

A BALANCING APPROACH TO STOCHASTIC ASSEMBLY LINES

أسلوب لاتزان خطوط التجميع ذات الأزمنة الاحتمالية

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يُقدم هذا البحث نموذجاً حسابياً يستخدم تقدير تكاليف التشغيل الكلية لتحليل واتزان وتقييم خطوط التجميع ذات الأزمنة الاحتمالية. يفترض في هذا النموذج أن أزمنة تنفيذ العمليات تتبع التوزيع الطبيعي وكذلك يمكن حساب كل من الوسط الحسابي والتباين لكل منها. ويقوم النموذج بتصميم الاتزان الأقل تكلفة وذلك بتعيين عناصر العمل للمطاطات على طول الخط بحيث يكون احتمال استكمال تنفيذ كل العمليات الخاصة بأي محطة خلال وقت دورة الانتاج أكبر من أو يساوي مستوى ثقة محدد مسبقاً وقد تم استخدام بارميتر جديد لاتخاذ القرار في عملية تعيين عناصر العمل وهذا البارميتر يساعد في عملية تنظيم السلوك التبايني لأزمنة تنفيذ العمليات على امتداد الوقت الدتاح للمحطة يتم تقييم الاتزان بحساب تكاليف التشغيل الكلية للخط في حالتي الاصلاح داخل وخارج الخطأ. وقد تم برمجة النموذج بلغة الفورتران وكذلك تم تطبيقه على حاله دراسيه من الصناعة الوطنية.

ABSTRACT

This paper presents a cost-oriented approach for analyzing, balancing, and evaluating stochastic assembly lines. Task times are assumed to be normally distributed random variables with known means and variances. The work is concerning with the design of an approximately minimum cost layout by assigning tasks to work stations along the line such that the probability of completing all tasks within the cycle time is greater than or equal to certain confidence level. A new decision parameter is used for assignment, this parameter helps in regulating the stochastic behavior of task durations along the span of each work station. A candidate line layout is evaluated by computing the total operating cost under off-line repair and on-line repair conditions. The procedure was programmed in 'FORTRAN-77' and applied to a case study problem from the national industry.

INTRODUCTION

Assembly line balancing problem concerns the assigning of tasks to a series of work stations manned with one operator or more. The objective is to minimize the total operating cost under several constraints; particularly, the precedence relations which restricts the order in which the tasks can be performed and the allowable cycle time which is the maximum work content time allowed to each work station along the line.

In real life there are significant variations in task performance

durations, thus, any considerable analysis of a paced assembly line must consider the probability of exceeding the cycle time which causes the incompleteness cost component. Some tasks are not completed because the time needed to complete them exceeds the time available at their stations. Other tasks may not be completed because previous dependent tasks have not been completed. Therefore, an analyst of line should expect violation in one or more stations and the incurred cost-incompleteness cost- has to be evaluated.

Because of the possibility that some tasks may not be completed within their stations, some remedial actions must be taken and the cost of the finished product will be affected by the type of action adopted. There are two main essential remedial actions, the first is to move the product down the line assembling the possible tasks regardless of the uncompleted tasks and repairing uncompleted units at the end of line (off-line repair). Second is to stop the entire line for time enough to complete all units in a particular cycle (on-line repair).

When task durations are considered stochastic, the operating cost for an assembly line consists of two components, normal operating cost and repair cost. By packing tasks tightly into stations, number of stations can be reduced which reducing the normal operating cost per unit. But packing stations will result in a higher probability to exceed the cycle time, thus incurs the remedial cost component.

In solving the assembly line balancing problem, the deterministic approaches neglected the probable dimension of incompleteness and tended to pack stations as possible (3), (5), (9), and (13).

Several line balancing techniques manipulate stochastic task duration problem. Ignall (6) recommended to assign the tasks with a specified percent of the cycle time. Moodie and Young (14) developed a heuristic approach for minimizing the number of work stations while maintaining a specified chance of completing the tasks. Mansoor and Ben Tuiva (12) have designed an incentive system which minimizes the labour cost per unit. Ramsing and Downing (15) developed a computer program to solve the problem at different levels of significance. Kottas and Lau (10) described an approach which assigns tasks to work stations on the basis of a marginal cost criterion. Sphicas and Silverman (18) evaluated a deterministic equivalent for stochastic assembly line balancing. Kao (7) proposed a preference order dynamic programming technique minimizes the labour cost under certain chance of completion. In another approach, Kao (8) developed a programmable strategy that minimizes the total operating cost. Maani and Hogg (11) proposed a stochastic network simulation model for analyzing the assembly line system. Shtub (16) developed a technique considers variable task durations and variable manning of work stations. Akagi, et al (2) developed an improved balancing method by assigning tasks to work stations according to its performance rate. Abdel-Shafi, A. A. A. (1) suggested a simulation approach determining the buffer level required between work stations to avoid incompleteness cost.

NOMENCLATURE

- b : Constant labour rate for off-line repair \$/unit time.
 C : Cycle time.
 $F(Z_k)$: Cumulative normal probability function for station k .
 $\phi(t)$: Normal probability density function of task times.
 $G(w)$: The distribution function of the random variable w .
 K : Number of work stations along the line.
 M_k : Sum of mean times of tasks assigned to station k .
 n : Number of task observations less than or equal to the mean.
 N : Total number of task observations.
 r : Normal labour rate \$/unit time.
 R_s : Station assignment ratio.
 R_t : Task time ratio.
 ρ : Repair penalty multiplier.
 T : Total work content time.
 w : Random variable of maximum time required for all stations of line to complete their tasks in a particular cycle.
 Z : Normal standard score corresponding to planned confidence level.
 Z_k : Number of standard normal deviations around mean time of station k .
 σ_k : Standard error (standard deviation) of station assigned time.

STOCHASTIC ALGORITHM

An assembly line balancing algorithm is designed, it has the ability to assign tasks with stochastic durations to work stations and minimizing two component cost function without violating the imposed restrictions such as: cycle time, precedence relationships, and confidence level.

ASSUMPTIONS

- Assembly line is operating continuously at a constant rate (Paced) producing identical units, and worker wages are the same.
- Task times are independent normally distributed random variables with estimable means and variances.
- Whenever a task is not finished during the cycle time in the assigned work station, there only one of two alternatives. First, the unit continues down the line to assemble the uncompleted remaining task(s) independent on missed task(s). All unfinished tasks are completed at the end of the line (off-line repair; (4)). Second, the line is stopped to complete the unfinished tasks in their assigned work stations (on-line repair; (17)).

PROCEDURE

The following steps describe the procedure whose flow chart is given in figure (1):

(1) Input data.

The data consist of task duration observations, precedence relations, cycle time, labour rates, and confidence level.

(2) Compute initial variables.

Those variables are task duration means and variances and task time ratios.

(3) Open a new work station.

Assign tasks to new station.

(4) Form precedence list.

Such list contains those tasks which satisfy the precedence constraints (all immediate predecessors are already assigned). If this list is empty, the balance is completed, go to step 7; otherwise go to next.

(5) Form assignment list.

This list comprises all tasks that will fit into the current station such that

$$M_k + Z \cdot \sigma_k \leq C \dots\dots\dots (1)$$

If this list is empty, go to step 3; otherwise go to next.

(6) Assigning a task.

A decision parameter called task time ratio R_i will be used to weight each task in the assignment list where $R_i = t_i/N$. The station is rationed by a percent R_s less than or equal to 50%. The first part is assigned the tasks which have higher probability to exceed their times or those have smallest task time ratio. The second part of the station is assigned contrary to the first. This because when the work is moved down the station, the probability of incompleteness is increased due to fatigue, monotony, and the accumulated variance. After each assignment go to step 4.

(7) Compute evaluation measures.

Different measures are computed to evaluate the current line design.

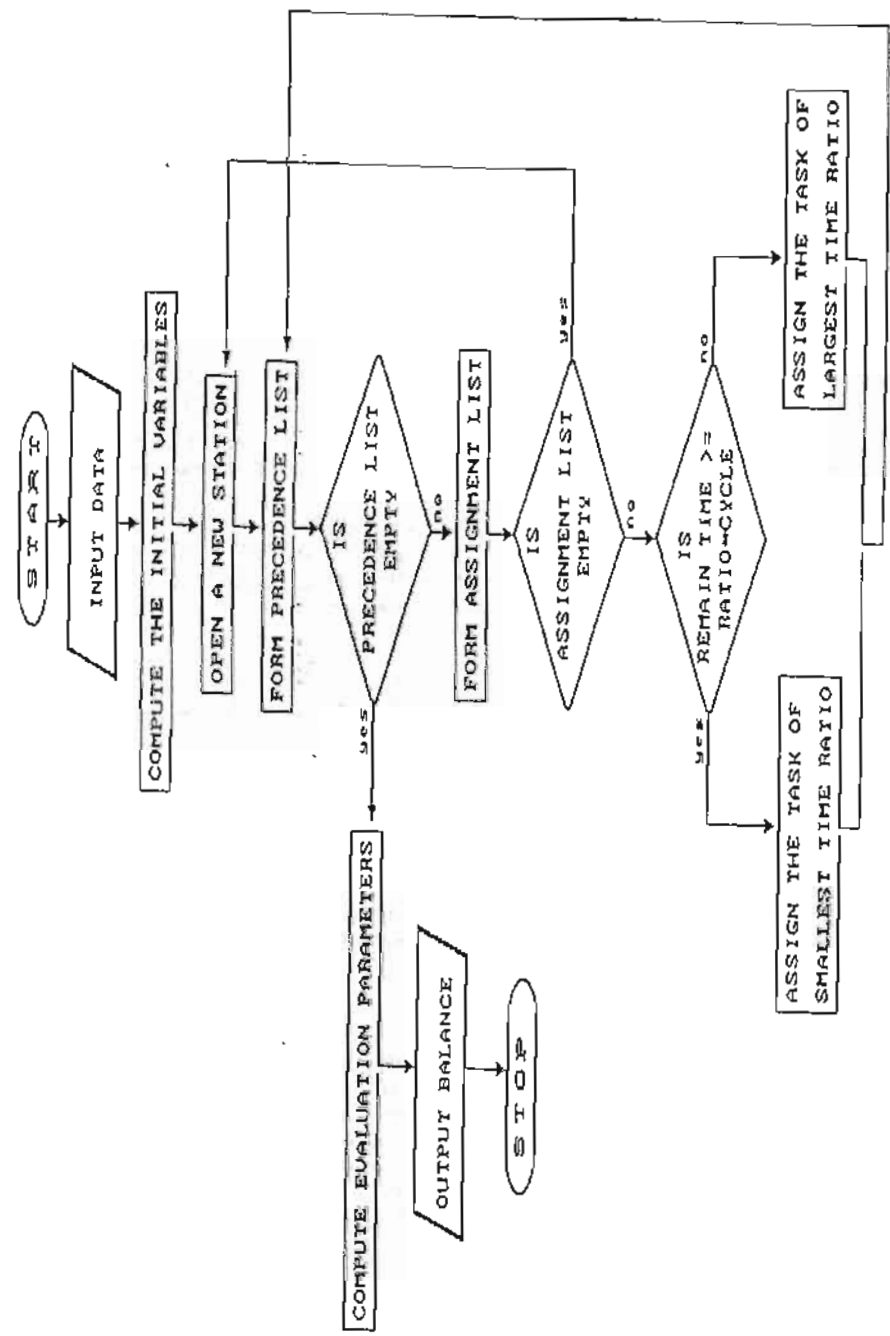


Fig. (1) Flow chart for stochastic balancing procedure.

$$(a) \text{ Balance delay \%} = \left(K.C - \sum_{k=1}^K M_k \right) / K.C \dots\dots\dots (2)$$

$$(b) \text{ Normal labour cost} = r.C.K \dots\dots\dots (3)$$

(c) Probability of exceeding cycle time for each station

$$= \int_c^{\infty} \phi(t).dt \dots\dots\dots (4)$$

(d) Probability of line completion

$$= \prod_{k=1}^K \left(1 - \int_c^{\infty} \phi(t).dt \right) \dots\dots\dots (5)$$

(e) Repair (incompletion cost). Two stochastic cost functions are used for that purpose, first when repair is made off-line and second when repair is made on-line.

$$\text{Expected off-line repair cost} = \left(1 - \prod_{k=1}^K F(Z_k) \right) . b.T \dots\dots\dots (6)$$

$$\text{Expected on-line repair cost} = \rho.K.C \int_c^{\infty} \left(1 - G(w) \right) . dw \dots\dots\dots (7)$$

$$(f) \text{ Expected total operating cost} = \text{Normal cost} + \text{Repair cost} \dots\dots (8)$$

(8) Selecting a balancing solution.

The procedure steps from 1 to 7 are to be repeated enough times to test the effect of stochastic task times, cycle time, station ratio, and confidence level on resulted balance.

CASE STUDY APPLICATION

In order to test the approach capability, a FORTRAN computer program is constructed for the procedure steps explained earlier and used to investigate a real life case study under a wide range of different conditions. The case study is an application conducted on a 20"-Colour T.V. set chassis produced at 'Arabic Company of Electronic Instruments-Telemisr (NEC)'. Observations for each task duration are recorded from the real life assembly work, task precedence relationships are analyzed and recorded (table 1). Normal labour rate ($r = \$0.004/\text{sec.}$) and repair penalty ($\rho = 5$).

Table (1) Task precedence relations and mean durations.

| I | II | III |
|----|-------|-----|
| 1 | 120.0 | - |
| 2 | 120.0 | 1 |
| 3 | 20.0 | 1 |
| 4 | 30.0 | 2 |
| 5 | 40.0 | 3 |
| 6 | 40.0 | 3 |
| 7 | 40.0 | 3 |
| 8 | 40.0 | 3 |
| 9 | 10.0 | 3 |
| 10 | 10.0 | 3 |
| 11 | 35.0 | 4 |
| 12 | 45.0 | 4 |
| 13 | 25.0 | 5 |
| 14 | 25.0 | 5 |
| | | 6 |
| | | 7 |
| | | 8 |
| 15 | 15.0 | 9 |
| | | 10 |
| 16 | 20.0 | 11 |
| 17 | 10.0 | 12 |
| 18 | 15.0 | 12 |
| 19 | 15.0 | 12 |
| 20 | 20.0 | 12 |
| 21 | 25.0 | 13 |
| | | 14 |
| 22 | 10.0 | 15 |
| 23 | 50.0 | 16 |
| 24 | 15.0 | 16 |
| 25 | 40.0 | 17 |
| 26 | 25.0 | 17 |
| | | 18 |
| | | 19 |
| 27 | 15.0 | 21 |
| 28 | 25.0 | 21 |
| 29 | 20.0 | 21 |
| 30 | 20.0 | 21 |
| 31 | 25.0 | 21 |
| 32 | 30.0 | 22 |
| 33 | 30.0 | 23 |

| I | II | III |
|----|------|-----|
| 34 | 45.0 | 23 |
| 35 | 60.0 | 23 |
| 36 | 20.0 | 23 |
| 37 | 15.0 | 24 |
| 38 | 15.0 | 24 |
| 39 | 20.0 | 24 |
| 40 | 30.0 | 20 |
| | | 25 |
| | | 26 |
| 41 | 30.0 | 27 |
| | | 28 |
| | | 29 |
| | | 30 |
| | | 31 |
| 42 | 15.0 | 21 |
| | | 32 |
| 43 | 40.0 | 33 |
| | | 34 |
| | | 35 |
| | | 36 |
| 44 | 60.0 | 40 |
| 45 | 25.0 | 40 |
| 46 | 40.0 | 40 |
| 47 | 50.0 | 40 |
| 48 | 10.0 | 41 |
| 49 | 10.0 | 42 |
| 50 | 15.0 | 42 |
| 51 | 10.0 | 42 |
| 52 | 20.0 | 44 |
| 53 | 20.0 | 48 |
| | | 49 |
| | | 50 |
| | | 51 |
| 54 | 25.0 | 37 |
| | | 38 |
| | | 39 |
| | | 45 |
| | | 46 |
| | | 47 |
| | | 52 |

| I | II | III |
|----|-------|-----|
| 55 | 45.0 | 53 |
| 56 | 40.0 | 48 |
| | | 54 |
| | | 55 |
| 57 | 30.0 | 43 |
| 58 | 40.0 | 56 |
| 59 | 15.0 | 56 |
| 60 | 20.0 | 56 |
| 61 | 25.0 | 56 |
| 62 | 30.0 | 56 |
| 63 | 10.0 | 56 |
| 64 | 20.0 | 56 |
| 65 | 25.0 | 56 |
| 66 | 10.0 | 57 |
| | | 58 |
| | | 59 |
| 67 | 10.0 | 60 |
| | | 61 |
| | | 62 |
| | | 63 |
| | | 64 |
| | | 65 |
| 68 | 20.0 | 66 |
| | | 67 |
| 69 | 45.0 | 68 |
| 70 | 25.0 | 69 |
| 71 | 200.0 | 69 |
| 72 | 100.0 | 70 |
| | | 71 |
| 73 | 20.0 | 72 |
| 74 | 10.0 | 73 |

I : Task number.
 II : Task mean duration(sec.).
 III: Task immediate predecessor(s).

RESULTS AND CONCLUSION

The developed computerized cost oriented approach has been used to investigate the case study problem. The results for different operating conditions are summarized in figures 2, 3,....., and 10. Based on these results, the following points may be concluded:

(1) Line completion probability is sensitive to the change in cycle time. so, cycle time has to be selected carefully to increase the probability of completion, consequently reducing repair cost.

(2) Balance delay and labour cost are sensitive to the change in cycle time in cyclic behavior, that is because cycle time affects packing tasks in work stations.

(3) Expected repair cost in case of on-line repair is always lower than off-line repair cost for any cycle time used, the different increases as cycle time gets larger. Off-Line repair method allows the unit with missed unassembled tasks to go through succeeding work stations where tasks dependent on the missed tasks can not be assembled, in turn, leads to more idle time in succeeding stations plus repairing time later; thus incurs more cost. On-Line repair method stops the line for a time enough to finish all work behind the schedule. Generally, repair cost depends on number of tasks in the job and the number of precedence relations.

(4) Expected total operating cost is sensitive to the change in cycle time in cyclic way for both method of repair, but the expected total cost for on line repair is lower than off-line repair.

(5) An improvement in expected total cost can be achieved by increasing confidence level. This is due to the reduction in the probability of incompleteness which, in turn, reduces the repair cost. In off-line repair case, the improvement in expected total cost tangible in unpredictable fashion. While in on-line repair case, the improvement is smaller in predictable fashion.

(6) The change in the specified station ratio R_s has an unpredictable effect on expected total cost for the two suggested repair methods. though the analysis can be used to select an acceptable line layout of minimum expected total cost.

The presented approach facilitates solving large industrial assembly line problems in which task times are stochastic, this exerts less effort, with minimum computations and computer storage. The suggested analysis is capable of producing comprehensive views for the assembly line balancing solutions and their expected total cost, which will be very helpful to the decision maker. It can be seen that the concept of assigning tasks to the stations on the basis of task time ratio and station ratio minimizes the expected total operating cost.

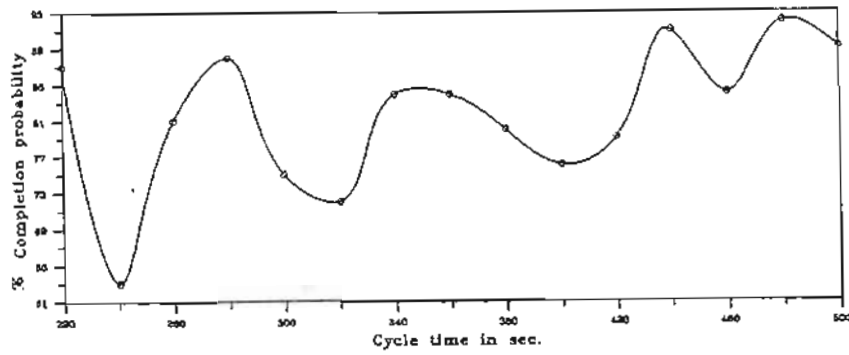


Fig. (2) Effect of cycle time on completion probability.

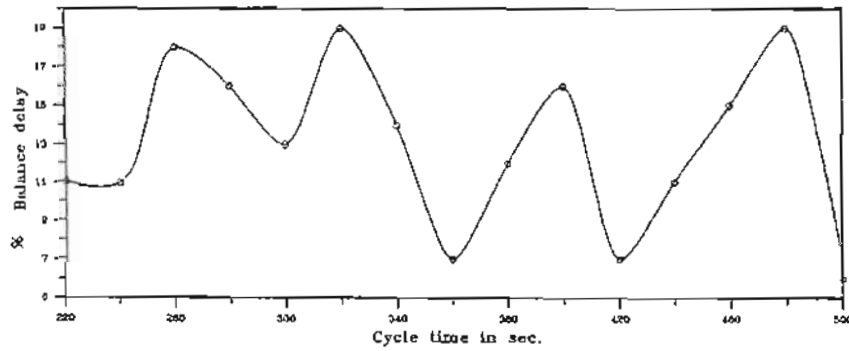


Fig. (3) Effect of cycle time on balance delay.

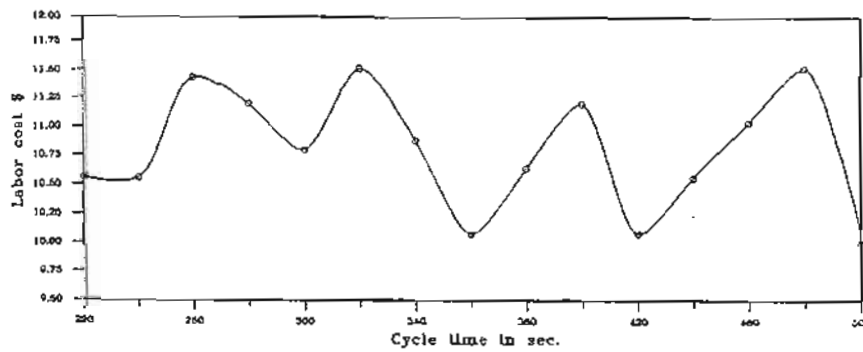


Fig. (4) Effect of cycle time on labour cost.

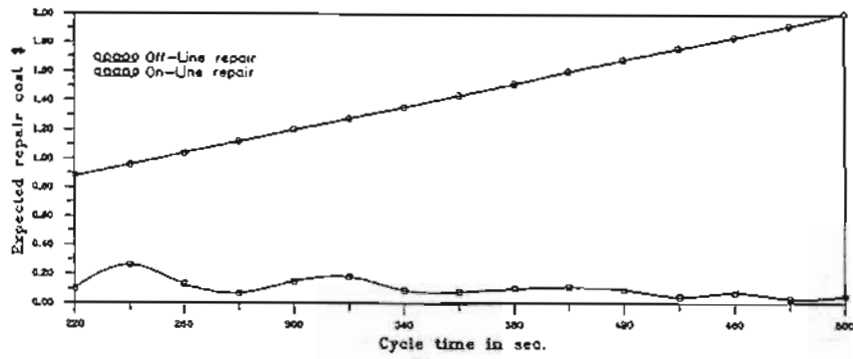


Fig. (5) Effect of cycle time on expected repair cost.

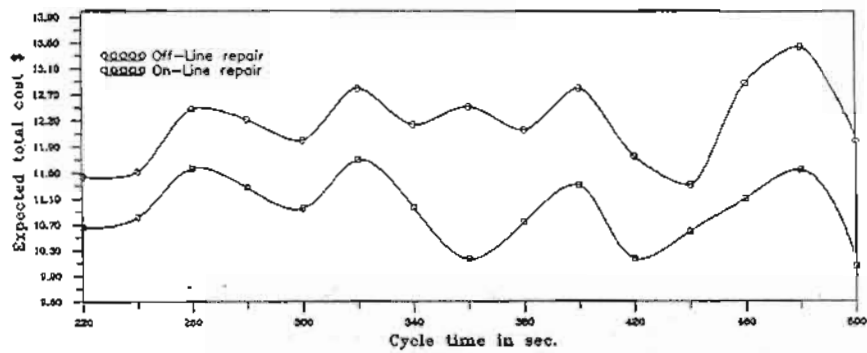


Fig. (6) Effect of cycle time on expected total cost.

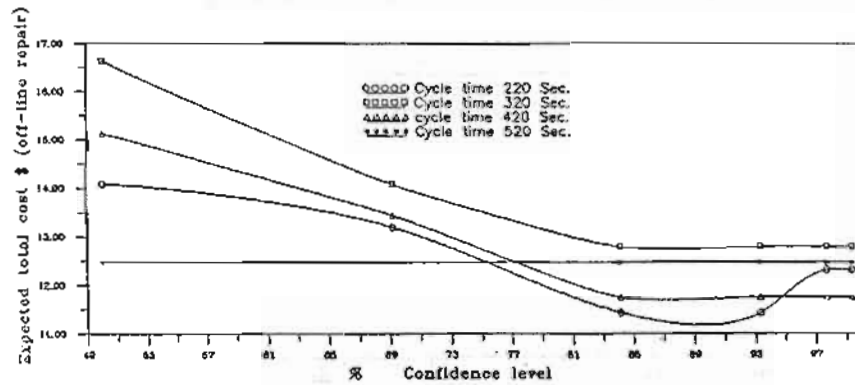


Fig. (7) Confidence level effect on off-line repair expected total cost

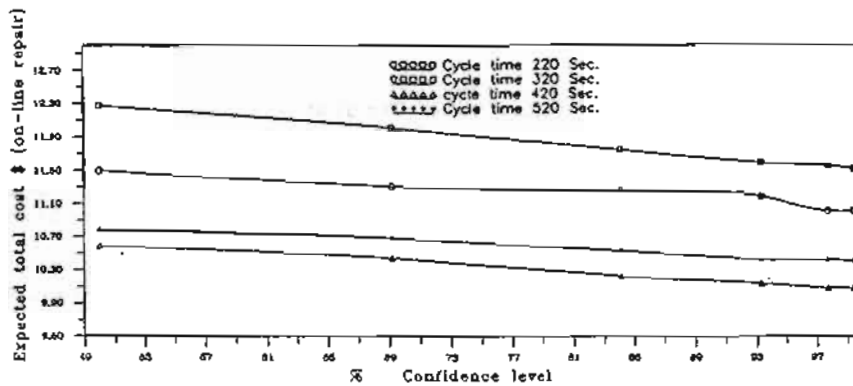


Fig. (8) Confidence level effect on on-line repair expected total cost

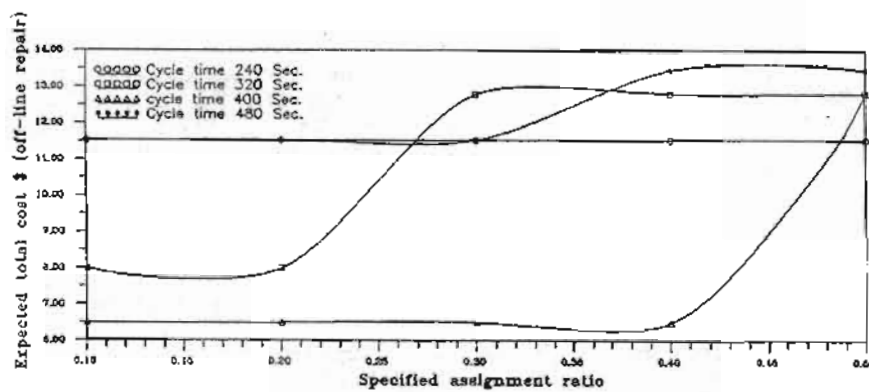


Fig. (9) Effect of specified station assignment ratio on expected total cost (off-line repair)

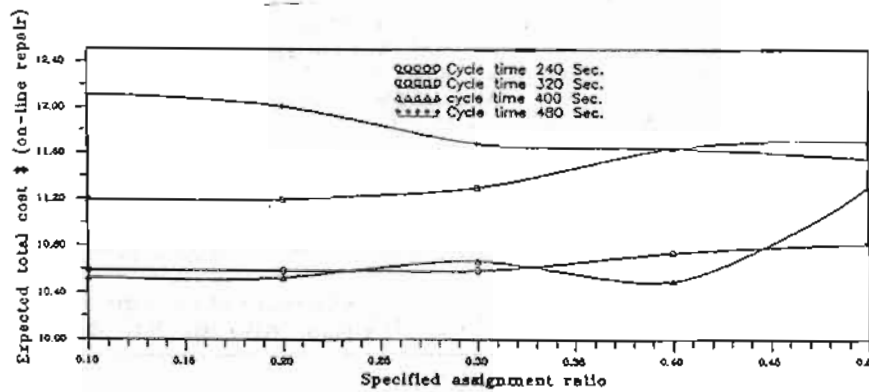


Fig. (10) Effect of specified station assignment ratio on expected total cost (on-line repair case).

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