

A LINEAR PROGRAMMING BASED APPROACH FOR GENERATION RESCHEDULING AND LOAD SHEDDING TO ALLEVIATE POWER SYSTEM OVERLOADS

مدخل البرمجة الخطية لإعادة جدولة قدرة وحدات التوليد وتخفيف الأحمال
لإزالة زيادة التحميل بنظم القوى الكهربائية

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المخلص: يقدم البحث مدخلا للبرمجة الخطية لإزالة زيادة التحميل بخطوط نقل القدرة الكهربائية وذلك عن طريق إعادة تصحيح القدرة الحقيقية المنتجة من وحدات التوليد وتخفيف الأحمال عند نقاط تحميل محددة. تعتمد طريقة الحل المقترحة على حل مجموعة من المعادلات الخطية للنظام، في حالة إعادة جدولة القدرة المنتجة من وحدات التوليد، نجد أن دالة الهدف هي تقليل تكلفة التشغيل لوحدات التوليد مع تحقيق قيود التحميل والتشغيل للنظام، بينما بالنسبة لمشكلة تخفيف الأحمال، نجد أن دالة الهدف تعتمد على تقليل الأحمال الكلية المراد تخفيفها مع تحقيق قيود التشغيل للنظام بالنسبة لأقصى قيمة للحمل المراد تخفيفه وأيضاً قيمة الحمل المخفف المسموح به في كل خطوة وذلك للحفاظ على مستوى معين من الخدمة لهذه الأحمال. نظراً لأن طريقة الحل المقترحة تأخذ في الاعتبار قيود التحميل والتشغيل لمكونات النظام، لذلك نجد أن حل مشكلة زيادة التحميل تؤدي إلى تشغيل النظام عند حالة تشغيل أخرى آمنة ومستقرة وبأقل تعديل للقدرة المنتجة من وحدات التوليد وبأقل قدر ممكن من الأحمال المخففة. تم تطبيق طريقة الحل المقترحة على نظم قوى كهربائية، النظام الأول مكون من 6 قضبان والثاني من 25 قضيب لبيان إمكانية وفعالية الطريقة المقترحة في إزالة زيادة التحميل بمكونات النظام، ويمكن الاستفادة من الطريقة المقترحة أيضاً في عمليات التخطيط وتحليل الأمان لنظم القوى الكهربائية، وتعتبر أيضاً وسيلة مهمة تساعد مشغل هذه النظم بمراكز التحكم.

Abstract: This paper presents a linear programming (LP) based approach for the alleviation of line overloads by generation rescheduling and load shedding. The algorithm is based on solving a linearized set of optimal equations. The objective function for generation rescheduling is to minimize the linearized system operating cost while satisfying the system equality and inequality constraints. In respect to load shedding scheme, the objective function is to minimize the total load demand to be shed taking into account the operating constraints of the maximum load that can be shed; to ensure a minimum service at each bus; and the step of load to be shed. The results for the 6-bus and 25-bus test systems are presented to show the effectiveness of the proposed algorithm.

Keywords: Power system overloads, Generation rescheduling, Load shedding, Contingency, Linear programming.

1. INTRODUCTION

The operating point of a power system changes continually due to various contingencies and disturbances on the system. If the system survives the outage, it will operate in a new steady state in which one or more transmission lines may be overloaded. The problem of power system line overloads has obtained more attentions in the modern power system. To meet the load demand in a power system while satisfying the stability and reliability criteria, either the existing transmission lines must be utilized more efficiently, or new lines should be added to the system. The first alternative provides an economically and technically attractive solution to the line overload problem by using some efficient controls, such as generation rescheduling, controllable series capacitors, phase shifters, and load shedding.

In the literature, many methods have been reported for determining a secure operating point [1-5]. All these methods use optimization techniques, which are complicated and time consuming from a computational point of view, especially for large systems. Under emergency conditions the operator, has to make quick decisions, with little concern for the optimality of the operating point. Hence an efficient, reliable and direct method is always required. Reference [1] developed such a direct method for generation rescheduling and load shedding to alleviate line overloads, based on the sensitivity of line overloads to bus power increments. This algorithm is suffered from unnecessary and excessive load shedding, and in some cases, a solution may not be obtained at all. Mohamed and Jasmon [2] also proposed a direct method for generation rescheduling and load shedding to alleviate line overloads. In this method, bus powers are modified at the terminal buses of the overloaded lines by an amount equal to the line overload during each iteration of the load flow solutions. Hence, the process will be time consuming, as a greater number of iterations are required for the final solution.

A number of research papers are available on the subject of base case optimization and corrective rescheduling. Such methods [5,6] employ various performance indices for optimization and utilize linear, nonlinear or quadratic programming techniques. Shandilya et al [6] presented a local optimization based method for generation rescheduling and load shedding to alleviate line overloads. The problem is solved by using the conjugate gradient method technique. This method is computationally expensive and the load shedding procedure is not introduced.

From the literature survey it appears that simple and efficient algorithms are not yet available for real time implementation. This is important because solution of this problem provides the starting point for overall security constrained optimization. For achieving this objective the following features are necessary:

- (i) Exploitation of weak P- δ and Q-V coupling to solve the real and reactive power separately. This reduces the dimension of the problem.
- (ii) Inexplicit representation of the network through loss formula and sensitivity coefficients. This eliminates the need for computationally expensive explicit network solutions during optimization process.
- (iii) All available algorithms in literature on security constrained optimization [2-5] show that there is a possibility of producing new violations while removing existing violations. In the proposed approach all important monitored quantities are included in the solution algorithm.

This paper handles power system line overloads by use of available controls such as generation rescheduling, and load shedding. The optimal solution is obtained by solving a linearized set of optimal equations for generation rescheduling and also for load shedding scheme. The proposed algorithm is tested on 6-bus and 25-bus test systems.

2. LIST OF SYMPOLS

- P_{G_i} : real power generated at bus i.
 $|V_i|$: voltage magnitude at bus i.
 δ_i : voltage phase angle at bus i.
 G_{ij} : branch conductance between bus i and j.
 B_{ij} : branch susceptance between bus i and j.
 $P_{f_{ij}}$: branch active power flow between bus i and j.
 C_{G_i} : cost coefficient of generator at bus i.
 $P_{G_i, \max}$: maximum active power generation limit.
 $P_{G_i, \min}$: minimum active power generation.
 $P_{f, \max}$: maximum limit active power flow.
 $P_{f, \min}$: minimum limit of active power flow.
 N_B : number of system buses.
 N_L : number of system lines.
 $P_{ij, \text{loss}}$: real branch power loss between bus i and j.
 a_j : coefficient of constraints.
 b_j : constraints bounds.
 C_i : cost coefficients.
 N_D : set of load buses.
 ΔP_{Dk} : load shedding at load bus k in one step.
 P_{Dk} : real power load committed at load bus k.
 $P_{Dk, \min}$: the lower limit of active power demand at bus k.
 $\Delta P_{D\max}$: the overall maximum active power demand that can be shed in one step.

3- PROBLEM FORMULATION

In this paper the constrained rescheduling model uses the submatrix $J_{P-\delta}$ of the full Jacobian matrix to exploit the advantage of decoupling between bus power and bus voltage. The LP-based algorithm with linearized cost function and system constraints has been proved to be fast and reliable [4,7].

3.1 System Objective Function and Constraints for Generation Rescheduling

The system objective function is to minimize the total system operating cost as:

$$\text{Min } f = C^T \cdot \Delta P_G \quad (1)$$

Subject to the following constraints:

(i) **Equality constraints:**

$$\Delta P_G - \Delta P_D - \Delta P_{\text{Loss}} = 0 \quad (2)$$

(ii) **Inequality constraints:**

$$\Delta P_{f,\min} \leq \Delta P_f \leq \Delta P_{f,\max} \quad (3)$$

$$\Delta P_{G,\min} \leq \Delta P_G \leq \Delta P_{G,\max} \quad (4)$$

In the Newton-Raphson method of power flows given in polar form, the linearized equations are expressed as follows:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = [J] \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad (5)$$

$$[\Delta P] = [J_{P-\delta}] [\Delta \delta] \quad (6)$$

$$[\Delta \delta] = [J_{P-\delta}]^{-1} [\Delta P] \quad (7)$$

or

$$[\Delta \delta] = [A] [\Delta P] \quad (8)$$

where $[A] = [J_{P-\delta}]^{-1}$. From equation (7), $\Delta \delta$ is solved by LU factorization without inverting the $J_{P-\delta}$ submatrix and stored in factored form and any required row of this submatrix can be calculated from the factored matrices.

The real power flow in a branch connected between bus i and j can be written as:

$$P_{ij} = |V_i|^2 G_{ij} - |V_i||V_j| (G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)) \quad (9)$$

The incremental flow can be represented as:

$$\Delta P_{ij} = \frac{\partial P_{ij}}{\partial \delta_i} \Delta \delta_i + \frac{\partial P_{ij}}{\partial \delta_j} \Delta \delta_j \quad (10)$$

Equation (10) can be written as follows:

$$[\Delta P_{ij}] = [C] [\Delta \delta] = [C][A] [\Delta P] = [D] [\Delta P] \quad (11)$$

where

$$C_{ij} = |V_i||V_j| (B_{ij} \cos(\delta_i - \delta_j) - G_{ij} \sin(\delta_i - \delta_j)) \quad (12)$$

and $[D] = [C][A]$

3.2 Incremental Real Power Loss

The real power loss at a branch between buses i and j can be represented as follows:

$$P_{ij \text{ loss}} = G_{ij} (|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos(\delta_i - \delta_j)) \quad (13)$$

Following a procedure similar to equations (9) and (10), we can write the incremental real power loss as follows:

$$[\Delta P_{\text{loss}}] = [F]\Delta\delta = [F][A][\Delta P] = [H][\Delta P] \quad (14)$$

where

$$F_{ij} = 2|V_i||V_j|G_{ij}\sin(\delta_i - \delta_j) \quad (15)$$

and $[H] = [F][A]$

The total incremental system real power loss can be written from equation (14) as:

$$\Delta P_{\text{loss}} = \sum_{k=1}^{NL} \Delta P_{k, \text{loss}} = \sum_{i=1}^{NB} [H_i] \Delta P_i \quad (16)$$

3.3 Calculation of Constraints limits

The limits for the incremental real power flow are given by:

$$P_{f, \text{min}} - P_f \leq \Delta P_f \leq P_{f, \text{max}} - P_f \quad (17)$$

The limits on control variables are as follows:

$$P_{G, \text{min}} - P_G \leq \Delta P_G \leq P_{G, \text{max}} - P_G \quad (18)$$

3.4 Problem Formulation as Linear Programming

The LP problem can be defined as to find a vector of unknown variables $X = [X_1, X_2, \dots, X_n]$ so as to minimize an objective function in form of equation (1) represented in vector form as:

$$\text{Min } f = C^T \cdot X \quad (19)$$

The constraint equations of the form shown in equations (2-4) are also represented in vector form as:

$$a_j^T \cdot X \geq b_j \quad (20)$$

3.5 Load Shedding Scheme

Load shedding can be defined as a coordinated set of controls which results in decrease of the system load. An optimal load shedding scheme will find a best stable operating point of disturbed system with a minimum amount of load to be shed. The load shedding problem can be represented by a linear optimization model as:

$$\text{Min } f_L = \sum_{k=1}^{ND} \Delta P_{Dk} \quad (21)$$

Subject to:

$$P_{Dk} - P_{Dk, \min} \geq \Delta P_{Dk} \quad (22)$$

$$\sum_{k=1}^{ND} \Delta P_{Dk} \leq \Delta P_{D \max} \quad (23)$$

Due to operating constraints there is a maximum limit to the load that can be shed at each bus to ensure a minimum service, and the load can only be shed in steps as for example 20, 40, 60% of the initial load [8,9]. The only unknown variables of the proposed load shedding model as shown by Eqns.(21-23) are load reductions ΔP_{Dk} . Their calculation is performed by LP technique till we alleviate the lines overload.

3.6 LP-Based Algorithm to Alleviate Lines Overload

To alleviate line overload the following sequence of control actions is expected from the operator:

- (i) Decrease the bus power injections at the sending end bus of the overloaded line. This is incorporated by decreasing the generation at this bus and/or at the buses feeding power to it.
- (ii) Maintain the bus power injections constant at the receiving end bus of the overloaded line by increasing the generation at this bus and/or at the buses feeding power to it.
- (iii) If the line overload is still not alleviated, curtail the load at the receiving end bus of the overloaded line and/or at the buses to which the power is being fed from this bus.

The proposed algorithm can be summarized as follows:

- 1- Input system data.
- 2- Given the contingency.
- 3- Solve load flow problem.
- 4- Calculate the branch flow.
- 5- Check the overloads? If 'No' stop, otherwise, go to step 6.
- 6- Solve LP problem and update control variables.
- 7- Check convergence? If 'No' apply the load shedding scheme and go to step3, otherwise go to 8.
- 8- Print optimal results.

The feature of developed algorithm is based on solving a linearized set of optimal equations which are solved very efficiently for real power corrections and load shedding. In the proposed algorithm the correction of generations is made first for the generators connected to the sending end of overloaded lines, or at the buses feeding the lines connected to the sending end of the overloaded lines. If the lines overload is still not alleviated then load shedding may have to be taken to remove overloads. Also, the procedure of load shedding is performed in steps first at the receiving end bus of the overloaded line and then at the buses to which the power is being fed from this bus.

4. APPLICATION AND TEST RESULTS

The algorithm developed in section 3 for alleviating line overloads has been implemented on 6-bus and 25-bus test systems. The detailed data of 6-bus system is given in appendix while the data of 25-bus test system is found in [1]. There are two approaches for testing the proposed algorithm to alleviate line overloads. These are:

- (i) Turn the base case into an overload system by setting line power limits below their base-case values.
- (ii) Induce overloads by removing loaded lines from the system.

4.1 The 6-Bus Test System

The base case overloads are created by reducing the lines power limits by 30% from its rated values. Table 1 shows the results of this case in respect to overloaded lines and the percentage of overload value. The control action taken in this case is to reschedule the generators. The corresponding corrective rescheduling is given in table 2. Table 3 shows the power flow in the overloaded lines with no load shedding and no line is overloaded in the system. Another contingency is applied to the 6-bus test system. This is represented by removing line #2. The results in table 4 show that line #1 is overloaded by about 24.55%. The control action in this case is to correct the generations as given in table 5 and to shed a load by 10.23 MW at bus #6 as shown in table 4.

Table 1 The overloads of 6-bus test system (Base case)

Overloaded line	Real power flow (MW)	Power limit (MW)	Overload %
4	-5.6	4.0	40
5	-20.3	15.0	35.33
6	16.18	15.0	7.86

Table 2 Generation rescheduling

Bus #	Base case P_G (MW)	Corrected value P_G (MW)
1	33.62	44.72
2	38.01	26.02

Table 3 Overloaded lines in new operating state

Line #	Line flow (MW)	Load shedding (MW)	Remarks
4	1.24	0.0	No line is overloaded
5	13.83		
6	11.27		

Table 4 The 6-bus system (Line # 2 is removed)

Overloaded line	Power flow (MW)	Power limit (MW)	Overload %	Line flow after correction (MW)	Load shedding (MW)
1	33.63	27.0	24.55	22.18	10.23

Table 5 Generation correction

Bus #	Base case P_G (MW)	Corrected value P_G (MW)
1	33.62	22.18
2	38.01	38.33

4.2 Results of 25-Bus Test System

The developed algorithm has also been employed for alleviating line overloads of the 25-bus test system. The system has 35 lines and 5 generator buses namely 1,2,3,4 and 5. The base case power limit is lowered by 10% for all lines. Table 6 presents the lines flows in the overloaded lines and the percentage of overload across each line. The generation correction is given in table 7 to alleviate the overloads shown in table 6 with no load shedding. Finally, table 8 gives the new operating state of the overloaded lines.

Table 6 Overloaded lines of 25-bus test system (Base case)

Overloaded lines	Real power flow (MW)	Power limit (MW)	Overload%
12	-48.6	43.0	13.02
17	57.8	53.0	9.056

Table 7 Generation correction

Bus #	Base case P_G (MW)	Corrected value P_G (MW)	P_G (MW)	
			Min	Max
1	267.3	272.3	50.0	300.0
2	99.3	93.6	20.0	125.0
3	147.9	151.3	30.0	175.0
4	39.1	48.0	10.0	75.0
5	193.0	178.4	40.0	250.0

Table 8 Overloaded lines in new operating state

Line #	Line flow (MW)	Load shedding (MW)	Remarks
12	-40.8	0.0	No line is overloaded in the system.
17	50.6		

5. CONCLUSION

A developed linear programming based algorithm for rescheduling of real power generation and load shedding to alleviate line overloads is presented. The feature of developed algorithm is based on solving a linearized set of optimal equations which are solved very efficiently for real power corrections and load shedding. Inclusion of inequality constraints on active power line flow limits and equality constraint on real power balance assures a solution representing a secure system. Transmission losses are also taken into account in the constraint function. Test results for the various test systems are presented. The obtained results clearly show that the

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proposed algorithm is superior, as a new secure operating condition is obtained with significantly less load shedding and with a little deviation from the pre-contingency system state. The developed algorithm can be successfully applied in operational planning, security analysis and will be a good aid to the load dispatcher.

6. APPENDIX

Data for the 6-bus system (at 100 MVA base)

Table 9: Lines data

Line #	Bus	to	Bus	Line parameters (p.u)	
				R	x
1	1		6	0.123	0.518
2	1		4	0.08	0.37
3	4		6	0.097	0.407
4	6		5	0.0	0.3
5	5		2	0.282	0.64
6	2		3	0.723	1.05
7	4		3	0.0	0.133

Table 10: Load data

Bus#	Demand (p.u)	
	P _D	Q _D
1	0.0	0.0
2	0.0	0.0
3	0.273	0.065
4	0.0	0.0
5	0.15	0.09
6	0.25	0.025

Table 11: Generators data

Generator Bus	P _G (p.u)	
	Max	Min
1	1.0	0.1
2	1.0	0.1

The operating costs of the generators are:

$$C_1 = P_{G1} + 0.05P_{G1}^2$$

$$C_2 = P_{G2} + 0.1 P_{G2}^2$$

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