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Stokes Flow of a Spherical Micropolar Fluid Droplet within Concentric Spherical Cavity Filled with Micropolar Fluid

Ahmed. G. Salem[*1], *M. S. Faltas***, *H. H. Sherief* **

**Department of Mathematics, Faculty of science, Mansoura University, 35516, Mansoura, Egypt*

***Department of Mathematics, Faculty of science, Alexandria University, Alexandria, Egypt*

**Corresponding authorE-mail address: a.g.salem@mans.edu.eg (Ahmed. G. Salem)*

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Abstract: An analytical study for the quasi-steady flow caused by a spherical micropolar or viscous fluid droplet translating at a concentric position in a second immiscible micropolar or viscous fluid within a spherical cavity is presented. The droplet translates along a diameter connecting their centers under the conditions of low Reynolds numbers. To solve the Stokes equations for the velocity fields inside and outside the droplet, general solutions are obtained in terms of spherical coordinates based on the concentric position. For the cases of the viscous droplet within a micropolar and micropolar droplet within a viscous fluid, a boundary condition related to vorticity with microrotation is used; while for the case of a micropolar drop within a micropolar fluid, continuity of microrotation and tangential couple stress are used. The normalized drag forces acted on the droplet for the different cases are represented through graphs for various values of relative viscosity, radii ratio of droplet and cavity, and the parameter that connects the vorticity with microrotation. The wall-corrected drag force is found to be a monotonic increasing function of the ratio of drop-to-cavity radii. The present work is motivated by its potential applications such as raindrop formation and rheology of emulsion.

keywords: Micropolar flow. Droplets and bubbles. Low Reynolds numbers. Spherical cavity**.**

1.Introduction

The area of moving drops and bubbles exists widely in nature. In nearly every situation, these fluid particles, which may lie within a continuum of another fluid, have an important significance on the physical behavior of the system. For instant, clouds are natural assemblages of tiny water droplets which coalesce and lead to rainfall. In addition, in industrial systems, such as in some chemical reactors, drops, and bubbles commonly occur as carriers of both reactants and products [1]. The earliest work in the regime of low Reynolds number is given by Stokes [2] who studied the motion of a solid spherical particle translating into a viscous fluid. One of the important parameters which have been evaluated is the drag force acting on the particle by the surrounding fluid. Naturally, this basic solution leads to the study of the motion of fluid spheres and particles of other shapes. The classical development after Stokes [2] was the solution of the slow motion of a fluid sphere given by Rybczynski [3] and independently by Hadamard [4]. They assumed that the velocity and the tangential stress are continuous at the interface of the fluid sphere and found the drag force exerted on the fluid sphere by the surrounding fluid. Extension of [3] or [4] to micropolar fluid was studied by Niefer and Kaloni [5]. The interfacial stresses acting at the drop surface tend to deform it. However, if the motion is sufficiently slow or the drop is small in size, the drop will in the first approximation be spherical. The distortion of the spherical shape of a droplet is studied in the literature by some authors e.g. [6–8].

In many studies in the literature under low Reynolds number conditions, drops and bubbles are not isolated and it is important to find out if the presence of neighboring drops and/or boundaries significantly affects the migration of a drop. The motion of a single particle embedded in a fluid only represents the case of a short distribution phase. The drop interaction problems have been studied widely by various exact, asymptotic, and numerical methods [10–24]. The system of a spherical drop within a spherical cavity can be viewed as an idealized model for oil or water droplets in the rock of the oil-containing formation composed of connecting spherical pores [25]. Faltas and saad [26] investigate investigate the creeping flow problems of a viscous droplet moving perpendicular to an impermeable bounding plane surface within a micropolar fluid and the motion of a viscous droplet within viscous fluid. Saad [27] studied the Stokesian flow of a spherical-shaped droplet which is halfway immersed in a semi-infinite phase of a micropolar fluid.

The well-known Navier–Stokes equations assume that the fluid particles do not have any internal structure. However, fluid particles may discover some microscopical effects such as turnover, constriction, or protraction for several fluids such as polymeric hang, and brute blood. That is the interior body should be considered for fluids whose particles have compound shapes. A well-accepted theory that considered the internal microstructure is the micropolar fluids which were initiated by Eringen [28-31]. In this theory here, specific particles can swirl independently from the rotation and locomotion of the fluid as a whole. Due to the above considerations, a new variable that represents the angular velocity of fluid particles and a new equation governing this variable should be added to the classical model. Ferrofluid is considered as a micropolar fluid because it consists of a stabilized colloidal suspension of Brownian magnetic particles in a non-magnetic liquid host [32]. The granular flow is considered also a micropolar flow because it has microstructure and rotation of particles [33-36]. Hayakawa [33] concluded that the theoretical study of certain boundary value problems agrees with relevant experimental results of granular flows.

Another class of microstructure fluids is the microstretch fluids in which each material volume element contains microvolume elements that can translate, rotate, and deform independently of the motion of the macro volume elements. Therefore, in microstretch fluids, material points are considered to stretch, expand, or contract, in addition to rotating about their centroids. This type of fluid is also known as Eringen fluids or micropolar fluids with stretch [37].

The Stokes axisymmetric microstretch streaming flow problem past a stationary viscous droplet and as well as the related problem of a viscous streaming flow past a stationary microstretch fluid droplet are studied by [38]. The droplets are considered either perfect spherical or deformed spherical in shape. For these flows, the microstretch scalar function is uncoupled from the stream function and microrotation component function.

The study aim of the problem is to find out the boundary effect on the quasi-steady motion of the drop. This article contains three hydrodynamical problems: (a) the quasi-steady translation of a classic viscous spherical drop in the concentric placement of a spherical cavity filled with micro-structure fluid of micro-polar type, (b) the quasi-steady translation of a micropolar drop in the concentric placement of a spherical cavity filled with viscous fluid, (c) the quasi-steady translation of a micro-polar drop in the concentric placement of a spherical cavity filled with micro-polar fluid. The fluids inside and outside the droplets are immiscible. Analytical solutions are found for each problem. Continuity of velocity and tangential stress is continuous at the surface of the droplet in addition, the no slip conditions and no spin at the inner surface of the cavity are used. The spin-vorticity relation is used in problems (a) and (b) at the surface of the droplet; while in problem (c), the continuity of microrotation and tangential couple stress are considered.

2. Micropolar Governing Equations

The equations governing the steady flow of an incompressible micropolar fluid in the absence of body forces and body couples as given by Eringen [29], under the conditions of low Reynolds numbers, are

$$\nabla \cdot \vec{q} = 0, \quad (2.1)$$

$$\nabla p = \kappa \nabla \wedge \vec{\nu} - (\mu + \kappa) \nabla \wedge \nabla \wedge \vec{q}, \quad (2.2)$$

$$2\kappa \vec{\nu} = \kappa \nabla \wedge \vec{q} - \gamma \nabla \wedge \nabla \wedge \vec{\nu} + (\alpha + \beta + \gamma) \nabla \nabla \cdot \vec{\nu}, \quad (2.3)$$

where $\vec{q}$, $\vec{\nu}$, and p are the velocity vector, microrotation vector, and fluid pressure at any point, respectively. μ is the viscosity coefficient of the classical viscous fluid and κ is the vortex viscosity coefficient. The remaining viscosity coefficients α, β and γ are gyro-viscosity coefficients. The constitutive equations defining the stress tensor Π and the couple stress tensor m are given by

$$\Pi = -p\boldsymbol{I} + \tfrac{1}{2}(2\mu+\kappa)\boldsymbol{\Delta} + \kappa\boldsymbol{\varepsilon}\cdot(\vec{\omega}-\vec{\nu}), \quad (2.4)$$

$$m = \alpha\boldsymbol{I}\cdot\nabla\vec{\nu} + \beta\nabla\vec{\nu} + \gamma\nabla^T\vec{\nu}, \quad (2.5)$$

where $\boldsymbol{I}$ is the unit tensor, $\boldsymbol{\varepsilon}$ is the unit alternating tensor, $\vec{\omega}\left(=\tfrac{1}{2}\nabla\wedge\vec{q}\right)$ is the vorticity vector and $\boldsymbol{\Delta}$ is the rate of deformation tensor, defined as

$$\boldsymbol{\Delta} = \tfrac{1}{2}\left(\nabla\vec{q} + \nabla^T\vec{q}\right), \quad (2.6)$$

Here the superscript T refers to the transpose of a tensor. The study is considered under the set of low Reynolds numbers of micropolar fluid.

3. Differential Equation Satisfied by the Stream Function

Let (r, θ, ϕ) be spherical coordinates with corresponding $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$ unit vectors. For an arbitrary axisymmetric particle translating steadily in an incompressible micropolar fluid along its axis of revolution, the velocity and microrotation vectors are of the form

$$\vec{q}(r,\theta) = q_r\vec{e}_r + q_\theta\vec{e}_\theta, \quad \vec{\nu}(r,\theta) = \nu\vec{e}_\phi. \quad (3.1)$$

With the help of equation (2.1), we can find the velocity components in terms of the Stokes stream function ψ as

$$q_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta}, \qquad q_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}. \quad (2.1)$$

Inserting (3.1) and (3.2) into the field equations (2.2) and (2.3), we

$$-\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\mu+\kappa)E^2\psi$$
$$+\frac{\kappa}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\nu) - \frac{\partial p}{\partial r} = 0, \quad (3.3)$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial r}(\mu+\kappa)E^2\psi$$
$$-\kappa\frac{\partial}{\partial r}(r\nu) + \frac{\partial p}{\partial\theta} = 0, \quad (3.4)$$

$$\gamma E^2(r\sin\theta\nu) - \kappa E^2\psi$$
$$-2\kappa(r\sin\theta\nu) = 0, \quad (3.5)$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\zeta^2}{r^2}\frac{\partial^2}{\partial\zeta^2}, \quad \zeta = \cos\theta. \quad (3.6)$$

Elimination the pressure p and microrotation ν from (3.3) - (3.5), we obtain

$$E^4\left(E^2-\ell^2\right)\psi = 0, \qquad \ell^2 = \frac{(2\mu+\kappa)\kappa}{(\mu+\kappa)\gamma}. \quad (3.7)$$

Let

$$\psi = \psi_1 + \psi_2, \quad (3.8)$$

where

$$E^4\psi_1 = 0, \quad \left(E^2-\ell^2\right)\psi_2 = 0. \quad (3.9)$$

Therefore, the microrotation component can be expressed in terms of the stream function as

$$\nu = \frac{1}{2r\sin\theta}\left(E^2\psi_1 + 2m\ell^2\psi_2\right), \quad m = \frac{\mu+\kappa}{\kappa}. \quad (3.10)$$

It should be noted that, the part of solution ψ_1 represents the solution of classical viscous fluid. The regular general solutions of the equations in are as follows:

$$\psi_1 = \sum_{n=2}^{\infty}\left(A_n r^n + C_n r^{(n+2)} + B_n r^{(-n+1)} + D_n r^{(-n+3)}\right)G_n(\zeta), \quad (3.11)$$

$$\psi_2 = \sum_{n=2}^{\infty}\sqrt{r}\left(F_n I_{n+\frac{1}{2}}(\ell r) + E_n K_{n+\frac{1}{2}}(\ell r)\right)G_n(\zeta), \quad (3.12)$$

where (I_n, K_n) are the first and second kind modified Bessel function of order n, respectively, G_n is the Gegenbauer first kind polynomial of order n and degree $-\tfrac{1}{2}$; $A_n, C_n, B_n, D_n, F_n, E_n,\ n \geq 2$ are unknown constants.

4. Motion of a spherical viscous droplet in spherical cavity filled with micropolar fluid

The movement of a spherical particle or droplet at the instant it passes the centre of a spherical cavity is of large significant as a leader to study the wall effects in the motion of a single particle or drop. Also, it gives a model of interactions between particles or droplets in unlimited multi-particle systems. An early work for Newtonian fluids was given by

Cunningham [39] for the quasi-steady motion of a solid spherical particle internal concentric spherical cavity filled by a viscous fluid. Haberman and Sayre [40] give the analogous solution for a liquid inner sphere. In both cases the outer container is assumed to be rigid, so that fluid adheres to it.

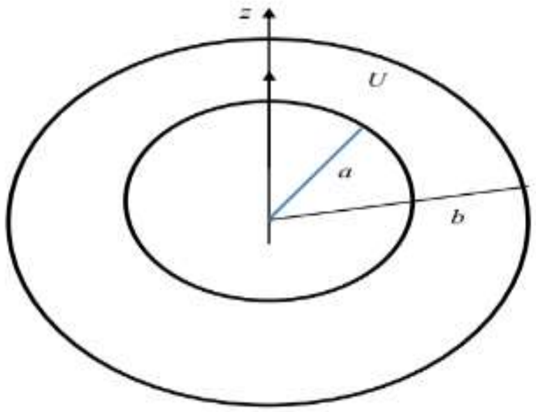

Fig. 1 Concentric spheres in relative motion.

Here, we assume the motion of a viscous spherical droplet of radius a at the instant it passes through the spherical cavity centre of radius b full of a micropolar fluid. The drop is moving with uniform velocity U, in the direction of positive $z-$axis, at the instant it passes the cavity centre, which is at sleep, see Fig.1. We must distinguish among the internal motion and the external motions. Let ψ be the stream function for the micropolar fluid inside the cavity and outside the droplet $a \le r \le b$ and ψ' be the stream function of the viscous flow inside the drop $r \le a$. The twain fluid phases are immiscible. The stream functions satisfy the following differential equations:

$$E^4\left(E^2-\ell^2\right)\psi=0, \qquad a \le r \le b \quad (4.1)$$

$$E^4\psi'=0, \quad r \le a \quad (4.2)$$

To complete the formation of this model, the conditions of the boundary at the internal surface of spherical cavity and at the interface among the viscous drop and the micro-polar fluid phase has to specified.

At the internal surface of the cavity ($r=b$):

Since the cavity is impervious, then

$$q_r=0, \quad r=b \quad (4.3)$$

We consider the no slip and the no spin boundary conditions at the internal surface of the cavity

$$q_\theta=0, \quad r=b \quad (4.4)$$

$$\nu=0, \quad r=b \quad (4.5)$$

At the interface of the droplet ($r=a$):

Since the twain fluid phases at the viscous drop interface and the micro-polar fluid enviroment it are immiscible, then no mass carry across the interface, that is the normal velocity components, on both sides together, should vanish. It is further considered that the tangential velocity is continuous across the interface. If we consider that the usual balance theory of interfacial tension is viable to the present model, the only one effect of interfacial tension is making non-continuity in the normal stress through the interface. Therefore, the duration of an interfacial tension does not, therefore, give the tangential stress, at the interface; hence, the tangential stress should be continuous at the interface. In addition to the above conditions, we shoud assign a boundary condition concerning the micro-rotation component of the micro-polar fluid phase. The almost suitable physical can frame down these circumstances is the micro-rotation of the micropolar fluid outside the drop is proportionate to the vorticity of the viscous fluid inner the drop [39]. The constant s which connects these quantities is called the coefficient of spin; it depends one and only on the nature of the twain fluid phases, and it varies from zero to one. When $s=0$, the micro-elements adjacent to the interface are incapable to rotate and when $s=1$, the vorticity of the viscous fluid phase equals to the micro-rotation of the micro-elements at the interface. Moreover, at the drop centre, the components of the velocity should exist. The above-stated boundary conditions physically may be framed mathematically as follows:

$$q_r=q_r'=U\cos\theta, \quad (4.6)$$

$$q_\theta=q_\theta', \quad (4.7)$$

$$t_{r\theta}=t_{r\theta}', \quad (4.8)$$

$$\nu=s\,\omega', \quad (4.9)$$

$$\lim_{r\to 0} q_r', \qquad \lim_{r\to 0} q_\theta' \quad \text{exist.} \quad (4.10)$$

Appropriate solutions of (4.1) and (4.2) that are compatible with the imposed boundary conditions and with the help of (3.11) and (3.12) are given, respectively, by

$$\psi = \left(Ar^2 + Cr^4 + Br^{-1} + Dr + FI_{\frac{1}{2}}(\ell r) + EK_{\frac{1}{2}}(\ell r)\right)G_2(\zeta), \quad (4.11)$$

$$\psi' = \left(Lr^2 + Nr^4\right)G_2(\zeta), \quad (4.12)$$

where $G_2(\zeta) = \frac{1}{2}\sin^2\theta$. To satisfy the boundary conditions, we need the following hydro-dynamical expressions:

$$\left.\begin{aligned} q_r = -\Big[&A + Cr^2 + Br^{-3} + Dr^{-1} \\ &+ r^{-\frac{3}{2}}\left(FI_{\frac{3}{2}}(\ell r) + EK_{\frac{3}{2}}(\ell r)\right)\Big]\cos\theta, \end{aligned}\right\} \quad (4.13)$$

$$\left.\begin{aligned} q_\theta = \tfrac{1}{2}\Big[&2A + 4Cr^2 - Br^{-3} \\ &+ Dr^{-1} + Fr^{-\frac{3}{2}}\left(\ell r I_{\frac{1}{2}}(\ell r) - I_{\frac{3}{2}}(\ell r)\right) \\ &- Er^{-\frac{3}{2}}\left(\ell r K_{\frac{1}{2}}(\ell r) + K_{\frac{3}{2}}(\ell r)\right)\Big]\sin\theta, \end{aligned}\right\} \quad (4.14)$$

$$\left.\begin{aligned} \nu = \tfrac{1}{2}\Big[&5Cr - Dr^{-2} \\ &+ m\ell^2 r^{-\frac{1}{2}}\left(FI_{\frac{3}{2}}(\ell r) + EK_{\frac{3}{2}}(\ell r)\right)\Big]\sin\theta, \end{aligned}\right\} \quad (4.15)$$

$$\left.\begin{aligned} t_{r\theta} = \tfrac{1}{2}(2\mu+\kappa)\Big[&3Cr + 3Br^{-4} \\ &+ Fr^{-\frac{5}{2}}\left(3I_{\frac{3}{2}}(\ell r) - \ell r I_{\frac{1}{2}}(\ell r)\right) \\ &+ Er^{-\frac{5}{2}}\left(3K_{\frac{3}{2}}(\ell r)\ell + rK_{\frac{1}{2}}(\ell r)\right)\Big]\sin\theta, \end{aligned}\right\} \quad (4.16)$$

$$q_r' = -\left(L + Mr^2\right)\cos\theta, \quad (4.17)$$

$$q_\theta' = \left(L + 2Mr^2\right)\sin\theta, \quad (4.18)$$

$$t_{r\theta}' = 3\mu' Mr\sin\theta, \quad (4.19)$$

$$\omega' = \tfrac{5}{2}Mr\sin\theta. \quad (4.20)$$

Inserting (4.13) - (4.20) into the boundary conditions (4.3) - (4.9), we obtain a set of simultaneous linear equations determining the unknown coefficients (A, C, B, D, F, E, L, M):

$$\left.\begin{aligned} &A + Cb^2 + Bb^{-3} + Db^{-1} \\ &\quad + b^{-\frac{3}{2}}\left(FI_{\frac{3}{2}}(\ell b) + EK_{\frac{3}{2}}(\ell b)\right) = 0, \end{aligned}\right\} \quad (4.21)$$

$$\left.\begin{aligned} &A + 4Cb^2 - Bb^{-3} + Db^{-1} \\ &\quad + Fb^{-\frac{3}{2}}\left(\ell b I_{\frac{1}{2}}(\ell b) - I_{\frac{3}{2}}(\ell b)\right) \\ &\quad - Eb^{-\frac{3}{2}}\left(\ell b K_{\frac{1}{2}}(\ell b) + K_{\frac{3}{2}}(\ell b)\right) = 0, \end{aligned}\right\} \quad (4.22)$$

$$\left.\begin{aligned} &5Cb - Db^{-2} \\ &\quad + m\ell^2 b^{-\frac{1}{2}}\left(FI_{\frac{3}{2}}(\ell b) + EK_{\frac{3}{2}}(\ell b)\right) = 0, \end{aligned}\right\} \quad (4.23)$$

$$\left.\begin{aligned} &A + Ca^2 + Ba^{-3} + Da^{-1} \\ &\quad + a^{-\frac{3}{2}}\left(FI_{\frac{3}{2}}(\ell a) + EK_{\frac{3}{2}}(\ell a)\right) \\ &\quad - L - Ma^2 = 0, \end{aligned}\right\} \quad (4.24)$$

$$\left.\begin{aligned} &2A + 4Ca^2 - Ba^{-3} + Da^{-1} \\ &\quad + Fa^{-\frac{3}{2}}\left(\ell a I_{\frac{1}{2}}(\ell a) - I_{\frac{3}{2}}(\ell a)\right) \\ &\quad - Ea^{-\frac{3}{2}}\left(\ell a K_{\frac{1}{2}}(\ell a) + K_{\frac{3}{2}}(\ell a)\right) \\ &\quad - 2L - 4Ma^2 = 0, \end{aligned}\right\} \quad (4.25)$$

$$\left.\begin{aligned} &A + Ca^2 + Ba^{-3} + Da^{-1} \\ &\quad + a^{-\frac{3}{2}}\left(FI_{\frac{1}{2}}(\ell a) + EK_{\frac{3}{2}}(\ell a)\right) = -U, \end{aligned}\right\} \quad (4.26)$$

$$\left.\begin{aligned} &3Ca + 3Ba^{-4} \\ &\quad + Fa^{-\frac{5}{2}}\left(3I_{\frac{3}{2}}(\ell a) - \ell a I_{\frac{1}{2}}(\ell a)\right) \\ &\quad + Ea^{-\frac{5}{2}}\left(3K_{\frac{3}{2}}(\ell a) + \ell a K_{\frac{1}{2}}(\ell a)\right) \\ &\quad - 3\lambda Ma = 0, \end{aligned}\right\} \quad (4.27)$$

$$\left.\begin{aligned} &5Ca - Da^{-2} \\ &\quad + m\ell^2 a^{-\frac{1}{2}}\left(FI_{\frac{3}{2}}(\ell a) + EK_{\frac{3}{2}}(\ell a)\right) \\ &\quad - 5sMa = 0, \end{aligned}\right\} \quad (4.28)$$

where $\lambda = \dfrac{2\mu'}{2\mu+\kappa} = \dfrac{2\sigma}{2+\kappa/\mu}, \quad \sigma = \dfrac{\mu'}{\mu}.$

The hydrodynamic drag force exerted on the viscous droplet is given by [42, 43]

$$F = 2\pi(2\mu+\kappa)D. \quad (4.29)$$

The expression (4.29) of the resultant drag force depends on the constant D which will be obtained from the solutions of the above system of equations. Since the values of the constant coefficients are lengthy, we record here only the value of D,

$$D = \frac{3c^3 U\Delta_1}{2\Delta}, \quad (4.30)$$

where

$$\left.\begin{aligned}\Delta_1 = 2\left(c_3c_8 - c_4c_7\right) - \left(2+3\lambda\right)\left(c_7c_{12} - c_8c_{11}\right)\\ +5s\left(c_3c_{12} - c_4c_{11}\right)\\ +\tfrac{1}{5}ma^2\ell^2c^{-\frac{3}{2}}\ell_3\Big[2\left(c_4c_5 - c_1c_8\right)\end{aligned}\right\}$$

$$\left.\begin{aligned}-\left(2+3\lambda\right)\left(cc_9 - c_5c_{12}\right) + 5s\left(c_4c_9 - c_1c_{12}\right)\Big]\\ +\tfrac{1}{5}ma^2\ell^2c^{-\frac{3}{2}}\ell_4\Big[2\left(c_1c_7 - c_3c_5\right)\\ -\left(2+3\lambda\right)\left(c_5c_{11} - c_7c_9\right) + 5s\left(c_1c_{11} - c_3c_9\right)\Big],\end{aligned}\right\}$$

$$\left.\begin{aligned}\Delta = \left(\tfrac{1}{5}c_1c^{-3} + c_2\right)\Big[\left(c_7c_{12} - c_8c_{11}\right)\\ +\tfrac{1}{5}ma^2\ell^2c^{-\frac{3}{2}}\left(\ell_3\left(c_8c_9 - c_5c_{12}\right)\right.\\ \left.+\ell_4\left(c_5c_{11} - c_7c_9\right)\right)\Big]\\ -\left(\tfrac{1}{5}c_5c^{-3} + c_6\right)\Big[\left(c_3c_{12} - c_4c_{11}\right)\\ +\tfrac{1}{5}ma^2\ell^2c^{-\frac{3}{2}}\left(\ell_3\left(c_4c_9 - c_1c_{12}\right)\right.\\ \left.+\ell_4\left(c_1c_{11} - c_3c_9\right)\right)\Big]\\ +\left(\tfrac{1}{5}c_9c^{-3} + c_{10}\right)\Big[\left(c_3c_8 - c_4c_7\right)\\ +\tfrac{1}{5}ma^2\ell^2c^{-\frac{3}{2}}\left(\ell_3\left(c_4c_5 - c_1c_8\right)\right.\\ \left.+\ell_4\left(c_1c_7 - c_3c_5\right)\right)\Big]\end{aligned}\right\}$$

Here $c_i,\ i = 1,2,...,12$, $\ell_i,\ i = 1,2,3,4$, $\ell'_1,\ \ell'_2$ are defined in Appendix A and $c = ba^{-1}$.

For compare, the drag force F_∞ acting on a spherical viscous drop in an unlimited micro-polar fluid is found to be [5]

$$F_\infty = \frac{6\pi Ua\left(2\mu+\kappa\right)\left(\mu+\kappa\right)\left(a\ell+1\right)\left(3\lambda+2\right)}{6\left(\mu+\kappa\right)\left(\lambda+2\right)\left(a\ell+1\right) - \kappa\left(3\lambda+2-5s\right)}. \quad (4.31)$$

The wall-correction factor $K = F/F_\infty$ is defined as the ratio among the drag in the presence of the cavity and the drag in an infinite medium:

$$K = \frac{6\left(\mu+\kappa\right)\left(\lambda+2\right)\left(a\ell+1\right) - \kappa\left(3\lambda+2-5s\right)}{2\left(2\mu+\kappa\right)\left(\mu+\kappa\right)\left(a\ell+1\right)\left(3\lambda+2\right)}\frac{c^3\Delta_1}{\Delta}. \quad (4.32)$$

For Newtonian fluid expression (4.32) reduces to the result given by Happel, and Brenner [44],

$$K = \frac{\left(\sigma+1\right)\left(\sigma+\frac{2}{3}-\left(\sigma-1\right)c^5\right)}{\left(\sigma+\frac{2}{3}\right)\left(\sigma+1-\frac{9}{4}\left(\sigma+\frac{2}{3}\right)c+\frac{5}{2}\sigma c^3-\frac{9}{4}\left(\sigma-\frac{2}{3}\right)c^5+\left(\sigma-1\right)c^6\right)}, \quad (4.33)$$

where $c = a/b$.

Some special cases of (4.33):

(1)Rigid sphere ($\sigma \to \infty$):

$$K = \frac{1-c^5}{1-\frac{9}{4}c+\frac{5}{2}c^3-\frac{9}{4}c^5+c^6}, \quad (4.34)$$

(2)Fluid sphere of vanishing viscosity ($\sigma \to 0$):

$$K = \frac{1+\frac{3}{2}c^5}{1-\frac{3}{2}c+\frac{3}{2}c^5-c^6}, \quad (4.35)$$

(2)Fluid sphere with viscosity equal to that of the external medium ($\sigma = 1$):

$$K = \frac{1}{1-\frac{15}{8}c+\frac{5}{4}c^3+\frac{3}{8}c^5}. \quad (4.36)$$

5. Motion of a spherical micropolar droplet in spherical cavity filled with viscous fluid

For the reverse problem that is when a micro-polar spherical drop at the instant it passes through the centre of a spherical cavity filled with a viscous fluid, the stream functions satisfy the following differential equations

$$E^4\psi = 0, \qquad a \le r \le b \quad (5.1)$$

$$E^4\left(E^2 - \ell'^2\right)\psi' = 0, \quad r \le a \quad (5.2)$$

where $\ell'^2 = \dfrac{\left(2\mu'+\kappa'\right)\kappa'}{\left(\mu'+\kappa'\right)\gamma'}$. This problem subject to the following boundary conditions:

$$q_r = q_\theta = 0, \qquad r = b \quad (5.3)$$

$$q'_r = q_r = U\cos\theta, \quad r = a \quad (5.4)$$

$$q'_\theta = q_\theta, \ r = a \quad (5.5)$$

$$t'_{r\theta} = t_{r\theta}, \ r = a \quad (5.6)$$

$$\nu' = s\,\omega, \ r = a \quad (5.7)$$

$$\lim_{r\to 0} q'_r, \quad \lim_{r\to 0} q'_\theta, \quad \lim_{r\to 0} \nu' \qquad \text{exist.} \quad (5.8)$$

Appropriate solutions of (5.1) and (5.2) that are compatible with the imposed boundary conditions and with the help of (3.11) and (3.12) are given, respectively, by

$$\psi = \left(\hat{A}r^2 + \hat{C}r^4 + \hat{B}r^{-1} + \hat{D}r\right)G_2\left(\zeta\right), \quad (5.9)$$

$$\psi' = \left(\hat{L}r^2 + \hat{M}r^4 + \hat{N}\sqrt{r}I_{\frac{3}{2}}\left(\ell' r\right)\right)G_2\left(\zeta\right). \quad (5.10)$$

The following hydrodynamical expressions are required:

$$q_r = -\left[\hat{A} + \hat{C}r^2 + \hat{B}r^{-3} + \hat{D}r^{-1}\right]\cos\theta, \quad (5.11)$$

$$q_\theta = \tfrac{1}{2}\left[2\hat{A} + 4\hat{C}r^2 - \hat{B}r^{-3} + \hat{D}r^{-1}\right]\sin\theta, \ (5.12)$$

$$\omega = \tfrac{1}{2}\left[2\hat{A}r^{-1} + 7\hat{C}r + 2\hat{B}r^{-4} + \hat{D}r^{-2}\right]\sin\theta, (5.13)$$

$$t_{r\theta} = \mu\left(3\hat{C}r + 3\hat{B}r^{-4}\right)\sin\theta \ \ (5.14)$$

$$q'_r = -\left(\hat{L} + \hat{M}r^2 + \hat{N}r^{-\frac{3}{2}}I_{\frac{1}{2}}\left(\ell' r\right)\right)\cos\theta, \ (5.15)$$

$$\left.\begin{aligned} q'_\theta = \tfrac{1}{2}\Big[&2\hat{L} + 4\hat{M}r^2 \\ &+\hat{N}r^{-\frac{3}{2}}\left(\ell' r I_{\frac{1}{2}}\left(\ell' r\right) - I_{\frac{3}{2}}\left(\ell' r\right)\right)\Big]\sin\theta, \end{aligned}\right\} (5.16)$$

$$\nu' = \tfrac{1}{2}\left[5\hat{M}r + m'\ell'^2 r^{-\frac{1}{2}}\hat{N}I_{\frac{3}{2}}\left(\ell' r\right)\right]\sin\theta, \ (5.17)$$

$$\left.\begin{aligned} t'_{r\theta} = \tfrac{1}{2}\left(2\mu' + \kappa'\right)\Big[&3\hat{M}r \\ &+\hat{N}r^{-\frac{5}{2}}\left(I_{\frac{3}{2}}\left(\ell' r\right) - \ell' r I_{\frac{1}{2}}\left(\ell' r\right)\right)\Big]\sin\theta, \end{aligned}\right\} (5.18)$$

where $\ell'^2 = \dfrac{\left(2\mu' + \kappa'\right)\kappa'}{\left(\mu' + \kappa'\right)\gamma'}$ and $m' = \dfrac{\mu' + \kappa'}{\kappa'}$.

Inserting (5.11) - (5.18) into the boundary conditions (5.3) - (5.7), we obtain a set of simultaneous linear equations determining the unknown coefficients $\left(\hat{A}, \hat{C}, \hat{B}, \hat{D}, \hat{L}, \hat{M}, \hat{N}\right)$:

$$\hat{A}b^3 + \hat{C}b^5 + \hat{B} + \hat{D}b^2 = 0, \quad (5.19)$$

$$2\hat{A}b^3 + 4\hat{C}b^5 - \hat{B} + \hat{D}b^2 = 0, \ (5.20)$$

$$\left.\begin{aligned} &\hat{A} + \hat{C}a^2 + \hat{B}a^{-3} + \hat{D}a^{-1} \\ &\quad - \hat{N}a^{-\frac{3}{2}}I_{\frac{3}{2}}\left(\ell' a\right) - \hat{L} - \hat{M}a^2 = 0, \end{aligned}\right\} (5.21)$$

$$\left.\begin{aligned} &2\hat{A} + 4\hat{C}a^2 - \hat{B}a^{-3} + \hat{D}a^{-1} \\ &-\hat{N}a^{-\frac{3}{2}}\left(\ell' a I_{\frac{1}{2}}\left(\ell' a\right) - I_{\frac{3}{2}}\left(\ell' a\right)\right) - 2\hat{L} - 4\hat{M}a^2 = 0, \end{aligned}\right\} (5.22)$$

$$\hat{A}a^3 + \hat{C}a^5 + \hat{B} + \hat{D}a^2 = -Ua^3, \quad (5.23)$$

$$\left.\begin{aligned} &3\lambda'\hat{C}a + 3\lambda'\hat{B}a^{-4} - 3\hat{M}a \\ &\quad - \hat{N}a^{-\frac{5}{2}}\left(3I_{\frac{3}{2}}\left(\ell' a\right) - \ell' a I_{\frac{1}{2}}\left(\ell' a\right)\right) = 0, \end{aligned}\right\} (5.24)$$

$$\left.\begin{aligned} &5\hat{M}a + m'\ell'^2 a^{-\frac{1}{2}}\hat{N}I_{\frac{3}{2}}\left(\ell' a\right) - 2s\hat{A}a^{-1} \\ &\quad - 7s\hat{C}a - 2s\hat{B}a^{-4} - s\hat{D}a^{-2} = 0, \end{aligned}\right\} (5.25)$$

where $\lambda' = \dfrac{2\mu}{2\mu' + \kappa'} = \dfrac{2\sigma'}{2 + \kappa'/\mu'}$, $\sigma' = \dfrac{\mu}{\mu'}$.

The hydrodynamic drag force exerted on the micropolar droplet is given by [42]

$$F' = 4\pi\mu\hat{D}. \ (5.26)$$

The constant $\hat{D}$ will be obtained from the solutions of the above system of equations, we record here only the value of $\hat{D}$,

$$\hat{D} = \frac{c^3 U \Delta'_1}{\Delta'}, \ (5.27)$$

where

$$\left.\begin{aligned} \Delta' = {} & m' a^2 \ell'^2 \left(1 - c^3\right)\left(c_2 c_6 - c_1 c_7\right) \\ & + c_3\left(c_4 c_7 - c_5 c_6\right) - c_3\left(c_2 c_6 - c_1 c_7\right), \end{aligned}\right\}$$

$$\left.\begin{aligned} \Delta'_1 = {} & \tfrac{3}{2}\left(2\lambda' + 3\right)c_7 \\ & + \left(3m' a^2 \ell'^2 c'_2 - 2s c'_3 c'_7\right)\left(c^{-3} - 1\right) \\ & + \tfrac{3}{2}c^{-3}c_3\left(5c^3 c'_7 - 2c'_2 - 2c'_5\right). \end{aligned}\right\}$$

Here c'_i, $i = 1, 2, ..., 7$, defined in Appendix B. For compare, the drag force F'_∞ acting on a micropolar drop in an unlimited viscous fluid is given by [5]

$$F'_\infty = 4\pi\mu U a \frac{a^2\ell'^2\left(3 + 2\lambda'\right)\left(\mu' + \kappa'\right)\left(a\ell' + \delta\right) + 15\kappa'\delta\left(\lambda' - 1\right)}{2a^2\ell'^2\left(\lambda' + 1\right)\left(\mu' + \kappa'\right)\left(a\ell' + \delta\right) - 5\delta\kappa'\left(s - 3\lambda' + 2\right)}, (5.28)$$

where $\delta = 3/\tanh a\ell' - 3/\left(a\ell'\right) - \ell'$.

The wall correction factor K' for this case is given by

$$K' = \frac{2a^2\ell'^2\left(\lambda' + 1\right)\left(\mu' + \kappa'\right)\left(a\ell' + \delta\right) - 5\delta\kappa'\left(s - 3\lambda' + 2\right)}{a^2\ell'^2\left(3 + 2\lambda'\right)\left(\mu' + \kappa'\right)\left(a\ell' + \delta\right) + 15\kappa'\delta\left(\lambda' - 1\right)}\frac{c^3\Delta'_1}{\Delta'}. (5.29)$$

6. Motion of a spherical micropolar droplet in spherical cavity filled with micropolar fluid

Here, we consider the motion of a spherical micropolar droplet of radius a at the instant it passes through the spherical cavity centre of radius b filled with a micropolar fluid. The drop is moving with uniform velocity U, in the positive $z-$axis, at the instant it passes the centre of a cavity which is at rest. Again, the two fluid phases are immiscible. The stream functions satisfy the following differential equations:

$$E^4\left(E^2 - \ell^2\right)\psi = 0, \qquad a \le r \le b \ \ (6.1)$$

$$E^4\left(E^2 - \ell'^2\right)\psi' = 0. \quad r \le a \ \ (6.2)$$

The solutions of (6.1) and (6.2) are respectively as follows

$$\left.\begin{aligned}\psi &= \Big[A^* r^2 + C^* r^4 + B^* r^{-1} + D^* r \\ &\quad + F^* I_{\frac{3}{2}}(\ell r) + E^* K_{\frac{3}{2}}(\ell r)\Big] G_2(\zeta),\end{aligned}\right\} \quad (6.3)$$

$$\psi' = \left(L^* r^2 + M^* r^4 + N^* \sqrt{r}\, I_{\frac{3}{2}}(\ell' r)\right) G_2(\zeta), \ (5.27)$$

where $A^*, C, B, D, F^*, E^*, L^*, M^*, N^*$ are nine constants' coefficients to be determined. The boundary conditions are as follows:

At the internal surface of the cavity $(r = b)$

The no mass flux, no slip and the no spin require

$q_r = q_\theta = \nu = 0.$ (6.5)

At the interface of the drop $(r = a)$:

$q'_r = q_r = U \cos\theta,$ (6.6)

$q'_\theta = q_\theta,$ (6.7)

$t'_{r\theta} = t_{r\theta}.$ (6.8)

In addition, since we have in this case two microstructure fluid phases, the boundary condition (4.20) or (5.7) should replaced by the continuity of microrotation at the surface of the droplet, that is

$\nu' = \nu.$ (6.9)

Moreover, the continuity of the tangential couple strass has to be added to complete the set of boundary conditions to determine the unknown constants coefficients uniquely, i.e.

$m'_{r\phi} = m_{r\phi}.$ (6.10)

Finally, the conditions of boundedness of the velocity and microtation at the centre of the drop must be added,

$\lim_{r\to 0} q'_r, \quad \lim_{r\to 0} q'_\theta, \quad \lim_{r\to 0} \nu'$ exist. (6.11)

The expression of the tangential couple stress component is given by

$$m_{r\phi} = \beta \frac{\partial \nu}{\partial r} - \frac{\nu}{r}. \quad (6.12)$$

The following hydro-dynamical components are needed to satisfy the boundary conditions:

$$\left.\begin{aligned} q_r &= -\Big[A^* + C^* r^2 + B^* r^{-3} + D^* r^{-1} \\ &\quad + r^{-\frac{3}{2}}\left(F^* I_{\frac{3}{2}}(\ell r) + E^* K_{\frac{3}{2}}(\ell r)\right)\Big]\cos\theta,\end{aligned}\right\} (6.13)$$

$$\left.\begin{aligned} q_\theta &= \tfrac{1}{2}\Big[2A^* + 4C^* r^2 - B^* r^{-3} + D^* r^{-1} \\ &\quad + F^* r^{-\frac{3}{2}}\left(\ell r I_{\frac{1}{2}}(\ell r) - I_{\frac{3}{2}}(\ell r)\right) \\ &\quad - E^* r^{-\frac{3}{2}}\left(\ell r K_{\frac{1}{2}}(\ell r) + K_{\frac{3}{2}}(\ell r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.14)$$

$$\left.\begin{aligned} \nu &= \tfrac{1}{2}\Big[5C^* r - D^* r^{-2} \\ &\quad + m\ell^2 r^{-\frac{1}{2}}\left(F^* I_{\frac{3}{2}}(\ell r) + E^* K_{\frac{3}{2}}(\ell r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.15)$$

$$\left.\begin{aligned} t_{r\theta} &= \tfrac{1}{2}(2\mu + \kappa)\Big[3C^* r + 3B^* r^{-4} \\ &\quad + F^* r^{-\frac{5}{2}}\left(3 I_{\frac{3}{2}}(\ell r) - \ell r I_{\frac{1}{2}}(\ell r)\right) \\ &\quad + E^* r^{-\frac{5}{2}}\left(3K_{\frac{3}{2}}(\ell r)\ell + r K_{\frac{1}{2}}(\ell r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.16)$$

$$\left.\begin{aligned} m_{r\phi} &= \tfrac{1}{2}\Big[5C^*(\beta - \gamma) + D^*(2\beta + \gamma) r^{-3} \\ &\quad + r^{-\frac{3}{2}} m\ell^2 F^*\left(\beta \ell r I_{\frac{1}{2}}(\ell r) - (2\beta + \gamma) I_{\frac{3}{2}}(\ell r)\right) \\ &\quad - m\ell^2 r^{-\frac{3}{2}} E^*\left(\beta \ell r K_{\frac{1}{2}}(\ell r) + (2\beta + \gamma) K_{\frac{3}{2}}(\ell r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.17)$$

$$q'_r = -\left(L^* + M^* r^2 + N^* r^{-\frac{3}{2}} I_{\frac{1}{2}}(\ell' r)\right)\cos\theta, \ (6.18)$$

$$\left.\begin{aligned} q'_\theta &= \tfrac{1}{2}\Big[2L^* + 4M^* r^2 \\ &\quad + N^* r^{-\frac{3}{2}}\left(\ell' r I_{\frac{1}{2}}(\ell' r) - I_{\frac{3}{2}}(\ell' r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.19)$$

$$\nu' = \tfrac{1}{2}\Big[5M^* r + m'\ell'^2 r^{-\frac{1}{2}} N^* I_{\frac{3}{2}}(\ell' r)\Big]\sin\theta, \ (6.20)$$

$$\left.\begin{aligned} t'_{r\theta} &= \tfrac{1}{2}(2\mu' + \kappa')\Big[3M^* r \\ &\quad + N^* r^{-\frac{5}{2}}\left(I_{\frac{3}{2}}(\ell' r) - \ell' r I_{\frac{1}{2}}(\ell' r)\right)\Big]\sin\theta,\end{aligned}\right\} (6.21)$$

$$\left.\begin{aligned} m'_{r\phi} &= \tfrac{1}{2}\Big[5(\beta' - \gamma')L^* \\ &\quad + \ell^2 m' r^{-\frac{3}{2}} N^*\Big(\beta' \ell' r I_{\frac{1}{2}}(\ell' r) \\ &\quad - (2\beta' + \gamma') I_{\frac{3}{2}}(\ell' r)\Big)\Big]\sin\theta,\end{aligned}\right\} \quad (6.22)$$

Applying the boundary conditions (6.5) - (6.10), we obtain a set of nine simultaneous linear equations for determining the nine unknown coefficients $A^*, C, B, D, F^*, E^*, L^*, M^*, N^*$:

$$\left.\begin{aligned}&A^{*}+C^{*}b^{2}+B^{*}b^{-3}+D^{*}b^{-1}\\&\quad+F^{*}b^{-\frac{3}{2}}I_{\frac{3}{2}}\left(\ell b\right)+E^{*}b^{-\frac{3}{2}}K_{\frac{3}{2}}\left(\ell b\right)=0,\end{aligned}\right\}$$

(6.23)

$$\left.\begin{aligned}&2A^{*}+4C^{*}b^{2}-B^{*}b^{-3}+D^{*}b^{-1}\\&\quad+F^{*}b^{-\frac{3}{2}}\left(\ell bI_{\frac{1}{2}}\left(\ell b\right)-I_{\frac{3}{2}}\left(\ell b\right)\right)\\&\quad-E^{*}b^{-\frac{3}{2}}\left(\ell bK_{\frac{1}{2}}\left(\ell b\right)+K_{\frac{3}{2}}\left(\ell b\right)\right)=0,\end{aligned}\right\}$$

(6.24)

$$\left.\begin{aligned}&5C^{*}b-D^{*}b^{-2}+F^{*}m\ell^{2}b^{-\frac{1}{2}}I_{\frac{3}{2}}\left(\ell b\right)\\&\qquad+E^{*}m\ell^{2}b^{-\frac{1}{2}}K_{\frac{3}{2}}\left(\ell b\right)=0,\end{aligned}\right\} \quad (6.25)$$

$$\left.\begin{aligned}&A^{*}+C^{*}a^{2}+B^{*}a^{-3}+D^{*}a^{-1}\\&\quad+F^{*}a^{-\frac{3}{2}}I_{\frac{3}{2}}\left(\ell a\right)+E^{*}a^{-\frac{3}{2}}K_{\frac{3}{2}}\left(\ell a\right)\\&\quad-N^{*}a^{-\frac{3}{2}}I_{\frac{3}{2}}\left(\ell' a\right)-L^{*}-M^{*}a^{2}=0,\end{aligned}\right\} \quad (6.26)$$

$$\left.\begin{aligned}&2A^{*}+4C^{*}a^{2}-B^{*}a^{-3}+D^{*}a^{-1}\\&\quad+a^{-\frac{3}{2}}F^{*}\left(\ell aI_{\frac{1}{2}}\left(\ell a\right)-I_{\frac{3}{2}}\left(\ell a\right)\right)\\&\quad-E^{*}a^{-\frac{3}{2}}\left(\ell aK_{\frac{1}{2}}\left(\ell a\right)+K_{\frac{3}{2}}\left(\ell a\right)\right)\\&\quad-a^{-\frac{3}{2}}N^{*}\left(\ell' aI_{\frac{1}{2}}\left(\ell' a\right)-I_{\frac{3}{2}}\left(\ell' a\right)\right)\\&\quad-2L^{*}-4M^{*}a^{2}=0,\end{aligned}\right\} \quad (6.27)$$

$$L^{*}+M^{*}a^{2}+N^{*}a^{-\frac{3}{2}}I_{\frac{3}{2}}\left(\ell' a\right)=-U, \quad (6.28)$$

$$\left.\begin{aligned}&5C^{*}a-D^{*}a^{-2}+m\ell^{2}a^{-\frac{1}{2}}F^{*}I_{\frac{3}{2}}\left(\ell a\right)\\&\quad+m\ell^{2}a^{-\frac{1}{2}}E^{*}K_{\frac{3}{2}}\left(\ell a\right)\\&\quad-m'\ell'^{2}a^{-\frac{1}{2}}N^{*}I_{\frac{3}{2}}\left(\ell' a\right)-5M^{*}a=0,\end{aligned}\right\}$$

(6.29)

$$\left.\begin{aligned}&3C^{*}a+3B^{*}a^{-4}\\&\quad+F^{*}a^{-\frac{5}{2}}\left(3I_{\frac{3}{2}}\left(\ell a\right)-\ell aI_{\frac{1}{2}}\left(\ell a\right)\right)\\&\quad+E^{*}a^{-\frac{5}{2}}\left(3K_{\frac{3}{2}}\left(\ell a\right)+\ell aK_{\frac{1}{2}}\left(\ell a\right)\right)\\&\quad-\lambda^{*}N^{*}a^{-\frac{5}{2}}\left(3I_{\frac{3}{2}}\left(\ell' a\right)-\ell' aI_{\frac{1}{2}}\left(\ell' a\right)\right)\\&\quad-3\lambda^{*}M^{*}a=0,\end{aligned}\right\}$$

(6.30)

$$\left.\begin{aligned}&5C^{*}\left(\beta\gamma^{-1}-1\right)+D^{*}\left(2\beta\gamma^{-1}+1\right)a^{-3}\\&+a^{-\frac{3}{2}}m\ell^{2}F^{*}\left(\beta\gamma^{-1}\ell aI_{\frac{1}{2}}\left(\ell a\right)-\left(2\beta\gamma^{-1}+1\right)I_{\frac{3}{2}}\left(\ell a\right)\right)\\&-m\ell^{2}a^{-\frac{3}{2}}E^{*}\left(\beta\gamma^{-1}\ell aK_{\frac{1}{2}}\left(\ell a\right)+\left(2\beta\gamma^{-1}+1\right)K_{\frac{3}{2}}\left(\ell a\right)\right)\\&=\gamma^{*}\Big[5\left(\beta'\gamma'^{-1}-1\right)M^{*}++\ell'^{2}m'a^{-\frac{3}{2}}N^{*}\times\\&\left(\beta'\gamma'^{-1}\ell' aI_{\frac{1}{2}}\left(\ell' a\right)-\left(2\beta'\gamma'^{-1}+1\right)I_{\frac{3}{2}}\left(\ell' a\right)\right)\Big],\end{aligned}\right\}$$

(6.31)

where

$$\left.\begin{aligned}&\lambda^{*}=\frac{2\mu'+k'}{2\mu+k}=\frac{2+k'/\mu'}{2+k/\mu}\sigma^{*},\\&\gamma^{*}=\frac{\gamma'}{\gamma}=\frac{\gamma'/\mu' a^{2}}{\gamma/\mu a^{2}}\sigma^{*},\quad \sigma^{*}=\frac{\mu'}{\mu}.\end{aligned}\right\}$$

The hydro-dynamic drag force exerted on the micropolar drop by the surrounding micropolar fluid is given by

$$F^{*}=2\pi\left(2\mu+\kappa\right)D^{*}. \quad (6.31)$$

The drag force F_{∞}^{*} acting on a micropolar drop in an unlimited medium occupied with another immiscible micropolar fluid is given by

$$F_{\infty}^{*}=\frac{6\pi\left(2\mu+\kappa\right)Ua\left(m_{1}n_{3}-m_{3}n_{1}\right)}{\left(m_{2}n_{3}-m_{3}n_{2}\right)+m_{1}\left(d_{3}n_{2}-d_{2}n_{3}\right)+n_{1}\left(d_{2}m_{3}-d_{3}m_{2}\right)},$$

(6.32)

where $d_2, d_3, n_1, n_2, n_3, m_2, m_3$ are defined in Appendix C.

The wall correction factor K' for this case is given by

$$K^{*}=\frac{\left(m_{2}n_{3}-m_{3}n_{2}\right)+m_{1}\left(d_{3}n_{2}-d_{2}n_{3}\right)+n_{1}\left(d_{2}m_{3}-d_{3}m_{2}\right)}{3\left(m_{1}n_{3}-m_{3}n_{1}\right)}D^{*}.$$

(6.33)

7. Results and discussions

In this section, we present the values of the wall correction factors for the three cases considered in this study.

Throughout our calculations, we consider the following values of the micropolar parameters:

$$\beta/\mu a^{2}=\gamma/\mu a^{2}=\beta'/\mu' a^{2}=\gamma'/\mu' a^{2}=0.3$$

(a) Expression (4.32) for the wall correction factor K of a viscous droplet immersed in micropolar fluid as a function of micropolarity parameter κ/μ, the viscosity ratio $\sigma=\mu'\backslash\mu$ and spin parameter s, $(0\le s\le 1)$.

(b) Expression (5.29) for the wall correction factor K' of a micropolar drop immersed in viscous fluid as a function of micropolarity parameter κ'/μ', the viscosity ratio $\sigma'=\mu\backslash\mu'$ and spin parameter s, $(0\le s\le 1)$.

(c) Expression (6.33) for the wall correction factor K^* of a micropolar drop immersed in micropolar fluid as a function of micropolarity parameters κ/μ and κ'/μ', and the viscosity ratio $\sigma^*=\mu\backslash\mu'$.

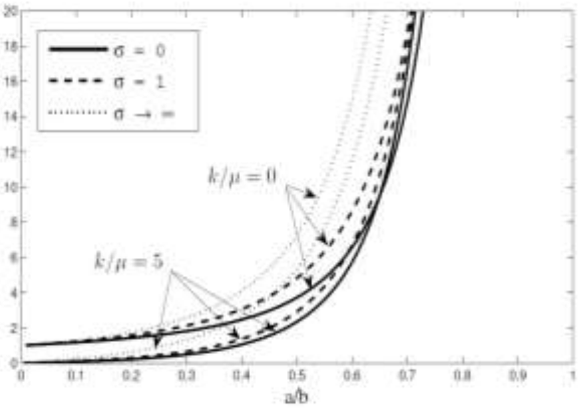

Fig.2: The wall correction factor K with the ratio a/b for different values of σ with $k/\mu=0,\ 5$, and $s=0.2$.

Table 1 and Fig.2 exhibit the correction factor K against the ratio a/b for the case of a viscous drop at the center of a cavity filled with micro-polar fluid. The plots in in Fig 2.2 and the numerical values of K in Table 2.1 indicate that the wall correction factor increases as the ratio σ increases with maximum values when the spherical droplet become solid $(\sigma\to\infty)$ and has minimum values for the case of gas bubble $\sigma\to 0$. As expected, keeping k/μ and σ fixed, the wall correction factor increases with a/b and $K\to\infty$ as $a/b\to 1$. For $\sigma\ne 0$ and fixed a/b, the wall correction factor K decreases with the increase of the micropolarity parameter k/μ. For the solid spherical particle case $(\sigma\to\infty)$, K decreases with the increase of the micropolarity parameter k/μ up a certain value of $a/b(=c_1$ say $)$ and for $a/b>c_1$, this behavior is reversed that is the wall correction factor increases with the increase of k/μ. It is noted that c_1 around o.65 for $k/\mu=5$; while it is around 0.1 for $k/\mu=3$, this means that the reverse behavior disappears for $k/\mu<3$.

Table 1: The wall correction factor K on the viscous drop at the concentric position of the spherical cavity for various values of a/b and σ with $s=0.2$ and $k/\mu=0,3$

$\frac{k}{\mu}$	a/b	K			
		$\sigma=0$	$\sigma=1$	$\sigma=10$	$\sigma\to\infty$
0	0.1	1.176469	1.228884	1.275381	1.286196
	0.2	1.428409	1.575101	1.719937	1.755845
	0.3	1.815186	2.126127	2.477682	2.572638
	0.4	2.468877	3.065980	3.864508	4.105934
	0.5	3.722224	4.830189	6.654733	7.294118
	0.6	6.569091	8.632597	13.06102	14.94813
	0.7	14.82553	18.78861	31.01511	37.82962
	0.8	50.77342	58.41122	102.0140	138.2237
	0.9	439.1557	431.7323	718.7807	1209.778
	0.98	60695.35	50758.07	58426.52	163427.8
3	0.1	0.236908	0.304353	0.537904	0.720867
	0.2	0.657138	0.817246	1.401903	1.865507
	0.3	1.473361	1.760704	2.927322	3.869409
	0.4	3.177104	3.607817	5.802297	7.621345
	0.5	7.065384	7.490829	11.61189	15.15734
	0.6	16.61415	17.22425	24.73756	32.09661
	0.7	42.44754	50.14650	60.82856	78.57493
	0.8	145.3872	204.2336	206.3463	1064.751
	0.9	1074.126	1568.819	2007.753	2050.927
	0.98	109861.1	185728.8	248460.3	293761.2

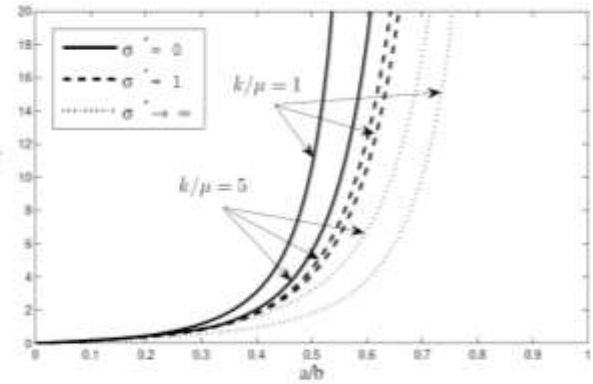

Fig.3: The wall correction factor K' with the ratio a/b for different values of σ' with $k/\mu=1,\ 5$, and $s=0.2$.

Table 2 and Fig.3 exhibit the correction factor K' against the ratio a/b for the case of a micropolar droplet at the center of a cavity filled with viscous fluid. The plots in Fig 3 and the numerical values of K' in Table 2 indicate that the wall correction factor decreases as the ratio σ' increases with maximum values when the spherical droplet become solid ($\sigma\to 0$) and has minimum values for the case $\sigma \to \infty$. Note that here the definition of σ' is the ratio of viscosities between external to internal fluid phases. Here also as expected, keeping k'/μ' and σ' fixed, the wall correction factor K' increases with a/b and $K' \to \infty$ as $a/b \to 1$. For fixed a/b, the wall correction factor K' increase with the increase of the micropolarity parameter k'/μ'. A comparison between tables 1 and 2 shows $K > K'$ for the corresponding values of all parameters.

Table 2: The wall correction factor K' on the micro-polar drop at the concentric position of the spherical cavity for various values of a/b and σ' with $s = 0.2$ and $k/\mu = 1,3$.

$\frac{k}{\mu}$	a/b	K'			
		$\sigma' = 0$	$\sigma' = 1$	$\sigma' = 10$	$\sigma' \to \infty$
0	0.1	0.169697	0.148015	0.147839	0.127193
	0.2	0.455952	0.399309	0.359468	0.320155
	0.3	1.165769	0.815550	0.747658	0.558586
	0.4	3.091914	1.787890	1.349758	0.964064
	0.5	10.53199	4.099761	2.545288	1.793005
	0.6	264.2645	10.55046	5.336549	3.818707
	0.7	5180.024	33.97130	13.50212	10.04362
	0.8	7826.971	178.6712	48.56526	38.13548
	0.9	12364.59	413.8146	346.0909	266.0226
	0.98	60753.94	54367.75	47981.56	26853.29
3	0.1	0.185078	0.158708	0.127630	0.118967
	0.2	0.472347	0.426363	0.364881	0.341835
	0.3	0.950504	0.912906	0.848108	0.824952
	0.4	1.943199	1.931467	1.877633	1.823610
	0.5	4.918337	4.662613	4.001639	3.612993
	0.6	14.58037	12.81052	9.377179	7.882092
	0.7	60.70186	45.13994	26.15953	20.62456
	0.8	1078.855	287.7909	101.9887	76.65208
	0.9	5291.419	1076.194	916.5998	679.1719
	0.98	119841.1	93174.27	28047.34	1300.788

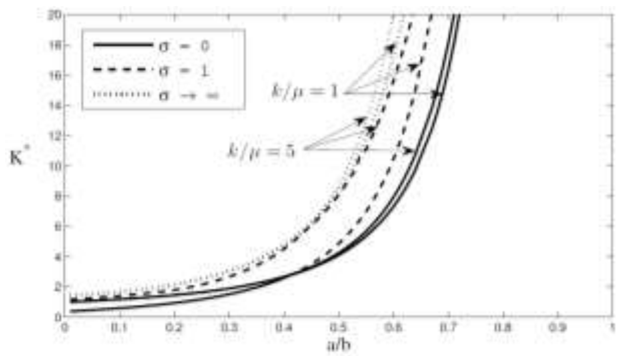

Fig. 4: The wall correction factor K^* with the ratio a/b for different values of σ with $k/\mu = 1, 5$, and $k'/\mu' = 1$.

Fig. 4 and Fig. 5 exhibit the wall correction factor K^* against the ratio a/b for the case of a micro-polar drop at the center of a cavity filled with micro-polar fluid. For the entire ranges of $k/\mu, k'/\mu', \sigma^*$, the wall correction factor K^* increases with a/b. For $\sigma^* \neq 0$ and for a given value of a/b, the wall correction factor increases with the increase of k/μ, or k'/μ'. a/b For $\sigma^* = 0$, the wall correction factor may increase or decrease as k/μ, or k'/μ' increases. Table 3 gives the same information as stated above.

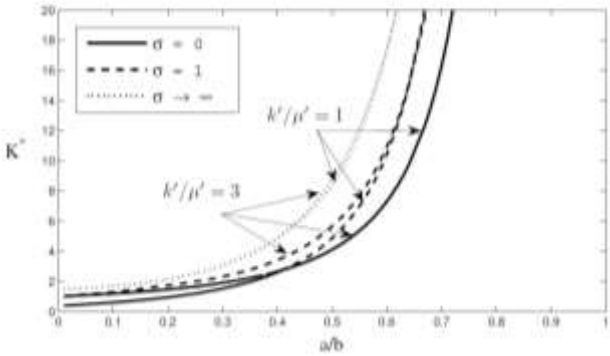

Fig.5: The wall correction factor K^* with the ratio a/b for different values of σ with $k'/\mu' = 1, 3$, and $k/\mu = 1$.

8. Conclusion

The axisymmetric motion of a spherical droplet at a concentric instantaneous position of a spherical cavity filled with another immiscible fluid has been examined in this work. Three cases are considered: viscous fluid droplet in a micropolar fluid, micropolar fluid droplet in a viscous fluid and micropolar fluid droplet in a micropolar fluid. For the first and second cases, the microrotion-vorticity boundary condition is used at droplet surface while for the third case, the continuity of microrotation and tangential couple stress have being used. The velocity fields are solved using spherical coordinates and the droplets velocities obtained for various values of the fluid properties for each case. The motivation of this study is to find the effect of boundary on the droplet. The effect of interaction between droplet and cavity expressed by finding expressions for the wall correction factors. It is fund that the interaction between the droplet and cavity can be very strong when their gap thickness approaches zero. For the special case the wall correction factor is stronger when it compered with the gas bubble.

Table 3: The wall correction factor K^* on the micropolar droplet at the concentric position of the spherical cavity for various values of a/b and σ with $k/\mu=1,5$ and $k'/\mu'=1,3$.

$\frac{k}{\mu}$	$\frac{k'}{\mu'}$	$^a/_b$	K^*			
			$\sigma=0$	$\sigma=1$	$\sigma=10$	$\sigma\to\infty$
1	1	0.1	0.5979	1.1903	1.2892	1.6743
		0.2	0.9434	1.4664	1.7792	2.1541
		0.3	1.5235	1.9021	2.6758	3.0589
		0.4	2.6012	2.6587	4.4641	4.8138
		0.5	4.1121	4.8686	8.3157	8.3669
		0.6	7.3132	10.462	15.228	17.291
		0.7	16.316	27.808	40.933	48.073
		0.8	54.849	106.76	111.73	129.80
		0.9	332.16	465.98	598.92	892.40
		0.98	1227.99	5384.64	32372	63534
1	3	0.1	1.1903	1.2892	1.4347	1.6577
		0.2	1.4664	1.7384	1.7792	2.1383
		0.3	1.9021	2.3141	2.6758	3.0699
		0.4	2.6587	3.4106	4.4640	4.9189
		0.5	4.1121	5.6614	8.3089	8.8032
		0.6	7.3132	10.934	17.497	17.087
		0.7	7.3132	10.934	17.087	17.497
		0.8	54.849	83.463	93.236	96.165
		0.9	465.98	1067.5	249.14	167.49
		0.98	290.64	3075.9	43630	63534
5	1	0.1	1.1889	1.2895	1.4122	1.6971
		0.2	1.4647	1.7799	1.8033	2.1934
		0.3	1.9029	2.4408	2.6803	3.0663
		0.4	2.6797	3.6325	4.5136	4.7094
		0.5	4.2543	6.6543	8.0911	8.7568
		0.6	7.9427	10.579	15.619	20.028
		0.7	18.071	24.545	46.183	54.837
		0.8	56.352	99.619	157.55	193.66
		0.9	407.67	898.37	1000	1341.4
		0.98	4503.26	200967	47614	82572

Table 4: Summary

Parameter	• K, the wall correction factor acting on a spherical viscous droplet in spherical cavity filled with micropolar fluid. • K', the wall correction factor acting on a spherical micropolar droplet in spherical cavity filled with viscous fluid • K^*, the wall correction factor acting on a spherical micropolar droplet in spherical cavity filled with micropolar fluid
a/b the relative separation distance	• K, K' and K^* are monotonic increasing functions of a/b • $a/b=0$, the droplet moving in an unbounded medium in the absence of the cavity wall, K, K' and $K^*\to 0$ • $a/b\to 1$, the droplet touches the cavity surface, K, K' and K^* become infinite. • K, K' and K^* are increases with a/b
k/μ, or k'/μ', The micropolarity parameter	• K, decreases with the increase of k/μ. • K', increase with the increase of k'/μ'. • K^*, increases with the increase of k/μ, or k'/μ'.
S, the spin parameter	• $s=0$ (no-spin), the microrotation component vanishes on the droplet surface.

	• K , K' and K^* are decreasing as s increases. • $s = 1$ (perfect spin), the microrotation component equals the vorticity at the droplet surface
σ, the ratio of viscous fluid viscosity to micropolar fluid viscosity	• $\sigma = \mu'/\mu$ • $\sigma \to \infty$, a solid sphere moves in a micropolar fluid. • $\sigma = 0$, a gas bubble moves in a micropolar fluid. • K , increases with an increase in σ .
σ', the ratio of micropolar fluid viscosity to viscous fluid viscosity	• $\sigma = \mu/\mu'$ • $\sigma' \to \infty$, a gas bubble moves in a viscous fluid. • $\sigma' = 0$, a solid sphere moves in a viscous fluid. • K' , decreases with an increase in σ' .
σ^*, the ratio of micropolar fluid viscosity to micropolar fluid viscosity	• $\sigma^* = \mu'/\mu$ • $\sigma^* \to \infty$, a solid sphere moves in a micropolar fluid. • $\sigma^* = 0$, a gas bubble moves in a micropolar fluid. • K^* , increases with an increase in σ^* .

Appendix A

$$\left.\begin{aligned} c_1 &= 3\left(1-c^3\right) - 3c^3\left(c^2-1\right) \\ &\quad -\left(\tfrac{9}{2}c^3\left(c^2-1\right)+3\left(1-c^3\right)\right)\lambda \\ c_2 &= -3\left(1-c\right)c^2 \\ &\quad +\left(\tfrac{3}{2}\left(1-c^3\right)-\tfrac{9}{2}\left(1-c\right)c^2\right)\lambda, \end{aligned}\right\} \quad (A.1)$$

$$\left.\begin{aligned} c_3 &= \left(3-a\ell\ell_1\right)\left(1-c^3\right)-3c^3\left(c^{-\frac{3}{2}}\ell_3-1\right) \\ &\quad +\left(\tfrac{3}{2}\left(3-a\ell\ell_1\right)\left(1-c^3\right)-\tfrac{9}{2}c^3\left(c^{-\frac{3}{2}}\ell_3-1\right)\right)\lambda \\ c_4 &= \left(3+a\ell\ell_2\right)\left(1-c^3\right)-3c^3\left(c^{-\frac{3}{2}}\ell_4-1\right) \\ &\quad +\left(\tfrac{3}{2}\left(1-c^3\right)\left(3+a\ell\ell_2\right)-\tfrac{9}{2}c^3\left(c^{-\frac{3}{2}}\ell_4-1\right)\lambda\right), \end{aligned}\right\} \quad (A.2)$$

$$\left.\begin{aligned} c_5 &= 5\left(1-s\right)\left(1-c^3\right)-\tfrac{15}{2}sc^3\left(c^2-1\right) \\ c_6 &= \tfrac{1}{2}\left(5s-2\right)\left(1-c^3\right)-\tfrac{15}{2}s\left(1-c\right)c^2, \end{aligned}\right\} \quad (A.3)$$

$$\left.\begin{aligned} c_7 &= \left(1-c^3\right)\left(ma^2\ell^2-\tfrac{5}{2}s\left(a\ell\ell_1-3\right)\right) \\ &\quad -\tfrac{15}{2}sc^3\left(c^{-\frac{3}{2}}\ell_3-1\right) \\ c_8 &= \left(1-c^3\right)\left(ma^2\ell^2+\tfrac{5}{2}s\left(a\ell\ell_2+3\right)\right) \\ &\quad -\tfrac{15}{2}sc^3\left(c^{-\frac{3}{2}}\ell_4-1\right), \end{aligned}\right\} \quad (A.4)$$

(A.5)

(A.6)
(A.7)

Appendix B

$$\left.\begin{aligned} c_1' &= \tfrac{3}{2}\left(\left(1-c^3\right)-3\left(1-c\right)c^2\right) \\ &\quad -3\left(1-c\right)c^2\lambda' \\ c_2' &= -\tfrac{3}{2}\left(2\left(1-c^3\right)+3c^3\left(c^2-1\right)\right) \\ &\quad +3\left(1-c^5\right)\lambda', \end{aligned}\right\} \quad (B.1)$$

$$\left.\begin{aligned} c_3' &= \tfrac{5}{2}\left(1-c^3\right)\left(a\ell'\ell_1'-3\right) \\ c_4' &= \tfrac{15}{2}\left(1-c\right)c^2-\left(\tfrac{5}{2}-s\right)\left(1-c^3\right), \end{aligned}\right\} \quad (B.2)$$

$$\left.\begin{aligned} c_5' &= 5\left(1-s\right)\left(1-c^3\right)+\tfrac{15}{2}c^3\left(c^2-1\right) \\ c_6' &= c^{-1}\left(\left(1-2c\right)\left(1-c^3\right)+\left(2c^3+1\right)\left(1-c\right)\right), \end{aligned}\right\} \quad (B.3)$$

$$\left.\begin{aligned} c_7' &= 2\left(2c^2-1\right)\left(1-c^3\right)+\left(2c^3+1\right)\left(c^2-1\right) \\ \ell_1' &= \frac{I_{\frac{1}{2}}\left(\ell'a\right)}{I_{\frac{3}{2}}\left(\ell'a\right)}. \end{aligned}\right\} \quad (B.4)$$

Appendix C

$$\left.\begin{aligned} d_2 &= \left(2-m^{-1}\hat{\ell}^{-1}\ell_1\right) \\ d_3 &= -\beta\gamma^{-1}\hat{\ell}\ell_1, \end{aligned}\right\} \quad (C.1)$$

$$\left.\begin{aligned} m_1 &= \left(3\lambda^*+2\right) \\ m_2 &= -\left(2+5\hat{\ell}\ell_1\right) \\ m_3 &= -\left(5m\hat{\ell}^2\left(\beta\gamma^{-1}\hat{\ell}\ell_1+\left(2\beta\gamma^{-1}+1\right)\right)\right. \\ &\quad \left.+5\gamma^*\left(\beta'\gamma'^{-1}-1\right)\right), \end{aligned}\right\} \quad (C.2)$$

$$\left.\begin{aligned} n_1 &= \left(3-\hat{\ell}'\ell_2\right)\left(\lambda^*-1\right) \\ n_2 &= \left(3-\hat{\ell}'\ell_2 - m'm^{-1}\hat{\ell}^{-1}\hat{\ell}'^2\ell_1\right) \\ n_3 &= -m'\hat{\ell}'^2\Big(\left(\beta\gamma^{-1}\hat{\ell}\ell_1+\left(2\beta\gamma^{-1}+1\right)\right) \\ &\quad +\gamma^*\left(\beta'\gamma'^{-1}\hat{\ell}'\ell_2-\left(2\beta'\gamma'^{-1}+1\right)\right)\Big), \end{aligned}\right\} \quad \text{(C.3)}$$

$$\ell_1 = \frac{K_{\frac{1}{2}}\left(\hat{\ell}\right)}{K_{\frac{3}{2}}\left(\hat{\ell}\right)},\ \ell_2 = \frac{I_{\frac{1}{2}}\left(\hat{\ell}'\right)}{I_{\frac{3}{2}}\left(\hat{\ell}'\right)}, \hat{\ell} = \ell a, \quad \hat{\ell}' = \ell' a. \quad \text{(C.4)}$$

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