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AN ALGORITHM FOR SOLVING VECTOR OP TIMIZATION ROBLEMS WITH PARAMETERS IN BOTH THE OBJECTIVE FUNCTIONS AND THE CONSTRAINTS BY USING INTERACTIVE APPROACHES

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ABSTRACT

In this work, an algoritihm for solving vector optimization problems with parameters in both the objective functions and the constraints is introduced. An interactive approach is used for this algorithm as the surrogat worth trade-off method, Also the modified Hybrid approach which combines the characteristics of both generalized Tcheby-cheff norm and the K-th objective Σ-constraints problem is used to scalarize the vector optimization problem. In this work also, the basic notions as the set of feasible paramerers, the solvability set and the stability set of the first kind are redefined for this problem, and the stability set is determined by using this algorithm, An example is given to clarify this algorthim.

INTRODUCTION

In earler work Nozidca et al. (5) and Osman et al. (6,7,8,9) gave notion of the stability set of the first kind, the set of all parameters corresponding to an optimal solution of parametric convex programming problems (or to an efficient solution of vector optimization problem (VOP)). Furthermore the STEP method given in (1) is an interactive scheme that profrssively elicits ingormation from the decision maker primarily to modify the weights for solving multiobjective linear programming problems. Also in [3]. Haims gave necessary and sufficient condition for the determination of the efficient solutions for (VOP) using the Hybrid approach which combiens the characteristics of bothe the nonnegative weighted sum problems and the k-th objective ε-constraints problem. And in [2], Bowman determine necessary and sufficient conditions for the determination of the efficient solutions for (VOP) using the generalized Tchebycheff norm. Also, Osman *et al.* in [9] introduced a modefied Hybrid approach for solving (VOP).

In this paper, an algorithm for determinig the stability set of the first kind for vector optimization problems with parameters in both the objective functions and in the constraints using interactive approaches is given.

2. PROBLEM FORMULATION

Let us consider the following parametric multiobjective nonlinear programming problem:

$$P(\lambda, v) \qquad \begin{array}{l} \min \ \{(f_1(x, \lambda), \ldots, f_m(x, \lambda)\} \\ \text{subject to} \\ M(v) = [x \in R^n / g_r(x, v) \le o, \ r = 1, 2, \ldots, k], \end{array}$$

where f_j, $j = 1, 2, \ldots, m$ and g_r, $r = 1, 2, \ldots, k$ are convex functions of class $C^{(1)}$ on R^n, and $\lambda \in R^l$, $v \in R^k$ are vector parameters.

Let us define the following scalarization of $P(\lambda, v)$ which will be called the modified Hybrid approach.

$$\min_{x \in M(v)} \ \max_j W_j[f_j(x, \lambda) - \bar{f}_j]$$

$P(\lambda, v, \varepsilon)$ subject to

$$f_j(x) \leq \varepsilon_j, j = 1, 2, ..., m,$$

whers $\bar{f} \in R^m$ is an ideal target, $\bar{f}_j = \min_{x \in M(v)} f_j(x, \bar{\lambda})$, and $w \in R^m_+$ (the positive orthant of the R^m - space). It will be seen that the noninferior solutions of P (λ, v) can be characterized in terms of the optimal solution of $P(\lambda, v)$ can be characterized in terms of the optimal solution of $P(\lambda, v, \varepsilon)$.

The problem $P(\lambda, v, \varepsilon)$ can be reformulated to take the following equivalent form :

$$\bar{P}(\lambda, v, \varepsilon) \quad \begin{array}{l} \text{nim } z \\ \text{subject to} \end{array}$$

$$N(\lambda,v,\varepsilon) = \{(x,z) \in R^{n+1}/w_j[f_j(x,\lambda) - \bar{f}_j] - z \leq o, f_j(x) - \varepsilon_j \leq o,$$

$$j = 1, 2, ..., m \quad \text{and} \quad g_r(x,v) \leq o, \quad r = 1, 2, ...,k \ ,\}$$

where $Z \in R$

It must be noted that roblem $\bar{P}(\lambda, v, \varepsilon)$ can be written in the equivalent form [3] :

$$\bar{P}_k(\lambda, v, \varepsilon) \quad \begin{array}{l} \min [f_k(x,\lambda) - \bar{f}_k] \\ \text{subject to} \end{array}$$

$$N_k(\lambda,v,\varepsilon) = \{(x,\lambda) \in R^{n+1}/w_i[f_i(x,\lambda) - \bar{f}_i] - f_k(x,\lambda) + \bar{f}_k \leq o,$$

$$i = 1, 2, ..., m, i \neq k, f_j(x) - \varepsilon_j \leq o, j = 1, ..., m, \quad g_r(x,v) \leq o,$$

$$r = 1, 2, ...,k \ \},$$

which is obtained by eleminating z from the first constraint of problem $\bar{P}(\lambda,v, \varepsilon)$.

Definition 1 : The set of feasible parameters of problem $\bar{P}_k(\lambda, \nu, \varepsilon)$ is defined by

$$U = \{(\lambda,\nu,\varepsilon) \in R^{l+n+k} / N_k(\lambda,\nu,\varepsilon) \neq \varnothing\}$$

Definition 2 : the solvability set of problem $P(\lambda, \nu, \varepsilon)$ is defined by

$$B = \{(\lambda,\nu,\varepsilon) \in U / P(\lambda,\nu) \text{ has efficient solution}\}$$

Definition 3 : Assume that the problem $\bar{P}_k(\lambda, \nu, \varepsilon)$ is solvable for $(\hat{\lambda},\hat{\nu},\hat{\varepsilon})$ with a corresponding optimal point $(\hat{x},\hat{z})$, then the stability set of the first kind corresponding to $(\hat{x},\hat{z})$ which is denoted by $S(\hat{x},\hat{z})$ is defined by

$$S(\hat{x},\hat{z}) = \left\{(\lambda,\nu,\varepsilon) \in B / \hat{z} = \min_{(\hat{x},\hat{z}) \in N_k(\lambda,\nu,\varepsilon)} [f_k(x,\lambda) - \bar{f}_k]\right\}$$

3. KUHM-TUCKER CONDITIONS AND STABILITY NOTION

From the assumption that the functions f_j, $j = 1, ..., m$ and g_r, $r = 1, 2, ..., k$ are convex on R^n and differentiable, then there exist

$\hat{\lambda} \in R^l$, $\hat{\mu} \in R^m$ and $\hat{\nu} \in R^k$ such that $(\hat{x},\hat{z})$ solves the following

Kuhn-Tucker problem :

$$\frac{\partial f_k}{\partial x_\alpha}(\hat{x},\hat{\lambda}) + \sum_{j\neq k} u_j w_j \frac{\partial f_k}{\partial x_\alpha}(\hat{x},\hat{\lambda}) +$$

$$\sum_{i=1}^{m} \mu_i \frac{\partial f_k}{\partial x_\alpha}(\hat{x}) + \sum_{r\notin S} v_r \frac{\partial g_r}{\partial x_\alpha}(\bar{x},\bar{v}) = o, \ \alpha = 1, ..., n \quad \text{..............(1)}$$

$$\Sigma u_j = 1 \qquad \ldots\ldots (2)$$

$$- z + w_j[f_j(\bar{x}, \bar{\lambda}) - \bar{f}_j \leq o, \qquad j = 1,\ldots,m, \; j \neq k \qquad \ldots\ldots (3)$$

$$f_i(\hat{x}) - \ddot{\varepsilon}_i \leq o, \qquad i = 1, \ldots, m \qquad \ldots\ldots (4)$$

$$g_r(\hat{x}, \hat{v}) < 0 \; r \in S \subset \{1, \ldots, k\} \qquad \ldots\ldots (5)$$

$$g_r(\hat{x}, \hat{v}_r) = o, \qquad k \notin S \qquad \ldots\ldots (6)$$

$$u_j\{-z + w_j[f_j(\hat{x}, \hat{\lambda}) - \bar{f}_j]\} = 0, \; j = 1, \ldots, m, \; j \neq k \qquad \ldots\ldots (7)$$

$$\mu_j[f_j(\bar{x}) - \hat{\varepsilon}_j] = o, \qquad j = 1, \ldots, m \qquad \ldots\ldots (8)$$

$$\mu_j, u_j, \nu \geq o \quad \forall\, i, j, r. \qquad \ldots\ldots (9)$$

In order to find the stability set $S(\hat{x}, \hat{z})$, let us consider the following set :

$$T = \{(I,J,S)/u_{j=o}, \; j \in J \subset \{1, \ldots, m\},$$

$$\mu_i = o, i \in I \subset \{1, \ldots, n\},$$

$$v_r = o, r \in S \subset \{1, \ldots, k\}$$

$$\{u_j > o, j \notin J, \mu_i > o, i \notin I, \text{ and } v_r > o, r \notin S\} \qquad \ldots\ldots (10)$$

then the set $S(\hat{x},\hat{z})$ takes the form :

$$S(\hat{x},\hat{z}) = \bigcup_{(I,J,S)} S_{(I,J,S)}(\hat{x},\hat{z})$$

$$= \{(\lambda, \nu, \varepsilon \in R^{l+m}/w_j(\bar{x}, \lambda) - \bar{f}_j] \leq \hat{z}, j \notin J,$$

$$w_j[f_j(\hat{x}, \lambda) - \bar{f}_j] = \hat{z}, j \notin J,$$

$$w_j[f_j(\hat{x}, \lambda) - \bar{f}_j] = \hat{z}, j \notin J,$$

$$g_r(\hat{x}, v) \leq o, r \in S \text{ and } g_r(\hat{x}, v) = o, r \notin S\} \quad \text{......(11)}$$

It is well known that if $(\hat{x}, \hat{z})$ is an optimal solution of $P(\lambda, v, \varepsilon)$ and $(\hat{x}, \hat{z}, \hat{u}, \hat{\mu}, \hat{v})$ solves the kuhn-Tucker conditions (1) - (9), where $u_j > o$, $\mu_j > o$, $v_k > o$ and f_j are strictly convex function on R^n, then x is an efficient solution of $P(\lambda, v)$.

4. INTERACTIVE WITH THE DM TO ELICIT PREFERENCE

This method modifies.

i) the constraint set of $P_k(\lambda, v, \varepsilon)$, and

ii) the weights w_j from the formula

$$w_j = \frac{f_j^* - \bar{f}_j}{\sum_{j \neq k} f_j^* - \bar{f}_j}, \quad \text{......................(12)}$$

$$\text{wher } f_j^* = \max_{x \in M(v^*)} f_j(x, \lambda^*) \text{ and } \bar{f}_j = \min_{x \in M(v^*)} f_j(x, \lambda^*),$$

At the r-th iteration, the DM is asked to evaluate the solution at the (r-1)-th iteration, and to compare the values $f_j(x^{r-1}), ..., f_m(x^{r-1})$ with the ideal $f_1^*, ..., f_m^*$.

He is asked to indicate which objective can be increased and by how much, so that other objective can be decreased from the current unsatisfactory levels. Suppose the DM chooses to sacrifice the $\hat{j} \neq k$ objective $f_{\hat{j}}$ by Δf_j. The constraint set for the r-th iteration is

$$M^r = M^r \cap \overline{M}^r, \qquad \text{..............................(13)}$$

where

$$\overline{M}^r = \Big\{x \in M(v^*)/f_{\hat{j}}(x) \le f_k(x^{r-1}) + \delta f_{\hat{j}}$$

$$f_k(x) \le f_k(x^{r-1}), + \overset{\circ}{\lambda}_{kj}\, \delta f_{\hat{j}}$$

$$f_l(x) \le f_l(x^{r-1}),\ l \ne k, \hat{j}\,,\ \delta f_{\hat{j}} > o$$

$$\text{and} \quad f_l(x) \le \overset{\circ}{\epsilon}_l,$$

$$f_j(x) \le \overset{\circ}{\epsilon}_j + \delta f_j, \quad j \ne \hat{j}, k \Big\}. \qquad \text{................(14)}$$

The weights should be modified accordingly setting

$$w_{\hat{j}} = o \text{ and } w_j = \frac{f_j^* - \bar{f}_j}{\sum_{i \ne j,k} f_j^* - \bar{f}_j}, \qquad \text{...............................(15)}$$

consequently, the programming problem $P(\lambda, v, \varepsilon)$ to be solved at the r-th iteration is

$$\min \quad z \text{..(16a)}$$

$$\text{s.t. } w_i\,[f_i(x,\lambda) - \bar{f}_i] - z \le o,\ i = 1, 2, ..., m,\ i \ne k, \hat{j}, \quad \text{.......(16b)}$$

$$f_j(x) - \varepsilon_j \le o,\ \ j = 1, 2, ..., m \quad \text{..(16c)}$$

$$g_r(x,v) \le o, \quad r = 1, 2, ..., k. \quad \text{..(16d)}$$

The process terminates when one of the following occurs :

i) the DM is satisfied with the current solution,

ii) there is no satisfactory objective in the current solution; or

iii) when r = n

5. THE ALGORITHM

Step 1 : Asking the DM to select any $(\lambda_1^*, v_1^*) \in U$ to obtain an efficient solution x_1^* of $P(\lambda, v)$. Also, selects f_k as a primary objective.

Step 2 : Compute the initial set of weights $w_1, \ldots, w_m$, set $r = 1$, $M^r = M$.

Step 3 : Asking the DM to select ε_j^1 , where each ε_j^1 should be selected in the range $[a_j, b_j]$ where

$$a_j = \min_{x \in M(v_1^*)} f_j(x, \lambda_1^*),\ b_j = \max_{x \in M(v_1^*)} f_j(x, \lambda_1^*).$$

Step 4 : Formulate (16a) - (16d) and solve to obtain x_1^*. Then compute $f_1(x_1^*, \lambda_1^*), \ldots, f_m(x_1^*, \lambda_1^*)$.

Step 5 : Using (10) and (11) to obtain the set of all parameters corresponding to x_j^*.

Step 6 : Asking the DM to compare $f_1(x_1^*, \lambda_1^*), \ldots, f_m(x_1^*, \lambda_1^*)$ with $\bar{f}_1, \ldots, \bar{f}_m$

(a) If the DM is satistfied with the current solution, stop-the best-compromise has been found.

(b) If there is no satisfactory objective, stop-no best-compromise solution can be found by this method.

(c) If there are some satisfactory objective, ask the DM to select one such objective $\hat{f}_j$ and the amount $\delta \hat{f}_j$ to be sacrificed (increased) in exchange for an improvement of some unsatisfactory objective.

Step 7 : If $r = n$, stop-no best-compromise solution can be found by this method.

Otherwise set $r = r + 1$, compute M^r, and modify the set of weights according to (13), (14) and (15), respectively. Then go to step 1, where

$$(\lambda_2^*, v_2^*) \notin S(\hat{x}_j, \hat{z}_1).$$

Step 8 : Analyzing the DM, and bathing through the questions qu. 1 and qu. 2 (Appendix (A)), one would expect that the solution x^{r+1} of a new problem $\bar{P}_k(\lambda_r^*, v_r^*, \varepsilon^r)$, where

$$\varepsilon_l^{r+1} = \varepsilon_l^r - \delta f_l \text{ and } \varepsilon_j^{r+1} = \varepsilon_j^r + \delta f_j,$$

$$(\delta f_j > o \text{ and } \delta f_l > o)$$

would be a better point than x^r according to the DM stop.

6. MUMERICAL EXAMPLE

Let us consider the following problem :

$$\min (f_1(x,\lambda), f_2(x,\lambda), f_3(x,\lambda)$$
subject to
$$x_1 + x_2 - v \le 2 \text{ and } x_1, x_2 \ge o,$$

where

$$f_1(x,\lambda) = \lambda_1(x_1 - 3)^2 + \lambda_2(x_2 - 2)^2,$$
$$f_2(x,\lambda) = \lambda_1 x_1 + \lambda_2 x_2,$$
$$f_3(x,\lambda) = \lambda_1 x_1 + 2\lambda_2 x_2, \text{ and } v \in [o, 1].$$

Step 1 : Asking the DM to select $(\lambda_1^*, v_1^*) = (1, 1, 1,\_ \in U$ to obtain an efficient solution x_1^*, also select f_1 as a primary objective.

Step 2 : The original set of weights computed from (12) is $w_2 = 1/3$, $w_3 = 2/3$, and the DM select $\varepsilon^1 = (7, 2, 4)$.

Step 3 : We solve the problem $\overline{P}_1(\lambda_1^*, v_1^*, \varepsilon^1)$, which yields the solution

$\widehat{x_1^*} = (0.4,\ 1.6)$,

$\overline{z}_1 = 0.67,\ f_1(x_1^*, \lambda_1^*) = 6.9,\ f_2\ (x_1^*, \lambda_1^*) = 2$

and $f_3\ (x_1^*, \lambda_1^*) = 3.6$.

Step 4 : Using (10) and (11) to obtain the set of all parameters $S(\widehat{x_1^*}, \hat{z}_1)$ where $I = \{3\},\ j = \{1, 3\},\ S = \phi$

$$S(\widehat{x_1^*}, \hat{z}_1) = \bigcup_{(I,J,S)} S(\widehat{x^*}_1, \hat{z})$$

$$= \{(\lambda, v, \varepsilon) : 6.7\,\lambda_1 + 0.16\,\lambda_2 \le 2.67$$

$$0.4\,\lambda_1 + 3.2\ \lambda_2 \le 0.96$$

$$\varepsilon_3 \le 1.6,\ 0.4\,\lambda_1 + 1.6\,\lambda_2 = 2.33$$

$$\varepsilon_1 = 6.9,\ \varepsilon_2 = 2,\ v = o\}.$$

Step 5 : Suppose the DM compares $f_1(x_1^*, \lambda_1^*),\ f_2\ (x_1^*, \lambda_1^*),\ f_3\ (x_1^*, \lambda_1^*)$ with the ideal (2, 0, 0) and is willing to give up (increase) f_2 by one unit from 2 to 3 to improve f_1, and then compute

$$\lambda_{12}^1 = -\frac{\partial f_1}{\partial f_2}\Big|_{x_1^*} = 9.6$$

Step 6 : The new constraints set M^2 becomes

$$M^2 = M^1 \cap M^2 = M\ \cap \overline{M}^2\,(v),$$

where

$$\overline{M}^2(v) = \{x \in R_+^2 / \,(x_1 - 3)^2 + (x_2 - 2)^2 \le 16.5,$$

$$x_1 + x_2 \le 3,\ x_1+2x_2 \le 3.6 \text{ and}$$

$$x_1 + x_2 - \nu \le 2,\ x_1+2x_2 \le 5,$$

$$(x_1 - 3)^2 + (x_2 - 2)^2 \le 7\}.$$

Take $w_2 = o \Rightarrow w_3 = 1$ and select $(\lambda_2^*, \nu_2^*), \notin S(\widehat{x_1^*}, \hat{z}_1^*)$

where $\lambda_2^* = (2.\ 2),\ \nu_2 = \frac{1}{2}$.

Step 7 : We solve

$$\min\ z$$

$$\text{s.t.}\quad x \in M^2,\ 2x_2 + x_2 \le z$$

$$2(x_1 - 3)^2 + 2(x_2 - 2)^2 - 4 \le z,$$

which yields $x_2^* = (1.\ 4,\ 1.1), \hat{z}_2 = 7.2$ and

$$f_1(x_2^*, \lambda_2^*) = 22.64, \quad f_2^* = 5, \quad f_3 = 7.2.$$

Step 8 : The set of all parameters corresponding to $(\widehat{x_2^*}, \hat{z}_2^*)$, where $I = \{1, 2, 3\}$, $J = \{1, 2\}$, $S = \{1\}$ takes the form

$$S(\widehat{x_2^*}, \hat{z}_2) = \bigcup_{(I,J,S)} S(\widehat{x_2^*}, \hat{z}_2)$$

$$= \{(\lambda,\nu,\varepsilon) : 1.4\ \lambda_1 + 2.2\ \lambda_2 = 7.2,$$

$$\varepsilon_1 \ge 3.37,\ \varepsilon_2 \ge 2.5,\ \varepsilon_3 \ge 3.6 \text{ and } n \ge 0.5\}.$$

APPENDIX (A)

Trade-off information [3].

Let $\lambda_{kj}(x^o)$, $j = 1, 2, \ldots, m$, $j \ne k$ be the Kuhn-Tucker multipliers corresponding to the ε-constraints of $\overline{P}_k(\lambda, \nu, \varepsilon)$ where x^o solves $\overline{P}_k(\lambda, \nu, \varepsilon)$:

(i) If all $\lambda_{kj}(x^o) < o$ for each j, then the efficient surface in the objective space around the neighborhood of $f^o = F(x^o)$ can be represented by $f_k = f_k(f_1, \ldots, f_k, f_{k+1}, \ldots, f_m)$ and

$$\lambda_{kj}\,(x^{\circ}) = -\frac{\partial f_k}{\partial f_j}\Big|_{F=F^{\circ}} \text{ for each } j,\ j = 1, 2, \ldots, m,\ j \neq k \quad (A_1).$$

Thus each $\lambda_{kj}(x^o)$ represents the efficient partial trade-off rate between f_k and f_j at F^o when all other objectives are held fixed at their respective values at x^o.

(ii) If $\lambda_{kj}(x^o) > o$ for some $j \neq k$ and $\lambda_{kl}\,(x^{\circ}) = o$, for some $l \neq k$, the efficient surface in the neighborhood of F^o can then be expressed as $f_k = f_k\,(\widehat{F})$, where $\widehat{F}$ is a vector consisting of all f_j with $\lambda_{kl}\,(x^{\circ}) > o$. Also, each $\lambda_{kl}\,(x^{\circ})$ that is strictly positive can be interpreted as a trade-off rate, that exhibits an exchange between f_k and f_j while each objective f_l such that $\lambda_{kl}\,(x^{\circ}) = o$ also changes. Thus if $\lambda^i_{kl} > o$ for each $l \neq k$, then λ^i_{kl} approximates a local partial trade-off at a point x^i where λ^i_{kl} is the Kuhn-Tucker multiplier associated with the constraints $f_l(x) \leq \varepsilon^i_l$. To move from x^i to some other locally efficient point in the neighborhood of x^i, λ^i_{kl} units of f_k will be given up per one unit gain of f_l (or vice versa), with all other objectives remaining constant at the level of $f_l(x^i)$, $l \neq k$ and j and therefore if $\lambda^k_{kl} > o$ for all $l \neq k$, we ask the DM for each $l \neq k$:

qu. 1 : "given that $f_j = f_j(x^i)$ for all $j = 1, \ldots, m$, how (much) would you like to decrease f_k by λ^i_{kl} units for each one unit increase in f_l will all other f_j remaining unchanged ?".

If we make a smal change of δf_j units in f_{li} and setting $w_j = o$, where $l \in J^i_n = \{j/1 \leq j \leq n, j \neq k, \lambda_{kj} > o\}$, then f_{ki} changes by $-\ \lambda^i_{kl}\,\delta f_l$, and each f_j, where $\lambda_{kj} = o$, also changes by $\lambda^i_{kl}\,\delta f_l$, and each f_j, where $\lambda_{kj} =$

o, also changes by $\left(\nabla f_j(x^i)\dfrac{\partial x(\varepsilon^i)}{\partial \varepsilon_l}\right) \partial f_l$ units,

qu. 2 : Given that $f_j = f_j(x^i)$ for all $j = 1, \ldots, m$, how (much) you like to decrease

f_k by λ^i_{kl} units and change f_j by $\nabla f_j(x^i) \frac{\partial x(\varepsilon^i)}{\partial \varepsilon_l}$ units, while increasing f_l by one unit ?.

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