, where z_{osi} is a random variable which can take on a value between zero and z_{os} (max.). These values can be obtained from manufacturer's data sheets. The expected value of z_{os} is considered to be a constant d.c. voltage for all time after the power switch is turned on. Therefore,

$$\underline{Z} = E[\underline{Z}_{os}] \quad (24)$$

Applying Eq.(23) into the LMS algorithm in Eq.(15) yields

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu e(n)\underline{X}(n) + \underline{Z}_{os} \quad (25)$$

Substituting for e(n) from Eq.(4-b), then

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu\underline{X}(n)[d(n) - \underline{X}^T(n)\underline{W}(n)] + \underline{Z}_{os} \qquad (26)$$

Taking the expected value of both sides of Eq.(26) and assuming $\underline{W}(n)$ to be fixed, then

$$\underline{W}(n+1) = \underline{W}(n) - 2\mu E[\underline{X}(n)\underline{X}^T(n)]\underline{W}(n) + 2\mu E[d(n)\underline{X}(n)] + E[\underline{Z}_{os}]$$

$$= [\underline{I} - 2\mu\underline{R}]\underline{W}(n) + 2\mu\underline{S} + \underline{Z} \qquad (27)$$

where $\underline{I}$ is an MxM unit matrix and $\underline{R}$ and $\underline{S}$ are as defined before. As the autocorrelation matrix $\underline{R}$ is positive definite, it can be expressed in normalised form as:

$$\underline{R} = \underline{Q}^{-1}\underline{\Delta}\,\underline{Q} \qquad (28)$$

where $\underline{\Delta}$ is a diagonal matrix of the eigenvalues of $\underline{R}$. The matrix $\underline{Q}$ is a square matrix called the modal matrix of $\underline{R}$. Its columns are assumed to be orthonormal eigenvectors of $\underline{R}$. Consequently [1]

$$\underline{Q}\underline{Q}^T = \underline{I} \quad \text{and} \quad \underline{Q}^{-1} = \underline{Q}^T \qquad (29)$$

Now, let us study the transient response of Eq.(27). First we make a rotation of coordinates [1] into the "primed" coordinates system such that:

$$\underline{W}(n) = \underline{Q}\underline{W}'(n) \qquad (30)$$

This will cause a rotation of coordinates into the principal axis of $\underline{R}$. Substituting Eq.(28), (29), and (30) into Eq.(27) and premultiplying both sides by $\underline{Q}^T$, Eq.(27) becomes

$$\underline{W}'(n+1) = [\underline{I} - 2\mu\underline{\Delta}]\underline{W}'(n) + 2\mu\underline{Q}^T\underline{S} + \underline{Q}^T\underline{Z} \qquad (31)$$

Define

$$S' = \underline{Q}^T\underline{S} = \begin{bmatrix} s_1 \\ s_2 \\ \cdots \\ \cdots \\ s_M \end{bmatrix} \qquad (32)$$

and

$$\underline{Z}' = \underline{Q}^T\underline{Z} = \begin{bmatrix} z_1 \\ z_2 \\ \cdots \\ \cdots \\ z_M \end{bmatrix} \qquad (33)$$

Therefore, Eq.(31) may be expressed as:

$$\underline{W}'(n+1) = [\underline{I} - 2\mu\underline{\Delta}]\underline{W}'(n) + 2\mu\underline{S}' + \underline{Z}' \qquad (34)$$

The general solution of Eq.(34) depends on the eigenvalues of the $\underline{R}$ matrix.

A scalar expression for each of the primed weights can be deduced from Eq.(34) as:

$$w'_i(n+1) = (1 - 2\mu\lambda_i)w'_i(n) + 2\mu s'_i + z'_i \tag{35}$$

where λ_i is the ith eigenvalue of $\underline{R}$ and s'_i and z'_i are the ith primed cross correlation and offset voltage error respectively. With initial weight vector $\underline{W}(0)$, n+1 iterations of Eq.(35) yield

$$w'_i(n+1) = (1 - 2\mu\lambda_i)^{n+1} w'_i + 2\mu s'_i \sum_{m=0}^{n} (1 - 2\mu\lambda_i)^m + z'_i \sum_{m=0}^{n} (1 - 2\mu\lambda_i)^m \tag{36}$$

If μ is made small enough so that the element $(1 - 2\mu\lambda_i)$ has magnitude less than one, then as the number of iterations increases, the limit [16]

$$\lim_{n\to\infty} (1 - 2\mu\lambda_i)^{n+1} \longrightarrow 0 \tag{37}$$

This requires

$$|1 - 2\mu\lambda_i| < 1$$

or

$$0 < \mu < 1/\lambda_i \tag{38}$$

Considering the second and third terms in the right hand side of Eq.(36) when μ satisfies condition (38) so that the element $(1 - 2\mu\lambda_i)$ is less than unity, it can be shown by summing the geometric series that

$$\lim_{n\to\infty} \sum_{m=1}^{n} (1 - 2\mu\lambda_i)^m \longrightarrow 1/2\mu\lambda_i \tag{39}$$

Substituting Eq.(37) and (39) into Eq.(36) yields

$$\lim_{n\to\infty} w'_i(n+1) = 0 + s'_i/\lambda_i + z'_i/2\mu\lambda_i$$

or

$$w'_i(n+1) = w^{*\prime}_i(n) + z'_i/2\mu\lambda_i \tag{40}$$

Hence for positive eigenvalue λ_i and μ satisfies Eq.(38), the effect of the input multiplier output offset voltage is to alter the steady state solution of the weight by a value equal to $z'_i/2\mu\lambda_i$. On the other hand, if λ_i is zero, the limit in Eq.(39) tends to and

$$w'_i(n+1) = w^{*\prime}_i(n) + z'_i \sum_{m=0}^{\infty} 1^m = \infty \tag{41}$$

Thus the input multiplier output offset voltages cause the weights corresponding to zero (or negative) eigenvalues to increase indefinitely. Therefore, these weights never reach a steady state solution, they saturate instead. Even those weights corresponding to positive eigenvalues differ greatly from the ideal. From Eq.(40) it can be seen that as μ approaches zero, w'_i approaches infinity. This is a very interesting result and contradicts all previous results assuming ideal case [1]. Let us examine this result more closely. Applying Eq.(12) into Eq.(9), the minimum MSE in the ideal case can be expressed as:

$$\xi_{min} = E[d^2(n)] + \underline{W}^{*T}\underline{R}\underline{W}^* - 2\underline{S}^T\underline{W}^*$$

$$= E[d^2(n)] - \underline{S}^T\underline{W}^* \tag{42}$$

Now, return to the problem of calculating the MSE when the input multiplier has a nonzero output offset voltage and recall from Eq.(40) that the weight vector was suboptimal. We therefore, want a means of expressing its deviation from optimum and the resulting increase in MSE. Define an error vector as:

$$\underline{V}(n) = \underline{W}(n) - \underline{W}^* \tag{43}$$

Substituting this value for W(n) in Eq.(9) yields

$$\xi = \xi_{min} + (\underline{W}(n) - \underline{W}^*)^T\underline{R}(\underline{W}(n) - \underline{W}^*) \tag{44}$$

When R is nonsingular, the weights approach a steady state solution which is found by substituting (S + Z/2 u) in Eq.(27) for S in Eq.(12), therefore,

$$\underline{W} = \underline{W}^* + (1/2\mu)\underline{R}^{-1}\underline{Z} \tag{45}$$

Hence the effect of the input multiplier output offset voltage is to shift the weights from their optimum point by an amount $(1/2\mu)\underline{R}^{-1}\underline{Z}$. From Eq.(45), we have

$$\underline{W} - \underline{W}^* = (1/2\mu)\underline{R}^{-1}\underline{Z} \tag{46}$$

Substituting Eq.(46) into Eq.(44) shows that the steady state MSE will be

$$\xi_{ss} = \xi_{min} + (1/2\mu)^2\underline{Z}^T\underline{R}^{-1}\underline{Z} \tag{47}$$

Eq.(47) reveals that the input multiplier output offset voltages increase the mean square error by an amount $(1/2\mu)^2\underline{Z}^T\underline{R}^{-1}\underline{Z}$ over its optimum value. Therefore, even when $\underline{R}$ is nonsingular there is an extremely large increase in MSE, which is inversely proportional to the square of μ. This result contradicts Widrow's theory [1,7], which states that the excess MSE is directly proportional to μ.

b)The Combined Effects of Input Multiplier Nonlinearities and Offset Voltages

Consider the input multiplier nonlinearities and assume all other circuit elements are ideal, therefore, Eq.(21) can be reduced to

$$V^1_{oi} = 2\mu e(n)x_i(n) + k_2x_i^2(n)\ k_3(2\mu e(n))^2 \tag{48}$$

Define

$$f_i(n) = k_2x_i^2(n) + k_3(2\mu e(n))^2 \tag{49}$$

As the nonlinear terms lumped together, then combining Eq.(22) and Eq.(49) gives the output of the ith multiplier as:

$$V^1_{oi} = 2\mu e(n)x_i(n) + \delta_i(n) \tag{50}$$

where $\delta_i(n)$ represents the noise in the multiplier output, i.e.,

$$\delta_i(n) = z_{osi} + f_i(n)$$

Define the vector $\underline{\Delta}_s$ as:

$$\underline{\Delta}_s = [\ E[\delta_1(n)]\quad E[\delta_2(n)]\ \ldots\ E[\delta_M(n)]\ ]^T \tag{51}$$

Following the same procedure as in the previous section, it is easy to obtain the steady state weighting vector as:

$$\underline{W}_{ss} = \underline{W}^* + (1/2\mu)\underline{R}^{-1}\underline{\Delta}_s \tag{52}$$

and

$$\xi_{ss} = \xi_{min} + (1/2\mu)^2 \underline{\Delta}_s{}^T \underline{R}^{-1} \underline{\Delta}_s \tag{53}$$

c)Effects of Integrator Offset Voltage and Bias Current

It is easily shown [12] that the integrator errors due to offset voltages and bias currents can be grouped together such that the steady state weight vector is given as:

$$\underline{W}_{ss} = \underline{W}^* + \underline{V}_{os} \tag{54}$$

where

$$\underline{V}_{os} = [\ E[v_{os1}]\quad E[v_{os2}]\ \ldots\ E[v_{osM}]\ ]^T$$

and v_{osi} is the integrator error due to offset voltage and bias current at ith weight. Substituting Eq.(54) into Eq.(43) yields

$$\xi = \xi_{min} + \underline{V}_{os}{}^T \underline{R}\, \underline{V}_{os} \tag{55}$$

That is, there is also excess MSE due to the integrator offset voltages and bias currents.

d)Summer Nonlinearities

The summer used to form the output of the adaptive filter can possess an offset voltage. This offset voltage will affect the steady state solution of the weight vector and MSE. In this case, the adaptive filter output will be given by:

$$y(n) = y^*(n) + \gamma \tag{56}$$

where $y^*(n)$ is the output of an ideal summer, and γ is a random variable representing the offset error. Therefore, the weight vector is expressed as:

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu \underline{X}(n)[d(n) - \underline{X}^T(n)\underline{W}(n) - \gamma] \tag{57}$$

Taking the expectations and rearranging terms yields

$$\underline{W}(n+1) = (\ \underline{I} - 2\mu\underline{R}\ )\underline{W}(n) + 2\mu(\underline{S} + \underline{\Gamma}) \tag{58}$$

where

$$\underline{\Gamma} = E[\underline{X}(n)\,\gamma\ ]$$

Following the same procedure given in section IV part (a), it is easy to show that

$$\underline{W}_{ss} = \underline{W}^* + \underline{R}^{-1}\underline{\Gamma} \tag{59}$$

and

$$\xi_{ss} = \xi_{min} + \underline{\Gamma}\, R^{-1} \underline{\Gamma} \tag{60}$$

V-SOLUTIONS

This section presents two techniques which offer promise in solving some of the problems resulting from adaptive filter internal circuit element imperfections.

a)Differential Integrator

A leaky integrator with a resistor added in the feedback loop in parallel with the integration capacitor provides a feedback path which may reduce the drift errors associated with the standard integrators. But this circuit still has two drawbacks; 1)the drift errors are not completly eliminated, and 2)the added resistor causes the integrator to have a finite memory. A better solution, known as differential integrator, is shown in

Fig. 3. This circuit can be used to minimise the effects of bias currents and offset voltages for an analog integrator.

The operation of the circuit shown in Fig. 3 is as follows: Assume the output of the top integrator is v_1, and the output of the bottom integrator is v_2. Since the input to the bottom integrator is zero, its output will only be the error terms due to bias currents and offset voltages. The output of top integrator will be $w_i(t)$ plus these same error terms. For identical OpAmps (a matched pair on a single integrated circuit), and identical resistor and capacitor values, the error terms from top and bottom integrators should be identical. After subtraction, the desired quantity $w_i(t)$ is left free of error terms and with infinite memory.

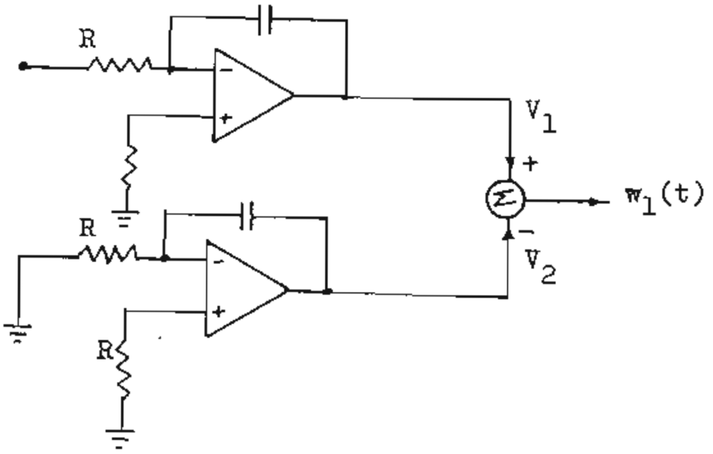

Fig. 3, Differential Integrator

b)Distributed Loop Gain Multiplier

The effects of the most dangerous errors, those caused by the input multipliers, are configuration dependent. Therefore, a modification in the standard configuration can reduce these errors. The new configuration called "distributed loop gain multiplier" is shown in Fig. 4. In this figure the noise is reduced by an amount K while the required output signal remains the same. The output from Fig. 4 is given as:

$$dw_i/dt = (2\mu/K)[Ke(t)x_i(t)] + (2\mu/K)[z_{osi} + k_2 x_i^2(t) + k_3(Ke(t))^2] \qquad (61)$$

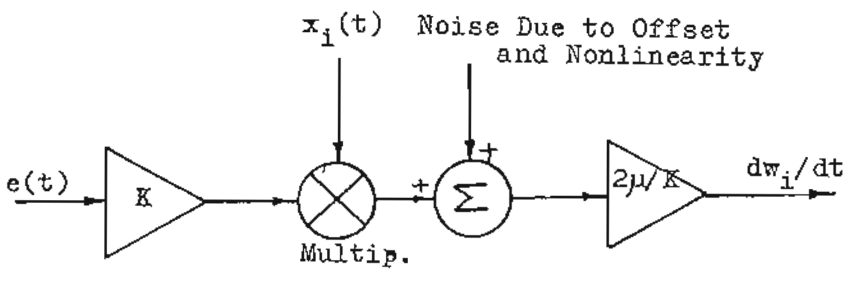

Fig. 4, Distributed Loop Gain Multiplier

The first term on the right hand side of Eq.(61) represents the required output signal and the second term represents the noise due to output offset and nonlinearities. This term will be referred to as:

$$\delta_{iD} = (2\mu/K)[z_{osi} + k_2 x_i^2(t) + k_3(Ke(t))^2]$$
$$= (2\mu/K)[z_{osi} + k_2 x_i^2(t)] + 2\mu k_3 K e^2(t) \qquad (62)$$

As $K \rightarrow 0$, $\delta_{iD} \rightarrow \infty$, because of the first two terms, and as $K \rightarrow \infty$, $\delta_{iD} \rightarrow \infty$, because of the last term. Therefore, there must be some value of K which is optimum for this configuration, i.e., optimum in the sense that it reduces the effects of the errors. To find this value of K which minimises the noise power, first square and take the expectations of both sides of Eq.(62), therefore,

$$E[\delta_{iD}^2(t)] = (2\mu/K)^2 E[z_{osi} + k_2 x_i^2(t)]^2 + E[2\mu k_3 K e^2(t)]^2$$
$$+ 8\mu^2 k_3 E[e^2(t)(z_{osi} + k_2 x_i^2(t))]$$

Taking the derivative of the above equation w.r.t. K and setting the result equal to zero yields

$$\partial E[\delta_{iD}^2]/\partial K = 0 = (-8\mu^2/K)E[z_{osi} + k_2 x_i^2(t)]^2 + 8\mu^2 k_3 K E[e^4(t)]$$

Therefore, the optimum value of K is given as:

$$K^* = (1/k_3)\sqrt{E[z_{osi} + k_2 x_i^2(t)]^2 / E[e^4(t)]} \qquad (63)$$

For this configuration, the weights approach a steady state solution given by

$$\underline{W} = \underline{W}^* + (1/K^*)\underline{R}^{-1}\underline{\Delta}_D \qquad (64)$$

and the steady state MSE is

$$\xi_{ss} = \xi_{min} + (1/K^*)^2 \underline{\Delta}_D^T \underline{R}^{-1} \underline{\Delta}_D \qquad (65)$$

where

$$\underline{\Delta}_D = [\; E[\delta_{1D}] \quad E[\delta_{2D}] \quad \ldots\ldots \quad E[\delta_{MD}] \;]^T$$

Comparing Eq.(53) and Eq.(65), it can be seen that the distributed loop gain configuration reduce the MSE by $(K^*/2\mu)^2$ over the standard configuration.

VI-CONCLUSIONS

This paper presented an analytical study on the implementation of analog and sampled data adaptive LMS filters. It was shown that for the standard configuration, imperfections of the input multiplier contribute an extremely large MSE to the adaptive system and the excess MSE is inversely proportional to the square of μ.

The MSE contributed by the bias currents and offset voltages found in the weight integrator was found to be substantial. A differential integrator configuration was presented to solve this problem.

The equations for the weights are dependent on the eigenvalues of R. Those weights corresponding to zero eigenvalue will saturate and will not provide a minimum MSE. The distributed loop gain configuration was introduced to overcome these problems.

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