ON LINE THREAT COMPUTATION FOR UNMANNED AIR VEHICLES

Jamal A. F. Azzam
M.Sc:Electrical Engineering, Portsaid Shipyard, Suez Canal Authority.

Prof.Dr. Hassan El_Deip
Electrical Engineering Dpt., Facaulity of Engineering, Suez Canal University.

Prof.Dr. Soliaman M.Sharr
Power And Machines Engineering Dp Facaulity of Engineering, Helwan, Helwan Uniirsty

احتساب التهديد على للطائرات بدون طيار أثناء الطيران

هناك اتجاهات قوية لاستخدام الطائرات بدون طيار UAVs في ميادين المعارك ، ومن المشاكل المهمة التي تواجه هذا الاستخدام هو انه يتعين على المركبة تخطيط المسار الأمثل لها من نقطة انطلاقها الى موقع الهدف. ويجب أن يكون لهذا المسار اقل تهديد وطول وقت الطيران واستهلاك للوقود. كما أن هذا المسار يجب أن يراعي القيدات الديناميكية للمركبة. أن سلامة المركبة (بتقليل التهديد الذي تتعرض له إلى اقل قيمة) له الأولوية المطلقة في عملية تخطيط المسار ولكن المشكلة الرئيسية هي : كيف يمكن احتساب التهديدات لمسار ما أثناء الطيران ؟ فليس هناك حل يعتمد عليه حتى الآن لهذا السؤال. فهناك العديد من الاقتراحات في المقالات العلمية المنشورة ولكل منها قصورها الخاص بها. فبعض هذه التقنيات تلجأ إلى تكامل دالة التكلفة للمسار وفي المقابل فان الثمن هو وقت احتساب للتهديد طويل بحيث يقضي على قدرة المركبة على ذاتية الحركة ومن ثم فلا يمكن استخدام هذه الطرق في وقت الطيران الحقيقي.

وبعض التقنيات الأخرى المستخدمة على نطاق واسع تضحي بدقة احتساب التهديد من اجل تخفيض وقت احتسابه وذلك بالطريقة المبسطة باحتساب التهديد عند ٣ نقاط فقط في كل مفصل (من المفاصل المتصلة المكونة للمسار) وهذه النقاط الثلاث مختارة طبقا للتشتت الأمثل لتكوين شبكة **Sukharev** ، وباختبار هذه الطريقة تبين انه لايمكن الاعتماد عليها حيث تتناقص قيمة التهديد بازدياد طول المفصل رغم احتفاظه بنفس البعد عن موقع الرادار. وفي كل هذه الطرق المشار إليها فان كل الاهتمام هو احتساب التهديد الناجم عن مواقع الرادارات فقط. وفي هذه المقالة مسح موجز لطرق احتساب التهديد وطريقة مقترحة جديدة تقدم إجابة على السؤال المطروح أعلاه . تقوم هذه الطريقة باحتساب التهديد الصادر عن مواقع الرادارات وأيضا الصادر عن مواقع الصواريخ المضادة للطائرات (SAMs) وذلك باستخدام صيغة **Chebchev** في إجراء عملية التكامل .

وقد تم مقارنة نتائج هذه الطريقة المقترحة مع كل من النتائج الأخرى بإجراء عملية التكامل باستخدام صيغة **Simpson** (وهي طريقة دقيقة لاحتساب قيمة التكامل لدالة ما) ومع النتائج المستخدمة بالطريقة المبسطة (وهي الطريقة الشائعة الاستخدام في الكثير من الابحاث المنشورة) وذلك من حيث قيمة التهديد المحتسب على مسار ما والوقت المستهلك لاحتسابه .

وقد أثبتت النتائج أن هذه الطريقة المقترحة دقيقة وسريعة جدا ومن ثم تحقق الشرطين الحاسمين : وهما الدقة والسرعة وميزة أخرى لها وهي أنه بزيادة عدد المفردات (كزيادة عدد الطائرات ومواقع الرادارات ومواقع SAMs) فانه يؤدي إلى زيادة طفيفة في وقت احتساب التهديد (وليست زيادة متسلسلة وغير محددة لدالة ما).

ABSTRACT:

Unmanned Air Vehicles (UAVs) have potential strengths to be implemented in the battle fields. A major problem for UAVs is to plan an optimal trajectory path from its starting position to the target position in real time. For a path to be optimal it has to minimize: threat, path length, fuel consumption, flying time, and preserves dynamic constructs of the vehicle.

The safety of the vehicle (i.e. minimizing the threats) has the first priority in the planning process. A major problem in this field is : how to compute the threats for a path on-line? A reliable solution for this problem is not settled yet. Different proposals are published, each has its own limitations. Some of these techniques integrate a threat cost function. The price is a long computation time that cancels the vehicle autonomy, since it can not be done in real time.

Other widely used techniques sacrifice the accuracy of the threat computation for the sake of smaller computation time. The threat is computed by this simplified method only at three points on each path leg (edge). These 3 points are the optimal dispersion in Sukharev

Accepted December 29, 2006

grid. Examining this method shows that it is ir-reliable since the threat value is decreasing with the increase of path length at the same closure to the radar site. In all those methods the concern is only about radar sites threats.

In this paper a short survey for threat cost computation methods is introduced. A proposed method that introduces an answer for the mentioned questions. This method computes the real threat not only from radars but also from surface to Air Missiles SAMs by integration using Chebychev quadratic formula in a very short time. The procedure is compared with results obtained by Simpson's Formula and with those get by the simplified technique (for computation time and accuracy). This simplified technique is commonly used in a lot of researches. The comparison proves that this methodology is accurate and very fast. It fulfills the two crucial goals : accuracy and swiftness. Moreover, the increment in computation time due to the increase in the arguments (UAVs, targets, radar sites) is trivial.

1. Introduction :

One of the critical problems of the evolving implementation of the UAVs is the trajectory planning (TP) in real time i.e. while the vehicle is flying. The optimal trajectory has to minimize a cost function J composed of path length, threat cost all over the path, flying time and fuel consumption. In the same time the optimal trajectory preserves the dynamic constraints of the vehicle (e.g. maximum speed, and maximum turning rate). Reported literatures for TP apply the outdoor robotic algorithms to UAV TP [1-4]. These techniques can be proudly classified as: grid based [5,6], graph based [7-9], virtual forces [10] and mathematical programming [11-13]. A common problem in all these techniques is : how to compute the threat cost of the feasible paths in real time. Accuracy and swiftness of the threat cost are the key issues in planning a trajectory. The published reports are concerned only about the threat of being detected by radar sites only. They can be classified as :

- Planning the path edges in equidistance from each two adjacent radar sites Fig. 1 (for grid based, graph based and virtual forces). Clearly, there is no guarantee for minimum threat since there may be large range coverage or multiple coverage. It may fit for a robot avoiding a fixed obstacles, but not useful for UAV penetrating enemies sites.
- Integrating the threat function all over the path (in conventional optimization and mixed integer linear programming techniques). They are off-line methods and can not be implemented in real time due to its large computation time.
- To reduce the computation time, a widely used simplified methodology is reported by computing the threat on 3 points on each edge of the path.

Investigating the simplified method proves that the gain of reducing the computation time is very costly since the accuracy of the computed threat is not guaranteed.

In this paper an analysis of optimization methods and the simplified one is introduced. A proposed method to compute the threats on-line accurately and swiftly is explained. The results of the proposed technique are compared with those obtained by using the simplified technique (which is widely implemented e.g. [1],[3],[8]). Another comparison with results get by implementing the Simpson's formula for integration. They are compared for : accuracy of the computed threat value and computation time. Thus, enable the vehicle to be autonomous. This is applied for the real threats a UAV is subjected to i.e. radar sites and SAMs sites. The threats are also integrated accurately by Simpson's Formula and the simplified method. A comparison of the threat computed values and times are shown and proves the strength of the proposed method.

2 Threats on UAV:

A UAV is subjected to different types of threats from soft weapons like jammers and decoyers or hard weapons such as: shoulder Lunched homing weapons, radar directed guns Surface to Air Missiles (SAMs) and early warning radars. The UAV is equipped with anti-jamming and anti-decoying sets. Flaying at altitude above 5000 ft defeats most radar directed guns and above 15000 ft defeats most shoulder launched homing weapons [1]. Still SAMs and Radar detection threat the UAV. SAMs are of different types of equipments, namely small range, medium range, large range and longer-range fire control sensors. Fire control sensors work as tracking and sensing tools and do not have any destructive capability [14].

2.1 Radars Threats

The general radar equation includes a large number of parameters explained in Equ.(1) [15]. The backscattered power received from a target of radar cross-section at the first mixer or preamplifier in the radar receiver site is:

$$P_r = \frac{P_T G_T L_T G_R L_R \lambda^2 L_P L_\alpha A_t}{(4\pi)^3 d_r^4} \qquad (1)$$

Where

- P_r is peak received power
- d_r the detection range of the desired target.
- P_T the peak transmit power (the average power during the pulse).
- G_T the transmit power gain of the antenna with respect to an omni directional radiator.
- A_e the effective aperture of the antenna which is equal to the projected area in the direction of the target times the efficiency.
- λ the wavelength of the radiation
- L_T the losses between the transmitter output and free space.
- G_R the receive power gain of the radar defined in a similar manner to the transmit gain
- L_R the receive antenna losses defined in a similar manner to the transmit losses may be included in the effective system noise temperature T_s.
- L_P the beam shape and scanning and pattern factor losses.
- L_α the two way pattern absorption or propagation losses of the medium.
- A_t the radar cross sectional area of the object that is being detected and is equal to

It is clear from Equ. 1 that the received signal P_r by the radar site is inversely proportional to the forth order of the Euclidian distance between the radar site and the vehicle so

$$\sigma_r = \frac{1}{d_r^4} \qquad (2)$$

Where

σ_r is the threat on the UAV due to a radar site.

2.2 Threats of SAMs Sites

Every SAM site has a range circle, the UAV path must be off the circumference of a safe circle. This circle has a diameter greater enough than that of the range circle Fig.1. It will be beyond the SAMs range. These safe circles of course have different diameters depending on SAMs range. However, it is still subjected to threats from these SAMs due to sources of errors (e.g. sensors error, time delays, position estimation errors,....etc). This threat is computed as [1]:

$$\sigma_s = \begin{cases} \dfrac{1}{(1+d_s)^2} & \text{if } d_s < r_s \\ 0 & \text{else where} \end{cases} \qquad (3)$$

Where

- σ_s : is the threat on the UAV due to a SAM site.
- d_s : is the distance from UAV position and the SAM site.
- r_s : is the minimum radius of the safe circle

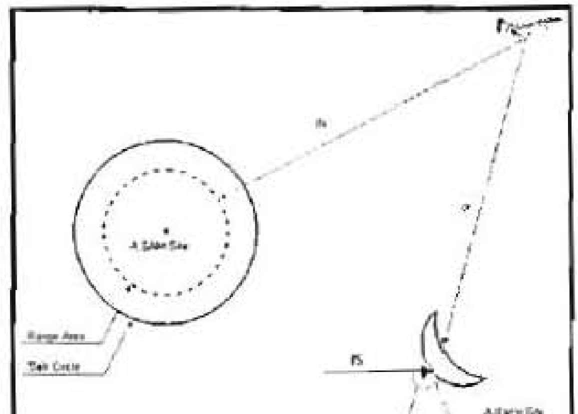

Fig 1 Considered Distances From SAMs And Radar Sites

3 Optimization Technique

For stealthy purpose the body of the UAV can be designed to reflect the incident radar signals into other directions to stray it away from the radar site. Fig 2 shows the body axis and angles. The threat function can be computed as [3] :

$$\sigma_r(\mu,\nu)=\begin{cases} 4\cos^2(3\nu) \text{ if } \nu \le 30^\circ \\ 4\cos^2(3\nu)\sin^2(2\mu) \text{ if } \nu > 30^\circ \end{cases} \quad (4)$$

Where

μ is the azimuth angle

ν the elevation angle measured with respect to Z_b axis

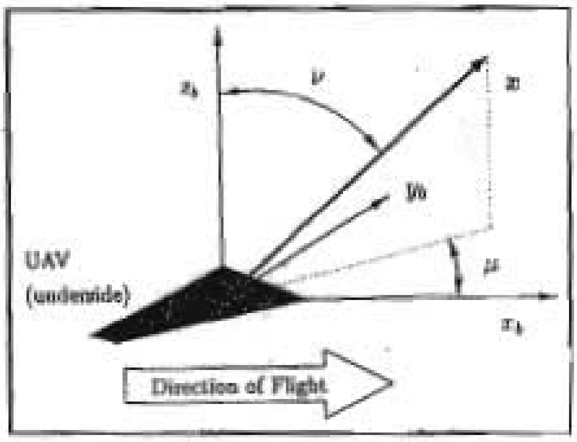

Fig. 2 The Effect of Flying Angles On Radar Signature

A Cost function formed by integrating the threat of Equ.4 plus the path length and tries to minimize the reflected radar signal from the UAV body to the radar site.

Another optimization techniques consider the earth axis x, y, z axis of the UAV body frame axis (x_b, y_b, z_b) as shown in Fig. 3. The UAV has 5 degrees of freedom 4 of them are considered in these methods :

- Translation about x_o, y_o.
- Yaw rotation about the z_b axis by an angle Ψ.
- Roll rotation about the x_b axis by an angle φ.
- Pitch rotation about the y_b (assumed zero here).

So, the following kinematic equations are used to model the UAV dynamic constraints [21]:

$$X = \cos\Psi$$

$$Y = \sin\Psi$$

$$\Psi = {}^{1}/_{6} \arctan u$$

The roll angle φ is constrained to an artificial input u as

$$\varphi = {}^{1}/_{3} \arctan u$$

A classical optimal control problem is to minimize a cost function of the form:

$$J(u)=\int_0^t \Big(\underbrace{1}_{\text{Path length}} + \underbrace{Q\sum_{i=1}^{n}\frac{S(x,y,\psi,u)}{r(x,y)}}_{K_a \text{ radar cost}} + \underbrace{\left(\frac{1}{3}u^2 + \frac{R2}{2}(u-\arctan u)^2\right)}_{\text{turning cost}}\Big)dt \quad (5)$$

Where

S is the threat cost of radars. It is represented in Equ. 4 as $\sigma_r(u, v)$.

$r = \dfrac{1}{d_r^4}$ distance from the radar site.

R_2 is a control constant > 0

Other techniques use integer linear programming and/or Mixed Linear Integer Programming (MLIP) [12] to minimize the cost. The major disadvantage of these techniques is that its computation cost is sufficiently high that it cannot be done in real time.

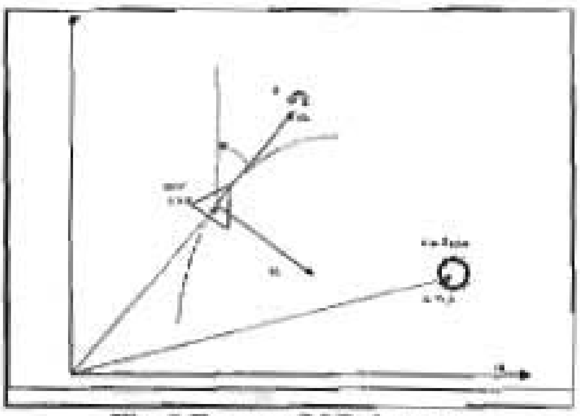

Fig. 3 Frames Of Reference

4. The Simplified Threat Computation

To avoid integration in real time, this a simplified method is widely used by computing the threat at three points only on the path. i.e. ($\frac{1}{6}$, $\frac{1}{2}$, $\frac{5}{6}$) of the path length, [8],[9],[14],[16] Fig. 4.

Threat costs are based on a UAV's exposure to adversary radars. Since the strength of a UAV's radar signature is proportional to $1/d_r^4$, where d_r is the distance between radar site and the UAV, the threat cost for traveling along an edge of path is proportional to the inverse of the distance to the forth power as explained in Equ.2.

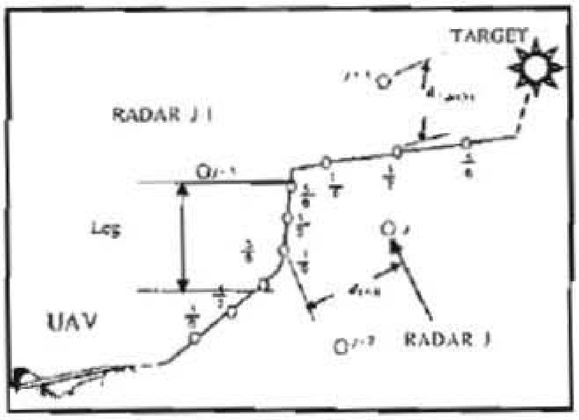

Fig. 4 Simplified Threat Cost From Vehicle To Target

Choice of these locations is dependent on the optimal dispersion in constructing Sukharev grid. The maximum dispersion obtained by the Sukharev grid is [17].

$$\delta(p) \geq \frac{1}{2[N_g^{1/d_g}]} \quad (6)$$

where N_g is the number of grid points
d_g is the number of grid dimension
while the next point in the grid can be computed as

$$\text{point number } k = \sum_{j=1}^{j=N_r} k_j\, g_j \quad (7)$$

These three locations namely, $L_i/6$, $L_i/2$, $5L_i/6$, where L_i is the length of the leg i. are depicted graphically in Fig 5.
The threat cost associated with the i^{th} edge is given by the expression [8],[9],[14],[16]:

$$J_{ti} = L_i \sum_{J=1}^{Nr}\left(\frac{1}{d^4_{\frac{1}{6},i,j}} + \frac{1}{d^4_{\frac{1}{2},i,j}} + \frac{1}{d^4_{\frac{5}{6},i,j}}\right) (8)$$

Where :
N_r is the number of radar sites
$d_{1/6\,ij}$ is the distance from the $1/6^{th}$ point on i th edge to the jth radar

By examining a lot of simulated examples a major disadvantage of this technique is that the performance of computed threat value is decreasing as the length of the edge is increasing as explained in (Fig.6), although its exposure to radar sites is lasting for more time. Consequently choosing the lowest cost edge may be improper .

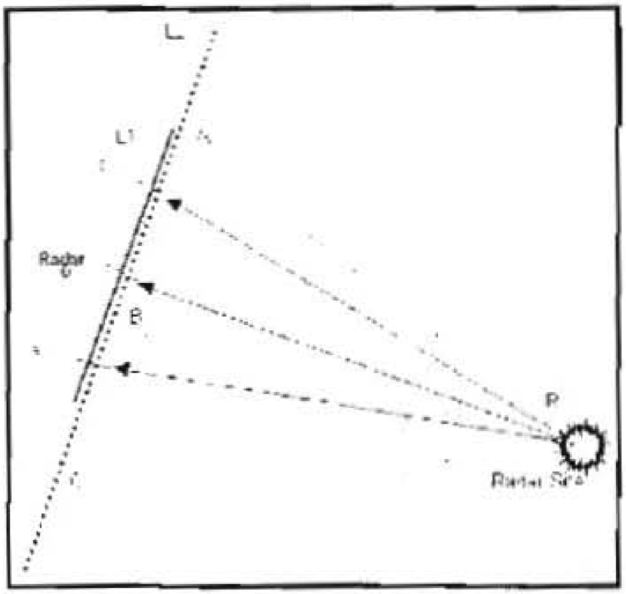

Fig 5 The Effect Of The Leg Length On The Threat value

In Fig 5 it is clear that the distances from R to A,B,C at ($\frac{1}{6}$, $\frac{1}{2}$, $\frac{5}{6}$) of the leg length L_2 is greater than those to a, b, c at ($\frac{1}{6}$, $\frac{1}{2}$, $\frac{5}{6}$) of the leg length L_1 although the two legs are on the same closer to the radar site i.e. RA > Ra, RB > Rb , RC > Rc.. Hence, the computation of the threat due to the radar site is reduced significantly if the leg length increased as shown in Fig. 6 as a consequent of this result, the comparison between the cost functions of different paths will be actually misleading and the searched optimal path with minimum cost will not be actually the minimum cost one.

In the remaining of this paper, an explanation of a more accurate technique based upon the computation of the integral over the path length. This computation is done using both the Simpson's formula and Chebychev's formula.

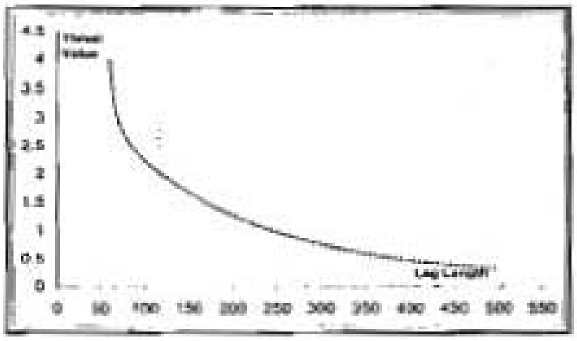

Fig. 6 Simplified Computed Threat vs. Leg Length (At The Same Close)

5 Simpson Formula :

To overcome this inaccuracy problem, the threat has to be integrated over the length of the leg. As a reference of the integrated value the Simpson's general formulas used.

Let n = 2m be an even number and let $f(x_i)$ ($i = 0, 1, 2, \ldots n$) be the values of the function $f(x)$ for equally spaced points $a = x_0, x, \ldots, x_n = b$, with spacing

$$h = \frac{b-a}{n} = \frac{b-a}{2m} \quad (9)$$

Simpson's rule:

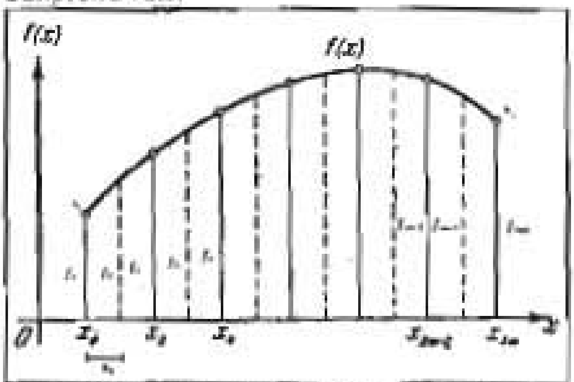

Fig. 7 Simpson's intervals of a general function

$$\int_{a}^{b} f(x)dx = \frac{h}{3}(y_0+y_n)+[4E_1+2E_2] \quad (10)$$

Where :

$E_1 = f_1 + f_3 + \ldots f_{2m-1}$

$E_2 = f_2 + f_4 + \ldots f_{2m}$

By implementing it to integrate the threat, the integrated values are more accurate but the computation time is very long (in order of minutes). This makes cost computation impractical and prevents the autonomous reactions of the vehicle. A comparison of the integrated, value and computation time are given in Fig. 11.

6. Chebyshev Quadratic Formula

The quadrate formula can be represented as [18]:

$$\int_{-1}^{1} f(t)\,dt = \sum_{i=1}^{n} B_i\, f(t_i) \quad (11)$$

Where B_i are constant coefficients. Chebyshev suggested choosing the abscissas t_i so that :

(1) The coefficients B_i are equal.

(2) The quadrature formula (11) is exact for all polynomials of degree up to n inclusive.

Let us show that the B_i and t_i can then be found. Setting

$B_1 = B_2 = \ldots = B_n = B$

And noting that for $f(t) \equiv 1$, we have

$$2 = \sum_{i=1}^{n} B_i = nB$$

Hence we obtain $B = \frac{n}{2}$ (12)

Consequently, the quadrature formula is of the form

$$\int_{-1}^{1} f(t)\,dt = \frac{2}{n} \sum_{i=1}^{n} f(t_i) \quad (13)$$

But the general case is to apply the integrals different from (-1, 1) so it is necessary to map the two boundaries (-1,1) to a general case. To apply the quadrature formula to an integral on a general interval of the form

$$\int_{a}^{b} f(x)\,dx$$

It is necessary to transform it by the substitution

$$x = \frac{b+a}{2} + \frac{b-a}{2}\, t_i$$

which carries the interval $a \le x \le b$ into the interval $-1 \le t \le 1$

applying the formula (11) to the transformed integral we get.

$$\int_{a}^{b} f(x)\,dx = \frac{b-a}{n} \sum_{i=1}^{n} f(x_i)$$

$$x_i = \frac{b+a}{2} + \frac{b-a}{2} t_i \qquad (14)$$

7. The Proposed Technique

The proposed technique considers the two types of threats which UAV are exposed to: radar signature and SAMs sites The threat values are computed by integrating the two threat functions over the path length using chebyshev's quadrature formula.

Computation Steps

For a single leg a-b Fig 7 which is a leg of the UAV path to the target position. We have to integrate the threat functions due to every radar site . The reflected signals from a plane body back to the radar site (hence the probability of detection) depends on multi factors such as the reflecting area, original signal strength, atmospheric conditions, and the main factor is the distance between the plane and the radar site. As explained by Equ 2 the attenuation is proportional to the forth power of the inverse distance. The following threat function has to be integrated:

To integrate these two functions Eqn. 2 and Eqn. 3, (Assuming the leg a-b, Fig. 7 is subjected to the two threats) as follows:

Step 1:

The leg length $L=\sqrt{(y_a-y_b)^2 + (x_a - x_b)^2}$ (15)

It's angle is $\theta = \tan^{-1}\left(\frac{(y_a - y_b)}{(x_a - x_b)}\right)$

Where

x_a, y_a are the coordinates of point a

x_b, y_b are the coordinates of point b

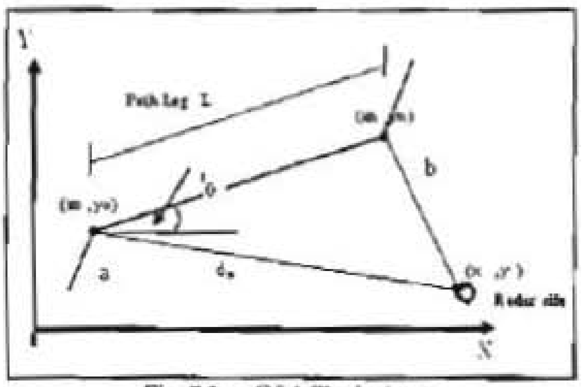

Fig. 7 Leg Of A Trajectory

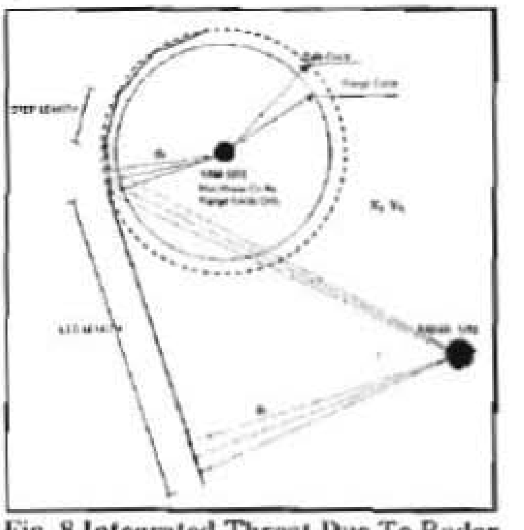

Fig. 8 Integrated Threat Due To Radar Sites and SAM Sites

Step 2:

For the best accuracy, the order of the quadratic formula is considered to be 7 which is the maximum solvable value. They are accurate for orders higher than since the derivation of them fulfills the second condition of Equ.11 so, we compute the roots of the x axis as x_i from Equ. 16 as follows :

$$x_i = \frac{x_b + x_a}{2} + \frac{x_b - x_a}{2} t_i \qquad (16)$$

Step 3:

If the distance between the radar site and the leg root i are considered d_{ri}, this can be computed as

$$d_{ri} = \sqrt{(y_r - y_i)^2 + (x_r - x_i)^2} \qquad (17)$$

or

$$d_{ri} = \sqrt{(y_r - y_i)^2 + (x_r - x_i)^2 + (z_r - z_i)^2}$$

for 3D coordinates

The radar threat at point i is :

$$\sigma_{ri} = \frac{1}{d^4_{ri}} ; i = 1,2,\dots 7. \qquad (18)$$

Step 4:

Getting σ_r the overall threat along the specified (a-b) leg is computed as :

$$\sigma_r = \int_{x_a}^{x_b} \sigma_{ri}\, dx \qquad \text{i.e.}$$

$$\sigma_r = \frac{x_b - x_a}{2} \sum_{i=1}^{n} \sigma_{ri}\, dx \qquad (19)$$

where :

x_n, x_o are the x coordinates of the start and end points of the leg.

σ_{ri} is the threat value of point i computed in Equ. (18)

Step 5:

Repeating the steps 1-4 to the other legs of the specified path from UAV starting position to the target position (n_{legs}) the radar threat all over the path P is summed as :

$$\sigma_{rp} = \sum_{i=1}^{n_{legs}} \sigma_{ri} \tag{20}$$

and for other radar sites the over all radars threat on path P is

$$(\sigma_{rp})_{total} = \sum_{j=1}^{n_r} \sum_{i=1}^{n_{legs}} \sigma_{rp} \tag{21}$$

Step 6:

Getting x_i, y_i in steps 2 the distances (d_{si}) (for the turning steps around the safe circle) the SAMs threats can be computed in a similar way as :

$$x_i = \frac{x_{stepo} + x_{stepn}}{2} + (\frac{x_{stepo} - x_{stepn}}{2})\ t_i$$

And its counter part

$$y_i = \frac{x_i}{Cos\,\theta_{stepi}}$$

$$d_{si} = \sqrt{(y_{sj} - y_i)^2 + (x_{sj} - x_j)^2} \tag{22}$$

x_i, y_i the coordinates of point i on the first turning step around SAM site j.

x_{stepo}, x_{stepn} the x – coordinates of the first and last point of the first step turning around SAMs j.

where

x_{sj} , y_{sj} are the coordinates of the jth SAM site.

j is the SAM site the path is turning around then

$$\sigma_{si} = \frac{1}{(1+d_{si})^2} \qquad \text{if } i\ d_{si} < r_{si} \tag{23}$$

$$= 0 \qquad \text{else where}$$

$$\sigma_s = \int_{x_o}^{x_n} \sigma_{si} \quad d_x$$

$$\text{so } \sigma_s = \frac{x_{stepn} - x_{stepo}}{2} \sum_{i=1}^{n} \sigma_{si} \tag{24}$$

so, the SAMs threat on the (turning part) if the path is

$$\sigma_{sp} = \sum_{i2=1}^{n_{steps}} \sigma_s \tag{25}$$

where n_{steps} : is the number of turning steps around the SAM site.

The only sites affecting Equ. 25 are the concerned ones. The effect of other sites (which the path do not turn around them) is zero. If the path is turning around a number of sites n_{turn} so the total SAMs threats will

$$(\sigma_{rp})_{total} = \sum_{i3=1}^{n_{turn}} \sum_{i2=1}^{n_{steps}} \sigma_{rp} \tag{26}$$

Step 7:

The overall threats targeting path P is

$$J_p = (\sigma_{rp})_{total} + (\sigma_s)_{total}$$

$$J_p = \underbrace{\sum_{j=1}^{n_r} \sum_{i1=1}^{n_{legs}} \sigma_{rp}}_{n_r \text{ radars threats}} + \underbrace{\sum_{i3=1}^{n_{turn}} \sum_{i2=1}^{n_{steps}} \sigma_{ssp}}_{n_{turn} \text{ SAMs thrat}} \tag{27}$$

The overall cost function of the path P is formed from this Threat function and the length of the path.

$$J_{P\,total} = k_{th}\ J_p + k_L\ J_{LP} \tag{28}$$

Where

J_{LP} is the length cost of the path P = $\sum_{i_1=1}^{n_{legs}} L_{i_1}$

k_{th} is a threat weighting factor to be estimated by the mission planner from (0 to 1):

k_L is a length weighing factor, $k_L = 1 - k_{th}$

8. Simulation And Results

Fig.9 shows the increase of the integrated value of the threat function with the length of the path legs, while it is decreasing in the simplified method as shown in Fig.6. In trajectory planning techniques, all feasible trajectories are computed. A cost function is computed for every one. The cost function is composed of two major costs: The first is the path length (consequently fuel consumption and flying time). The second is the threat cost. Then the planner searches among all these feasible trajectories for the minimum cost one. Hence, if the simplified method is used to compute the threat, it results a lower value than the actual. Consequently, the optimal trajectory searched is not actually the optimal. Fig 10 & Fig 11 show two trajectories A,B, for a UAV through 6 SAMs sites along with 6 radar sites in Fig. 10, and 4 radar sites in Fig.11. the results are summarized in table 1

To compare the optimal trajectory which threat is computed by the simplified method (path A), and that computed by the proposed technique (path B):

- In Fig 10 path A (σ_L=2050, σ_{th}=1.04) is shorter than path B (σ_L=2788, σ_{th}=0.9), while path B is less in threat than path A. this is due to the last leg of path A is closer to the radar site while path B avoided that radar and gets away around SAM site number 5.
- In Fig. 11 path B (σ_L=4210, σ_{th}=1.52), is shorter than path A (σ_L=4469, σ_{th}=1.5), with trival increase in threat value. This increase is due to the more accuracy of the proposed technique over the simplified one..

Fig. 9 Increased Threat With Length

Table 1 Simulation Results

	Simplified Tech.			Proposed Tech.		
	Path A			Path B		
	Time Sec.	σ_L	σ_{th}	Time Sec.	σ_L	σ_{th}
Fig. 10	13.75	2050	1.04	4.75	2788	0.90
Fig. 11	14.5	4469	1.5	5.5	4210	1.52

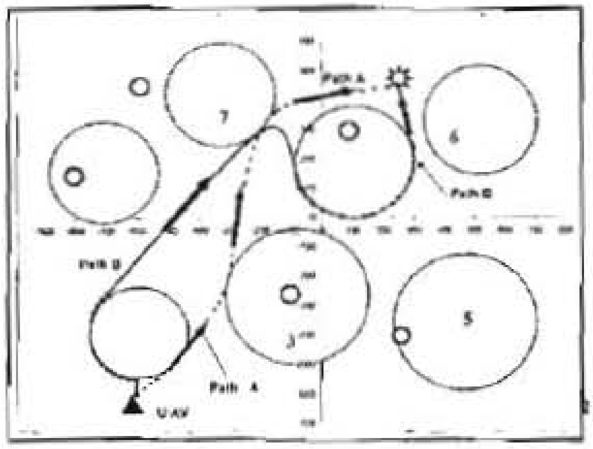

Fig. 10 Path A by the Simplified Method, Path B by the Proposed Method

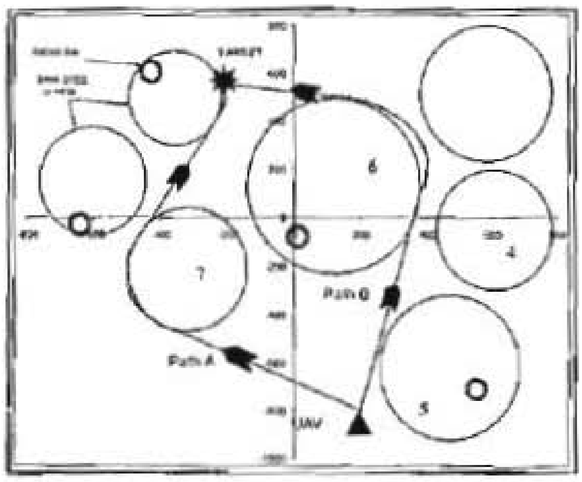

Fig. 11 The Minimum Threat Path A Computed By The Simplified Technique And Path B Computed by The Proposed Techniques.

To check the efficiency of the proposed technique, a lot of simulations are performed for different situations, The threat computation is done by 3 different methods:

a. The simplified technique explained in Section 4.
b. By Simpson's method to integrate the threat along the path length explained in Section 5 taking the interval = 1 km.
c. The proposed technique.

A comparison of the results regarding: computation time and the computed threat

1- The Computation cost of the proposed technique is the minimum among the three methods. While Simpson's method requires a significant time cost which makes it unfeasible for autonomous system Fig. 12.

2- The threat value integrated by the proposed techniques and the Simpson's technique are close to each other, while both are higher than the simplified technique Fig. 13. This shows the inaccuracy of the simplified one.

3- The difference of the threat computed value by the simplified technique and the other two methods differs in some paths than the others. So, it cannot be reliable in to compare among paths in searching for the optimal.

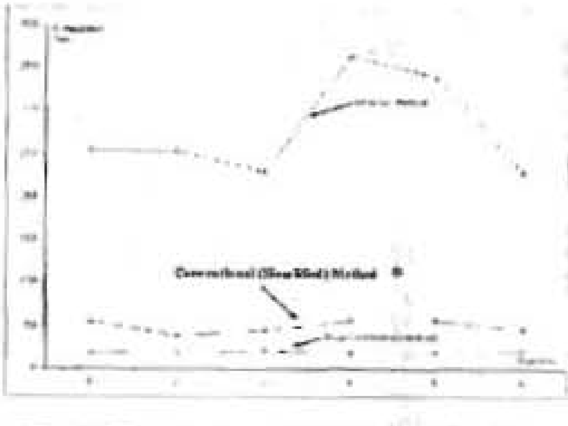

Fig. 12 Comparison of Computation Time

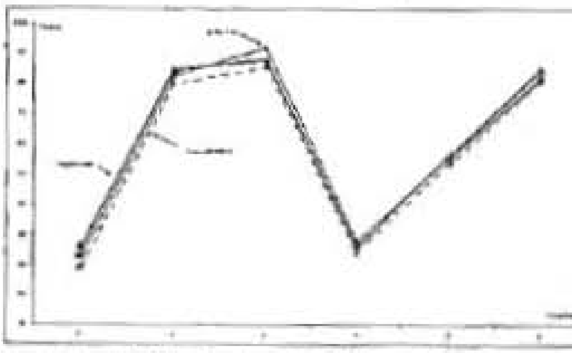

Fig. 13 The Integrated Threat Value

9. Conclusions

The importance of the developed technique is clear, it provides a solution for an accurate threat computation in a very short time. Other techniques may be accurate (as in Simpson's integration) but infeasible because of its high computation cost, or less in accuracy to reduce its computation cost (as in simplified techniques). Also, it considers for the two threats from SAMs and radars sites. Low computation cost enables the vehicle to be autonomous i.e. planning and (re-planning) of its trajectory in real time.

The cost function for any trajectory of the UAV is composed of two parts length and threat. The threat computed by the simplified method (used in other techniques) is misleading in specifying the minimum cost one, since it is devaluated as the path length is increased. Consequently, the minimum threat cost trajectory chosen does not actually have the minimum threat. The proposed methodology introduces a unified, accurate bases in comparison among paths to get the optimal trajectory by integrating the threat function along the trajectory length. It has these advantages:

(1) It provides a significant solution for the two contradicting conditions of the threat computation : accuracy and swiftness.

(2) It considers for both radar sites and the SAMs sites.

(3) It gives real bases for comparison between paths to choose the minimum cost one, and a unified base for threat computation among different trajectories.

(4)The computation cost is very low enabling the vehicle autonomy.

(5)The computation time is irrelevant to paths lengths (like other integration techniques)

(6)The increase of computation time due to the increase in the number of arguments (SAMs, Targets, radars) is trival in the proposed technique, while it is increases in multiplication order in the others.

Refrences

1. Chandler, P., Rasmussen, S., and Pacher, M.,"UAV cooperative path planning". Proc. GNC, 2000.
2. Myungsoo Jun,Raffaello D'Andrea," Path Planning for Unmanned Areal Vehicles in Uncertain and Adversarial Environments'. In S. Butenko, R. Murfey, and P. Paradolas, Editors," Cooperative Control Model Algorithms, Chapter 6" Page (95-111), Kluwer, 2002.

* Used in [1], [3], [8]

3. Scott A. Bortoff, "Path planning for Unmanned Air Vehicles" American Institute of Aeronautics & Astronautics-2001.
4. SomL o.j., Sokolov A., v.u kanyin v., " Intelligent Robot Control. The Automatic Trajectory Planning ", hu/ Elekiinet. 1999 .
5. Mark H. Overmars,"Recent Developments in Motion Planning", Computational Since ICCS 2002: International Conference, Amsterdam, The Netherlands, April 21-24, 2002 proceedings, Part III.
6. Yong k.Hwang, Narendra Ahuja, "Gross Motion Planning-A Survey", ACM Computing Surveys. Vol. 24, No.3, 992 .
7. McLain, T. and Beard, R. "Trajectory planning for coordinated rendezvous of unmanned air vehicles.", Proc. GNC,2000.
8. Timothy W. McLain ,"Coordinated Control of Unmanned Air Vehicles ", Air Force Research Laboratory, Airvehicles Directorate, Wright-Patterson AFB, Ohio, 1999.
9. Timothy W. McLain,Randal W. Beard ," Trajectory Planning For Coordinated Renendezvous of Unmanned Air Vehicles ",American Institute of Aeronautics and Astronatics,Inc.,2000 .
10 Khatib O , "Real Time Obstacle Avoidance for Manipulators and Mobile Robots". IEEE international conference on robotics and automation, st. louis, PP. 500-505. 1985.
11. John B., Arthur R., Jonathan, P.how, " Receding Horizon control of Autonomous Aerial Vehiches ", MIT, 2003.
12. Tom S. Bart M., Eric F., Jonathan H. " Mixed Integer Programming for Multi – Vehicle Path Planning", European control conference, 2001 .
13. Tom S., Eric F., Jonathan H., " Safe Receding Horizon Path Planning for Autonomous Vehicles ", 40^{th} Allerton Conference On Communication, Control, and Computation, 2002.
14. Theju Maddula, Ali A.Minai,Marios M. Polycarpou,"Multi-Target assignment and Path Planning for Groups of UAVs", In Cooperative "Control and Optimization", S. Butenko, R. Murphey and P. Pardjloc (eds), Kluwer Academic Publishers, PP. 261-772, 2004.
15. Bassem F. Mahafxa, "Radar Systems Analysis and Design using MATLAB", Charman & Hall / CRC, 2005.
16. Beard, R., McLain, T., and Goodrich, M., "Coordinated target assignment and intercept for unmanned air vehicles.", Proc. ICRA, 2000.
17. LaValle S.M."Planning Algorithms", Cambridge University Pre SS, 2006.
18. B.P. Demidovich L. A. MARON, Computation Mathematics, translated from the Russian by George Yankovsky Mir Publishers. Moscow, 1981. Approximation", 1957