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DIAGNOSIS OF REACTOR CORE IMPACT USING WAVELET TECHNIQUE

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ABSTRACT

This paper describes a new method for diagnosis of reactor cores using noise signal processing method. Detection and diagnosis of faults are based on the readings of signals coming from a set of detector tubes, which are surrounded by fuel assemblies in the reactor core. Vibrations of the detector tubes arise from the strong flow of coolant water in the reactor. If the vibration is large enough, the detector tube may impact on the nearby fuel assemblies, which in turn will execute a short, damped oscillation after each hit. This oscillation produces transient in neutron noise signal that is too small to be observed. The traditional frequency analysis is not an effective tool for *finding out transient in non-stationary signal and needs a reference or impact-free signal, that is* often impossible to obtain in practical applications. In this work, we suggest a method to detect impact or transient in the signal, using one time the discrete wavelet transform (DWT) and in the other an adaptive wavelet filter. The adaptive wavelet filter is found to be very effective in detection of detector vibration and impact problem early before its damage; therefore, the on-line diagnoses of the reactor will detect this error and if necessary alert the operator. The reactor with detector tube impact model is developed using Matlab-Simulink. The results show the effectiveness of the proposed method.

هذه الورقة تصف طريقة جديدة لتشخيص الاعطال في قلب المفاعل بواسطة تحليل اشارة الضوضاء. ان عملية اكتشاف و تشخيص الاعطال تتم علي اساس قراءة الاشارات القادمة من الكواشف المحاطة باعمدة الوقود النووي داخل المفاعل ، وهذه الكواشف تتذبذب عندما يتدفق تيار الماء المبرد القوي داخل قلب المفاعل. وينشا تصادم ما بين هذه الكواشف و اعمدة الوقود النووي عندما يكون التذبذب كبيرا. ويحدث عند كل تصادم ان تتولد نبضة صغيرة جدا وهي صعبة الملاحظة تضاف الي اشارة ضوضاء النيترون . وقد تبين ان الطريقة القديمة و هي تحليل التردد اداة غير فعالة في ايجاد النبضة الناشئة عن التصادم في الاشارة غير الثابتة وتحتاج الي اشارة قياسية خالية من التصادم للمقارنة بها و هذه الاشارة القياسية غيرممكنة الحصول عليها عمليا . في هذا التطبيق تم اقتراح طريقة لاكتشاف التصادم من خلال تحليل الاشارة مستخدما طريقتين ، اولاهما: طريقة تحويل المويجة المتقطعة ، وثانيها: باستخدام مرشح المويجة المتكيف. وقد وجد ان مرشح المويجة المتكيف فعال جدا في اكتشاف تذبذب و تصادم الكواشف داخل المفاعل مبكرا قبل تدميرها ، و هكذا يتم التشخيص الفوري لهذا الخطا و يقوم بتنبيه مشغل المفاعل اذا كان ذلك ضروريا . لقد تم عمل محاكاة للتصادم الذي يحدث داخل المفاعل بواسطة برنامج Matlab. النتائج اظهرت فاعلية الطريقة المقترحة . وتم اقترح عمل معالج نيترون الضوضاء في المستقبل مبني علي مصفوفة البوابات المبرمجة FPGA .

Keyword :Diagnostic, Neutron noise processor, Reactor monitoring ,Wavelet.

1. Introduction

Wavelet techniques were successfully used in various diagnostic problems involving non-stationary processes as in [1-3]. In nuclear reactor diagnosis; there have been several methods proposed and tested in the past for detecting the impacting of detector tubes, Pazsit I. and Glockler[4 and 7] proposed methods include monitoring of the magnitude of the include of the vibration peak in the measure; peak broadening.; distortion of the phase between two detectors in the tube; appearance of higher harmonics[5]; finally higher moments of the detector Amplitude Probability Distribution (APD) and the "decay ratio" of the detector signal[6]. All the above methods consider the signals as stationary random signal but it is really non-stationary. Besides, non of them is absolute, e.g. comparison with data from the same core but without impacting.

In this paper, we proposed an alternative method which is free from the signal need, by using the wavelet. The advantage of the wavelet analysis is the possibility of local evaluation of different scale frequency characteristics of time series, which is essential in identification of failures.

The wavelet analysis is capable of providing both time- domain information and frequency-domain information simultaneously. Similar to wavelet function, the transient feature components of vibration signals. They have local energy distributions in both the time domain and the frequency domain. Wavelet functions can be used for detection of transient feature components because they have similar time–frequency structures. Since different types of wavelets have different time–frequency structures, we should use the wavelet whose time–frequency structure matches that of the transient component the best, in order to detect the transient component effectively.

In some applications, noise is considered to be something unwanted or disturbing, the noise in radio broadcasting or disturbing noise which can occur when speaking in a cell-phone. In order to get a clear, noise-free signal, the noise is often filtered away or de-noising. However, in other application as in noise diagnoses it is the other way around, the static signal is filtered out and the noise is considered as the important part of the signal. Thus, the noise is extracted as in reactor diagnosis rather than filtered away. During the last few years, DWT has been used for neutron noise diagnosis [4,5]. It has been shown that in diagnostics of controlled objects wavelet analysis may be widely used, which makes possible an accurate examination of scale invariant dynamics of complex technical systems. DWT uses dyadic discretization. The structures of both the decimated DWT and the un-decimated DWT are too rigid for them to provide a good match to the structure of the transient component.

This paper presents the reactor with detector tube impact model. At first, simulating the signal of a vibrating detector with the presence of an impacting–induced fuel box vibration and the noise elimination method are demonstrated. Then, the method of determining the DWT and the necessary discrimination threshold from the measured signal is described. The "noisy" (i.e. stationary) part of the original signal can be removed, whereas the transients will survive. An adaptive wavelet filter based on Morlet wavelet is proposed. The shape of the wavelet filter (time–frequency resolution) is automatically adjusted to extract periodic impulses immersed in noisy signals for impact recognition. When the time series of the wavelet are convoluted with the signal, the filtered result is obtained.

Morlet wavelet is selected because it is a cosine function with exponential decay on both sides and is very much like an impulse, which is suitable for impact detection. To get good performance of the filtering result, the scale and the time–frequency balance parameter for Morlet wavelet is selected carefully. The vibration and impact signal are detected successfully by using this method.

2. REACTOR CORE WITH DETECTOR TUBE IMPACT MODEL

The reactor core consists of a detector tube together with the surrounding fuel box assemblies. The vibrations arise from the strong flow of the coolant water in the reactor and the fact that the detector tubes, which are roughly four meters long, are fixed only in their ends. If the vibration is strong enough, the tube may impact on the nearby one or more fuel assemblies surrounding the detector tube. The situation is illustrated for physical model as in Fig. 1, and also, the mathematical model as in Fig. 2.

Although the vibration in the X-Y plane is two dimensional, no qualitative difference was found between 1-D and 2-D Simulations [4,6 and 7]. Because of this fact, a simplified one dimensional simulation has been used in the current work to study the possibility to detect impacting of a detector tube against surrounding fuels boxes, as shown in Fig. 3.

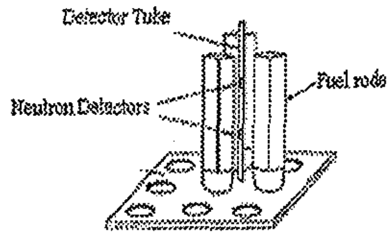


Fig.1. Detector tube and fuel rods.

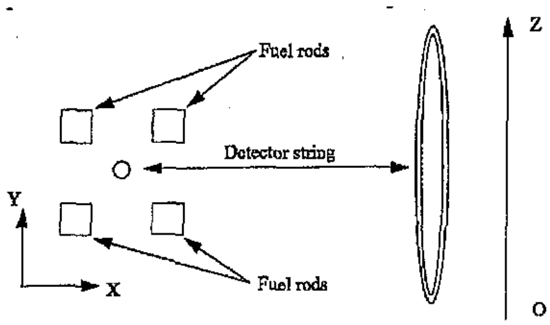


Fig.2. Reactor Core Mathematical Model.

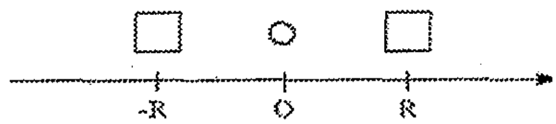


Fig. 3. 1-D Model

Following [4 and 5], we assume that the damped oscillations of a detector guide tube may be described by using (1).

$$\ddot{y}_d(t)+2\theta.\dot{y}_d(t)+\omega_0^2\, y_d(t)=f(t) \qquad (1)$$

With $<y_d(t)> = 0$ being the equilibrium position, standing for the damping factor and f(t) being a random driving force. It was found reasonable [5] to model the stochastic force f(t) by a discrete series of impulses arriving at regular times $t_n = n.\Delta t$

$$f(t) = F_c \sum_{n=-\infty}^{n=\infty} r_n \delta(t - t_n) \qquad (2)$$

Where r_n is a normal random variable with mean = 0 and standard deviation = 1. The parameter F_c, also called the "force coefficient", describes the strength of the driving force. The pulse repetition frequency must be chosen much higher than that of the oscillator, i.e.

$$\frac{1}{\Delta t} >> \frac{\omega_0}{2\pi}$$

Impacting is simulated by confining the detector tube motion within distance (-R,+R). Whenever |x(t)|, in the course of simulation, exceeds R, the velocity $\dot{y}_d(t)$ is reversed, i.e. $\dot{y}_d(t)$ change to $-\dot{y}_d(t)$, which models elastic reflection from an infinite mass without energy loss. Also we assume that the neutron noise $\delta n(t)$ is linearly related to the mechanical vibration, i.e

$$\delta n(t) = c_1 . y_d(t) \qquad (3)$$

where c_1 is constant

The model used in the current work also involves the fuel box vibration that obeys the equation:

$$\ddot{y}_f(t) + 2\theta_1 \dot{y}_f(t) + \omega_1^2 y_f(t) = F_b(t) \qquad (4)$$

where the force $F_b(t)$ is induced by impacting. At each impact, the detector tube transfers a certain impulse, $2m.|\dot{y}_d(t)|$, to the fuel rod. Thus if the box/tube mass ratio is $M/m = k$, then, at impacting, one has $|y_f| = 2.|\dot{y}_f|/(k+1)$. This will lead to the representation:

$$F_b(t_n) = 2\frac{M}{k+1}.\delta(t-t_n)|\dot{y}_d(t_n)| \equiv c.\delta(t-t_n)|\dot{y}_d| \qquad (5)$$

Where tn is the time of the n th impact and $|\dot{y}d(t)$ is the speed of the impacting tube. The damping vibration of the fuel box was chosen such that vibrations decay between two consecutive impacts if the impacting is not excessive. No confinement of the fuel box vibration is assumed.

It is assumed that the vibration of the fuel box leads to neutron flux fluctuations, which are added to the detector signal. Event here It is sufficient in this study to assume a linear relationship between the displacement of the fuel box and the induced noise. Thus in the extended model, the noise from the vibrating and impacting detector will be given as

$$\delta n(t) = c_1 . y_d(t) + c_2 . y_f(t) \qquad (6)$$

The parameters c_1 and c_2 are not given any physical interpretation here. Actually, only the ratio c_2/c_1 has some significance regarding the analysis. This value was chosen such that the simulated spectra resembled to the measured ones in that the peak corresponding to the fuel box eigenfrequency became rather low in the power spectrum of $\delta n(t)$.

As was described in the above, the statistical properties of the stationary random process $y_d(t)$ change when impacting occurs. It was these changes that were hitherto used for detecting impacting. The process $y_f(t)$, describing the effect of the transient vibrations of the fuel box on the detector signal, has not been considered before. This latter process is however non-stationary, and it cannot be efficiently handled by spectral analysis methods. Wavelet techniques are used to detect impact in neutron noise signal.

According to the above model, the neutron noise signal $\delta n(t)$ consists of a stationary process $(c_1.y_d(t))$ from detector tube signal which is disturbed by a non-stationary process $(c_2.y_f(t))$ from fuel box signal on impact as shown in fig. 4. Wavelet analysis, as opposed to Fourier decomposition, has reportedly proved to be a powerful tool in dealing with non-stationary signal [9].

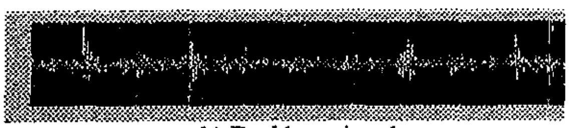
3. WAVELET TRANSFORM

Wavelet transforms are inner products between signals and the wavelet family, which are derived from the mother wavelet by dilation and translation. Let $\psi(t)$ be the mother wavelet, the daughter wavelet will be ,

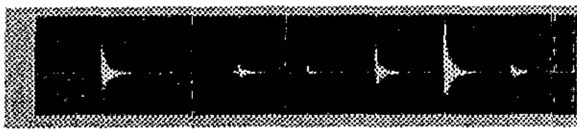
$$\psi_{a,b}(t) = \psi((t-b)/a) \qquad (7)$$

where a is the scale parameter and b is the time translation. By varying the parameters a and b, we can obtain different daughter wavelets that constitute a wavelet family. Wavelet transform is to perform the following operation:

a) Detector tube signal



b) Fuel box signal



c) Total signal (detector tube and fuel box)

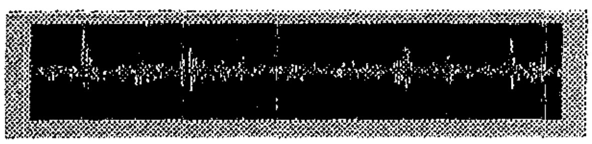


Fig.4. Simulated Reactor core signals

$$W(a,b) = \frac{1}{\sqrt{a}} \int x(t) \psi^*_{a,b}(t) dt \qquad (8)$$

Where '*' stands for complex conjugation.

Calculating wavelet coefficients at every possible scale is a fair amount of work,, if we choose scales and positions based on powers of two, so called dyadic scales and positions, then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform (DWT).

A. *Discrete Wavelet Transform (DWT)*

The signal is decomposed into approximations and details. The approximations are the high scale, low frequency components of the signal, the details are the low-scale, high-frequency components. In general wavelet analysis gives a more flexible way of de-noising the signal by recursively repeating the basic decomposition into approximations and details in multi-level decomposition.

It is shown in [8] that any continuous function *f(t)* can be represented by the following expansion, defined in terms of a given scaling function and its wavelet derivatives:

$$f(t) = \sum_k C_L(k)\phi_L(t) + \sum_k \sum_{j=1}^{L} dj(k)\Psi j(t) \qquad (9)$$

where: $\phi_L(t)$ is the L-th level scaling function.
$\Psi j(t)$ for j=1,2,3 ,...L are wavelet functions.

The function *f(t)* can be mapped into the wavelet domain and represented at different resolution levels using (9).

In order to work directly with the wavelet transform coefficients, the relationship between the detailed coefficients at a given level in terms of those at previous level is used [9]. In general, the discrete signal is assumed the highest achievable approximation sequence, referred to as 0-th level scaling coefficients. It is shown that the approximation and detail sequences at level j. 1 are related to the approximation sequence at level j by

$$c_{j+1}(k) = \sum_m h_0(m-2k) c_j(m) \qquad (10)$$

and

$$d_{j+1}(k) = \sum_m h_1(m-2k) c_j(m) \qquad (11)$$

Equations (10) and (11) state that approximation sequence at higher scale (lower level index), along with the wavelet and scaling filters, $h_o(k)$ and $h_1(k)$ respectively, can be used to calculate the detail and approximation sequences (or discrete wavelet transform coefficients) at lower scales.

In practice, a discrete signal, at its original resolution is assumed the 0-th level approximation sequence; i.e., $c_0(k) = f(t)$.

For a given wavelet system, with known wavelet filters $h_o(k)$ and $h_1(k)$, it is possible to use (10) and (11), in a recursive fashion, to calculate the discrete wavelet transform coefficients at all desired lower scales (higher lever). In most engineering applications, the wavelet systems are chosen such that the two wavelet filters have finite number of non-zero coefficients. In signal processing terminology, these filters are referred to as finite impulse response (FIR) filters. Under this assumption, and by using ideas from multirate signal processing literature [10], it is possible to calculate the two summations in (10) and (11) by using two FIR filters.

For each level from 1 to L, select a threshold and apply soft or hard thresholding to the detail coefficients.

The proper value of threshold (τ) can be determined by help the Lemma in Yang and Shamma[11] which relates the variance of a normal process $z_i \in N(0,\sigma^2)$ to its transform. However, a threshold τ is calculated as in (12).

$$\tau = 4\sqrt{N}.\sigma \qquad (12)$$

which cancels almost all noise if σ is set to the standard deviation of the noise that is present in the signal. Determination of this standard deviation is not so easy in general and it requires knowing of the process involved in neutron noise signal. The neutron noise signal y(t) consists of stationary (S), transient (T) and noisy (N) components as in (13).

$$y(t) = S(t) + T(t) + N(t) \qquad (13)$$

As known, the noise component N(t) has higher frequency than stationary S(t) or transient T(t) component as shown in Fig. 4. With this information one can estimate the parameter σ by applying a high pass filter (HP) with a cut-off frequency that is higher than the mechanical eigenfrequencies stationary or transient components.

$$y_{HP} \approx HP\{N\} \qquad (14)$$

The standard deviation σ can be estimated by :

$$\sigma_N{}^2 = Var[y_{HP}] \qquad (15)$$

This is the principle of the method that was used in this paper. With a proper choice of the threshold, the "noisy" (i.e. stationary) part of the original signal can be removed, whereas the transients will survive.

B. *Adaptive Morlet Wavelet Filter*

Morlet wavelet is one of the most popular non-orthogonal wavelets. The definition of Morlet is :

$$\psi(t) = \exp(-\beta^2 t^2 / 2)\cos(\pi t). \qquad (16)$$

It is a cosine signal that decays exponentially on both the left and the right sides. This feature makes it very similar to an impulse. It has been used for transient isolation and mechanical fault diagnosis through the performance of a wavelet de-noising procedure [12].

A daughter Morlet wavelet is obtained by time translation and scale dilation from the mother wavelet, as shown in the following formula [13]:

$$\psi_{a,b}(t)=\psi\left(\frac{t-b}{a}\right)=\exp\left[\frac{\beta^2(t-b)^2}{2a^2}\right]\cos\left[\frac{\pi(t-b)}{a}\right] \qquad (17)$$

where a is the scale parameter for dilation and b is the time translation. It can also be looked at as a filter.

To identify the immersed transient by filtering, the location and the shape of the frequency band corresponding to the transients must be determined first. Scale a and parameter b control the location and the shape of the daughter Morlet wavelet, respectively. As a result, an adaptive wavelet filter could be built by optimizing the two parameters for a daughter wavelet. Several researchers have reported on how to select the mother wavelet that adapts the best to the signal to be isolated. Details on how to select b in Morlet wavelet to make the mother wavelet match the signal to be isolated are provided in [12].

In this paper, Kurtosis is used due to its sensitivity to sharp variant structures, such as impulses. The bigger the impulse in signals is, the larger the kurtosis. As a result, kurtosis can be used as the performance measure of a Morlet wavelet filter. The definition of kurtosis is

$$kurtosis\ (y) = (m_4 / m_2^{\ 2}) - 3 \qquad (18)$$

where $$m_2 = \sum (y - mean)^2 / N$$

$$m_4 = \sum (y - mean)^4 / N$$

The kurtosis measures a kind of departure of y from Gaussianity, it can be considered as an index of peakedness or flatness of a distribution.

The procedure to perform the adaptive wavelet filtering is as follows:

(1) Vary the parameters a and b within preselected intervals to produce different daughter wavelets.
(2) Perform wavelet filtering using each daughter wavelet and calculate the kurtosis of each outcome.
(3) Compare the kurtosis value. The parameters a and b that correspond to the largest kurtosis are the best parameters to use to reveal the hidden transient.

4. ANALYSIS AND RESULTS

Three simulated signals are shown in fig.4 (the first one,($c_1.y_d(t)$), comes from the detector tube, the second signal, ($c_2.y_f(t)$) is induced by the fuel box, and finally, the third plot displays ($\delta n(t)$) the total detector signal). It should be noted here that the amplitude of the fuel box signal, is much smaller than detector tube signal, and thus it is completely invisible in the total signal. The following two methods are used to identify the transient component $c_2.y_f(t)$ in total signal $\delta n(t)$.

First method:

The DWT is used to analyze neutron noise signal by recursively repeating the basic decomposition into approximation and detail. Different types of mother wavelets (bior1.5, bior1.3, db2, sym2 and haar) are tested, with applying a level dependent threshold as in (12) to each of detailed coefficients, as described in section III. The impacting can be detected through recovering transients existing in details signal. A good result for detecting the presence of the impacts in the signal being obtained using Daubechies (db2) mother wavelet in the level 4 details of the wavelet decomposition as shown in Fig. 6.

To check how the wavelet transform performs with respect to detecting impacts, a series of the simulation were made first by heavy impacting (Fc=40) and then gradually decreasing the impacting through medium (Fc=30) and weak (Fc=20). The results are shown in Fig. 6-8.

By using (detect_impact_wavelet and get_count) software as Matlab programs that applied to threshold detail coefficients to calculate Impact Rate(IR). The IR gives the number of spikes in the signal due to the fuel rode vibration. The IR gives the number of spikes in the signal due to the fuel rode vibration. A high value means severe impacting as in Fig. 9. By using the IR, there is no need for an expert judgment to detect the impacting frequency.

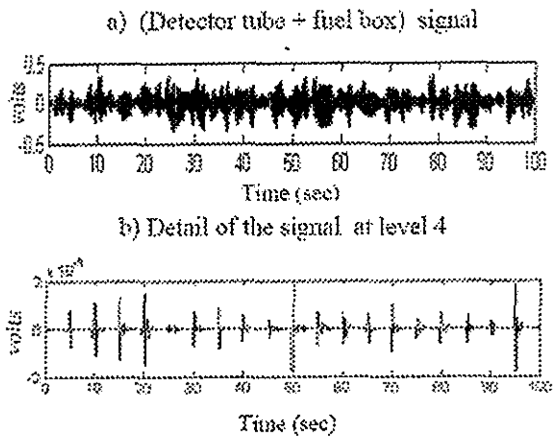


Fig. 6 Simulation with a large force coeffient (Fc=40) heavy impacting.

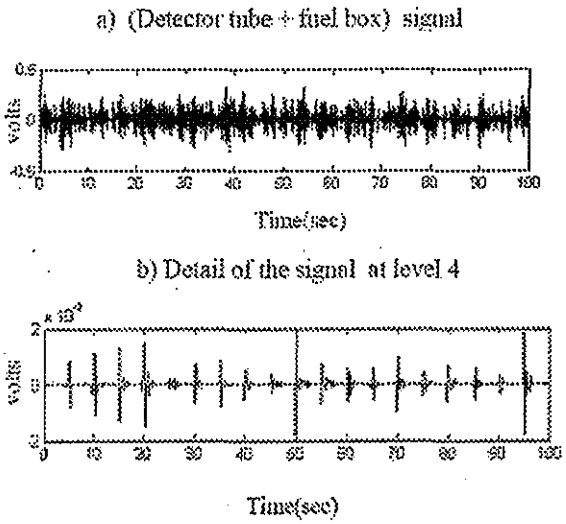


Fig.7. Simulation with medium force coefficient (Fc=30) medium impacting

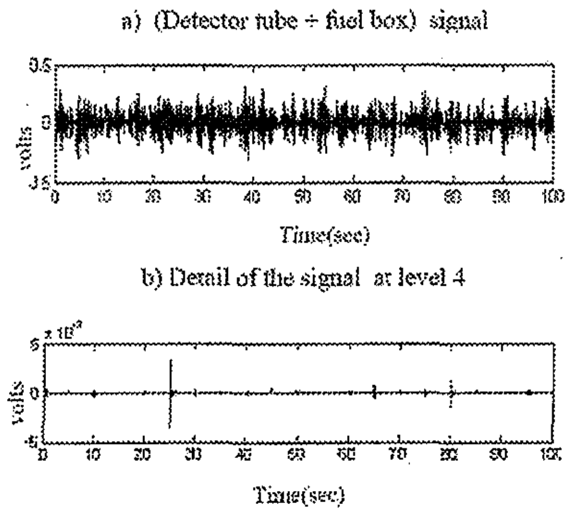


Fig. 8. Simulation with a weak force coefficient (Fc=20) very few impacting

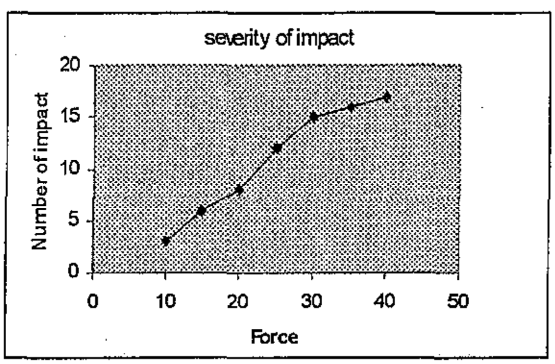


Fig.9. Dependence of the severity impact on the force coefficient F in reactor simulation.

Second method:

As introduced in section (3 - b) Morlet wavelet used to obtain the adaptive wavelet filter. We changed the value β from 1 to 7 with step 0.1, the scale varies from 1 to 45 with a step size of 1.

Figures 10 and 11 show the waveform of neutron noise signal with impacting for wavelet filter with different values for β, a, and Kurtosis. We notice that with increasing the kurtosis value, the number of appearing impacts in wavelet filter increased. This shows that the wavelet filter obtained with the largest kurtosis value can extract the transients (i.e. impact) from neutron noise signal and provides the best filtering results.

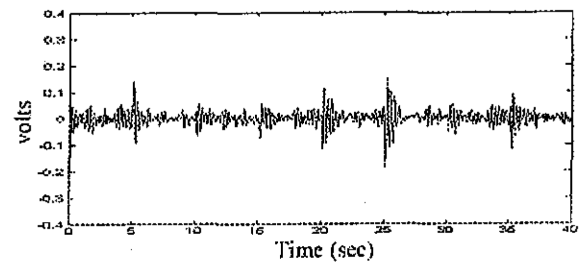


Fig.10 The filter result with β=3 and a= 15 for the wavelet filter

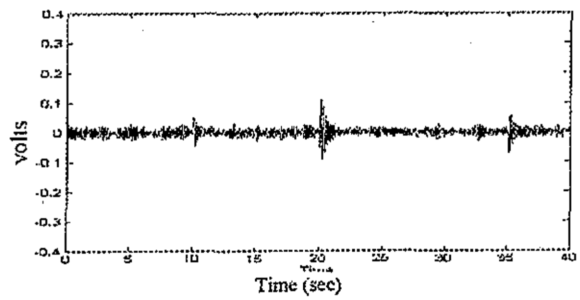


Fig.11 The filter result with β=7 and a= 32 for the wavelet filter

5. CONCLUSION

The ability to monitor and detect damage in detector tube at the earliest possible stage is very important and considered a safety factor in the reactor operation. In this paper, a neutron noise based method is proposed for the detection of impacting of detector tube in the reactor core. The basic idea relies on the assumption that non–stationary transient (e.g. fuel box vibration) may be induced at impacting. However, their presence in the detector signal can be detected by wavelet analysis. Two methods are proposed; the first uses discrete wavelet decomposition. With good selection of the mother wavelet and proper threshold, the analysis shows that this method has significant potential in detecting detector tube impacting. A general method was suggested to determine a suitable threshold during the inverse transformation.

The concept of the severity parameter was introduced, which varies from weak to very heavy. This method doesn't detect impact and vibration only, but also measures the degree of impact by severity parameter (no vibration, vibration but no impact or indication of impacting) before detector tube damage. In addition, another new method was tested in the present work, based on adaptive filter. Morlet wavelet is used for impact (transient) detection. To identify transient hidden in noisy signals by filtering, the daughter wavelet should match the time-frequency structure of the transient well.

An adaptive Morlet wavelet filter based on kurtosis maximization is proposed to detect transient automatically for recognition detector impact. Compared with the first method wavelet decomposition, the result obtained by this method is more effective for impact detection. The two methods are important for increasing the confidence in the decision of core diagnostic problems. In future, the above mentioned work will be hardware implemented using Field Programmable Gate Array (FPGA).

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