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MERITS OF USING THE NON – LINEAR PROGRAMMING IN SOLVING THE OPTIMIZATION PROBLEMS OF GEODETIC NETWORKS

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ABSTRACT

The present study concentrates on the method used in solving the optimization problem of geodetic networks. In this context, there are two solution techniques that can be used in the solution of the optimization problem; namely: least squares technique and non- linear programming method. Hence, the main objective of the present paper is to evaluate the solution of the optimization problem using non- linear programming. An overview of both least squares technique and non-linear programming method, as applied to the optimization problem will be given .The results will be discussed and analyzed. The obtained results showed the superiority of the non-linear programming method

يعنى هذا البحث بدراسة استخدام البرمجة غير الخطية فى حل مسائل التصميم الأمثل للشبكات وقد تم فى هذا البحث التعرض بشكل مستفيض للبرمجة غير الخطية وكيفية استخدامها فى حل مسائل التصميم الأمثل للشبكات .ولبيان أهمية استخدام البرمجة غير الخطية في حل مسائل التصميم الأمثل للشبكات فقد تم حل مثال عددى بطريقتين مختلفتين، استخدم فى إحداها نظرية أقل مجموع لمربعات الأخطاء كطريقة تقليدية شائعة الاستخدام واستخدم فى الطريقة الأخرى طريقة البرمجة غير الخطية وقد وصلت نسبة التحسن فى الدقة التى تم الحصول عليها عند استخدم البرمجة غير الخطية إلى 125% (precision) مقارنة بنظرية أقل مجموع لمربعات الأخطاء ، مما يوضح أهمية استخدم البرمجة غير الخطية فى حل مسائل التصميم الأمثل للشبكات الجيوديسية .

1.INTRODUCTION

All the strategies used for the solution of the optimization problems, and especially the second-order design problems, are considered as one of two categories: Computer simulation and analytical methods. The concept of computer simulation can be summarized as (e.g., Cross and Whiting, 1980; Cross, 1985; Asal, 1994). The disadvantages of the simulation process are that, the success of the method depends on some extent to the skill and experience of the designer. Also, the optimum solution may not be achieved, thus the resulting network, even if it satisfies the requirements, may not be the optimal one, especially in terms of the cost. In the analytical methods, two basic approaches are used in the solution. The first is based on the concept of best–fitting the user precision, in least–squares sense. The other uses the techniques from the operation research called mathematical programming. The mathematical programming concept is classified according to the nature of the functions of the problem into: linear programming and non-linear programming techniques. This paper deals with the case of analytical solution. Also, the main objective of the present research is to investigate the statistical and practical validity of the non- linear programming method, as applied to solving the SOD problem. The variance covariance matrix of the estimated coordinates will be considered as a measure of precision. The realization of the results will be performed for a two- dimensional trilateration network (numerical example).

2. THE SIMPLE LEAST-SQUARES SOLUTION

In general, the simple least-squares approaches can be classified into three kinds, namely:

1– **The direct approaches:** which directly approximate the criterion matrix.

2– **The canonical design:** which uses the concept of the singular value decomposition of the inverse criterion matrix.

3– **The iterative approach:** which starts by initial weight vector p^{o}, then, it is updated through an iterative manner, until reaching the desired one.

This paper discusses the solution of the second-order design problem of geodetic network by the direct approaches.

2.1 THE DIRECT APPROACH

2.1.1 SOLUTION OF THE SOD PROBLEM FOR CORRELATED OBSERVATIONS

Bossler et al. (1973) proposed the solution of the SOD problem, depending on the calculus of the generalized inverse of a matrix, based on, e.g., the work of Rao & Mitra (1971). Let the design matrix (A), be given and the criterion cofactor matrix Q_x is also given, then:

$$Q_x = (A^T P A)^+ \qquad (1)$$

then, $A^T P A = Q^{-1}_x$ (2)

Using the Khatri-Rao product, equation (2) can be rewritten as (Nincov, 1982; Crosilla et al., 1989) :

$$(A^T_{n,u} \Theta A^T_{n,u})\, p^{\backslash}_{n,1} = q_{u^2,1} \qquad (3)$$

where:

$p^{\backslash}$: contains the diagonal elements of the weight matrix P

q : formed by ordering the rows of Q^{-1}_x in a column.

The SOD problem, which considers the diagonal weight matrix is called, the diagonal SOD and most of the SOD problems are performed as the diagonal designs. All the following solutions of the problem will consider the diagonal SOD only. (Kamal et al, 2000; Rahil et al, 2000)

2.1.2 SOLUTION BY DIRECT APPROXIMATION OF THE CRITERION MATRIX ITSELF

The matrix equation:

$$A^T P A = (Q_X)^+ \qquad (4)$$

Multiplying both sides from left and right by Q_X, one gets:

$$Q_X A^T P A Q_X = Q_X (Q_X)^+ Q_X \qquad (5)$$

Put: $K = Q_X A^T$ (6)

Finally :

$$K P K^T = Q_X \qquad (7)$$

Converting (7) to a set of linear equations, using the Khatri-Rao product:

$$(K \Theta K)\, p = q^{\backslash} \qquad (8)$$

where :

$$p = \text{vec}(P),\; q^{\backslash} = \text{vec}(Q_X) \qquad (9)$$

(vec) : is the operation by which the column of a matrix can be stacked one under another starting each column from the diagonal element.

The general solution of Equation (8) is:

$$p = (K \Theta K)^+ q^{\backslash} \qquad (10)$$

2.1.3 THE DIRECT APPROXIMATION OF THE INVERSE CRITERION MATRIX

Starting by the cofactor matrix equation:

$$A^T P A = (Q_X)^+ \qquad (11)$$

Using the Khatri–Rao product, the system (11) can be converted to a system of linear equations for the vector p, thus:

$$p = (A^T \Theta A^T)^+ q \qquad (12)$$

with: $p = \text{vec}(P)$ and $q = \text{vec}(Q_x)^+ = \text{vec}\, P_X$ (13)

The solution of (12) is obtained by (e.g., Schmitt 1978 ,1979, 1985):

$$p = (A^T \Theta A^T)^+ q \qquad (14)$$

2.2 THE CANONICAL DESIGN OF THE SECOND-ORDER DESIGN PROBLEM

Schaffrin et al. (1977) presented the canonical formulation of the SOD problem. This is based on the singular value decomposition of the inverse of the criterion cofactor matrix Q_X, as follows:

$$(Q_X)^+ = E D E^T \qquad (15)$$

where:

$D = \text{diag}(d_1, d_2, \ldots, d_r, 0, \ldots, 0)$ is the *diagonal matrix of the eigenvalues of P_X, with rank* $(P_X) = r$.

E = the orthogonal matrix of eigenvectors of P_X.

From the cofactor matrix equation, then:

$$A^T P A = P_X = E D E^T \qquad (16)$$

Multiplying the equation (16) by $E^T E$, then:

$$E^T A^T P A E = E^T E D E^T E \qquad (17)$$

Remember that: $E E^{-1} = I$ (identity matrix) and $E^T = E^{-1}$, then:

$$(A E)^T P (A E) = D \qquad (18)$$

put : $Z = A E$, (19)

then: $Z^T P Z = D$ (20)

Using the Khatri–Rao product, Eq. (20) can be transformed to a system of linear equations by:

$$(Z^T \Theta Z^T) p = d \qquad (21)$$

where:

p and d: are the vector mappings of P and D

The system (21) can be solved also by the pseudo–inverse as:

$$p = (Z^T \Theta Z^T)^+ d \qquad (22)$$

2.3 THE ITERATIVE APPROACH

Considering the equation of the solution vector (x), resulting from the least–squares adjustment, which is of the form:

$$x = (A^T P A)^+ A^T P L \qquad (23)$$

Wimmer (1981, 1982) applied the covariance law to eq. (22), to get the cofactor matrix, which yields:

$$Q_X = (A^T P A)^+ A^T P * Q_L * [(A^T P A)^+ A^T P]^T \qquad (24)$$

$$= (A^T P A)^+ A^T P * P^+ * P A(A^T P A)^+$$

put: $H = (A^T P A)^+ A^T P$ (25)

then: $H P^+ H^T = Q_X$ (26)

Using the Khatri–Rao product, eq. (25) can be transformed to a system of linear equations by:

$$(H \Theta H) p^\backslash = q^\backslash \qquad (27)$$

where:

$p^\backslash = vec (P^+)$

$q^\backslash = vec (Q_X)$

Equations (25, 27) are solved iteratively, because H contains an actual weight matrix P, which is updated *to the new* P^+. *This iterative solution can be* expressed symbolically as:

$$p^{(i+1)} = (H^{(i)} \Theta H^{(i)})^+ q^\backslash \qquad (28)$$

$$H^{(i)} = (A^T P^{(i)} A)^+ A^T P^{(i)} \qquad (29)$$

where:

$$p_i = \begin{pmatrix} \frac{1}{p_i^+} \\ 0 \end{pmatrix} \quad for \quad \begin{pmatrix} p_i^+ = 0 \\ p_i^+ = 0 \end{pmatrix} \qquad (30)$$

A suitable initial solution of P, is the unit matrix (I), *as proposed by Wimmer (1982).* Kuang (1992) reported that, depending on the comparative studies, the iterative solution does not converge in every case, and sometimes it is not able to yield a solution at all.

3. THE MATHEMATICAL PROGRAMMING METHOD

Recall that, the optimality of the least-squares solution is achieved from the fact that, the resulting weight vector will have a minimum norm, i.e., the sum of the squares of the weights is minimum. This indicates a minimum cost. The simple approaches of the SOD problem consider only –usually– the precision criteria, which are expressed through the criterion matrix. In these approaches, the other criteria, as reliability and cost, and the required considerations of the design, can be checked after the design. Then, the design can be changed according to these requirements, if they were not achieved. In such a case, negative weights are possible (Cross and Fagir,1982). Clearly, negative weight have no physical meaning, hence it is difficult to interpret them. In the mathematical programming method, the other criteria and considerations can be formulated as a risk function or constraints. Examples of these considerations are: the non-negative of the values of the resulting weights, the cost functions, and the achievable accuracy requirements.

3.1 THE LINEAR PROGRAMMING METHOD

The linear programming is a branch of mathematics, which deals with the optimization of linear functions, subjected to set of linear constraints. Here is some forms of the linear programming functions, proposed by some studies, when solving the SOD problems.

To avoid the negative weights, Equation (13) which represent the direct and canonical designs, can be written as inequalities with additional restrictions for the non negative weights as (Schmitt, 1985):

$$H P \leq q \qquad (31)$$

with:

$$P \geq 0 \qquad (32)$$

in which:

$H = (A^T \Theta A^T)$ (In eq. 14)

The risk function may be the weighted sum of the observational weights, thus:

$$\sum_{i=1}^{n} P_i = \min \quad or \quad \sum_{i=1}^{n} C_i P_i = \min \qquad (33)$$

which indicates the minimum cost.

This form of the risk function and the constraints allows the application of the simplex algorithm. The disadvantage of the simplex algorithm is that a lot of observations are resulting with zero weights. Thus, in nearly 50% of the optimal designs in the examples in Grafarend (1975), there is no redundant observation.

Cross and Thapa (1979) suggested that the sign of the constraints:

$$(A^T \Theta A^T)\, p \{ \geq = \leq \}\ q \qquad (34)$$

can be determined as follows :

- All rows of ($A^T \Theta A^T$) which correspond to a diagonal element (variance) of Q_X, will be:

$$(A^T \Theta A^T)_i\, p \geq q \qquad (35)$$

- All those corresponding to the off–diagonals are:

$$(A^T \Theta A^T)_j\, p \leq q \qquad (36)$$

In summary, the complete form of the problem will be:

$$Min. \sum_{i=1}^{n} P_i \qquad (37)$$

subject to:

$$H\,P \leq q \ \ (\text{where: } H = A^T \Theta A^T) \qquad (38)$$

$$P \geq 0 \qquad (39)$$

But the disadvantage of the Simplex design by the above method of determining the sign in the relation (18), is that the resulting network sometimes does not satisfy the design criteria. This is because the simple reversal of the inequality sign due to the inversion of the criterion matrix, is not valid (Cross & Fagir, 1982). Thus, if the inversion of the criterion matrix is avoided, the Simplex design will be more effective. For this reason, Cross and Whiting (1980), suggested a solution method without inverting the criterion matrix, but aim to obtain the required variances of observations, instead of the required weights.

3.2 THE NON-LINEAR PROGRAMMING METHOD

The non-linear programming methods are used, when any of the constraints or the objective function is non–linear function. Several studies, (e.g., Milbert (1979) and Schaffrin (1981)) had formulated the SOD problem as non–linear programming problem. For example, the approach supposed by Schaffrin (1981), and another form of the non-linear programming method, was proposed by Kuang (1992). This method used the least–squares principle, and a set of approximate values of the weight vector p, in a similar manner, as the observation equation adjustment process. The approximation is done with respect to the criterion matrix itself, not its inverse. The solution depends on equating the resulting covariance matrix Q_X, by the given criterion matrix Q_S. This leads to a system of redundant non–linear and inconsistent equations, to be solved for unknown weights. The solution can be summarized as follows:

considering the parametric least-squares adjustment, using approximate values of the weights, the criterion matrix is:

$$Q_X = (A^T P A + D D^T) - G (G^T D D^T G)^{-1} G^T \qquad (40)$$

where:

G: is the inner constrained network datum matrix.

D : is the minimum constrained network datum matrix.

It is clear that, the elements of Q_X are non–linear functions of the observational weights, assuming a fixed configuration. Given a set of approximate observational weights, Q_X can be linearized using the Taylor series expansion by (Kuang, 1992) and Doma (2004):

$$Q_X = Q_X^o + \sum_{i=1}^{n} \frac{\partial Q_X}{\partial p_i} \Delta p_i \qquad (41)$$

where:

$$Q_X^o = \left\{ (A^T P A + D D^T)^{-1} - G (G^T D D^T G)^{-1} \right\}_{p^o} \qquad (42)$$

with:

p^o : are given approximate weights

and:

$$\frac{\partial Q_X}{\partial p_i} = \left\{ -(A^T P A + D D^T)^{-1} \left(A^T \frac{\partial P}{\partial P_i} A \right) (A^T P A + D D^T)^{-1} \right\}_{p^o} \qquad (43)$$

Considering the case of uncorrelated observations, the partial derivatives of the weight matrix P, with respect to the individual weight p_i ($i = 1, \ldots\ldots, n$) is a matrix containing zeroes except for identity at the (i, j) –th position, i.e.,

$$\frac{\partial P}{\partial p_i} = (a_{kj}) \quad \text{with } a_{kj} = \begin{cases} 1.0 & \text{if } k = j = i \\ 0.0 & \text{elsewhere} \end{cases} \qquad (44)$$

In the present study, the partial derivatives $\left(\frac{\partial Q_X}{\partial P_i} \right)$ can be computed as the following matrix named (F matrix):

$$\frac{\partial Q_X}{\partial P_i} = (F_{m,m})_i = \begin{pmatrix} f_{1,1} & f_{1,2} & f_{1,3} & \ldots & f_{1,m} \\ f_{2,1} & f_{2,2} & f_{2,3} & \ldots & f_{2,m} \\ f_{3,1} & f_{3,2} & f_{3,3} & \ldots & f_{3,m} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ f_{m,1} & f_{m,2} & f_{m,3} & \ldots & f_{m,m} \end{pmatrix}_i \qquad (45)$$

We can rewrite Eq. (43) as:

$$Q_X - Q_X^o = \sum_{i=1}^{n} \frac{\partial Q_X}{\partial p_i} \Delta p_i \qquad (46)$$

then:

$$Q_S - Q_X^o = \begin{pmatrix} q_{1,1} & q_{1,2} & q_{1,3} & \cdots & q_{1,m} \\ q_{2,1} & q_{2,2} & q_{2,3} & \cdots & q_{2,m} \\ q_{3,1} & q_{3,2} & q_{3,3} & \cdots & q_{3,m} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ q_{m,1} & q_{m,2} & q_{m,3} & \cdots & q_{m,m} \end{pmatrix} \qquad (47)$$

where:

$q_{11} = Q_{X11} - Q^o_{X11}$, $q_{12} = Q_{X12} - Q^o_{X12}$, ... , $q_{m,m} = Q_{Xmm} - Q^o_{Xmm}$.

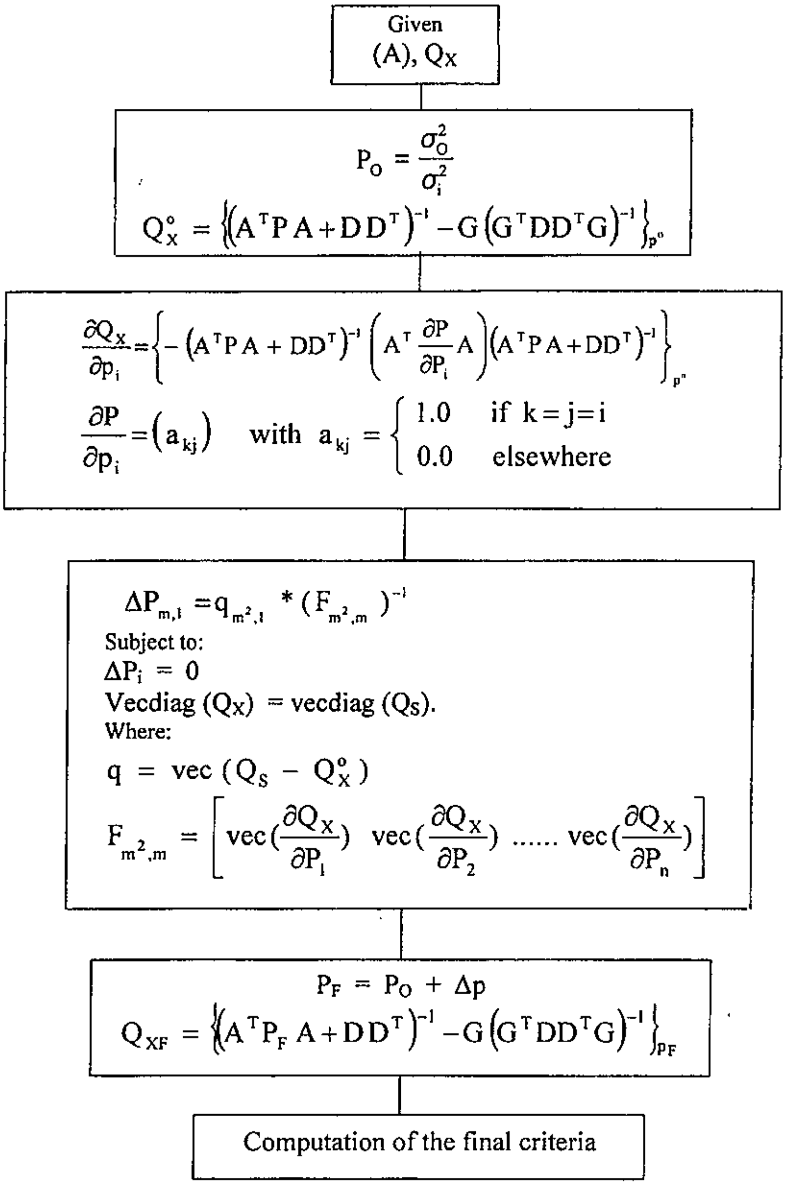

Fig. 1 Diagram for the solution steps of the case study that will be solved using the non-linear programming method.

then:

$$\begin{pmatrix} q_{1,1} & q_{1,2} & \cdots & q_{1,m} \\ q_{2,1} & q_{2,2} & \cdots & q_{2,m} \\ q_{3,1} & q_{3,2} & \cdots & q_{3,m} \\ \cdots & \cdots & \cdots & \cdots \\ q_{m,1} & q_{m,2} & \cdots & q_{m,m} \end{pmatrix} = \begin{pmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,m} \\ f_{2,1} & f_{2,2} & \cdots & f_{2,m} \\ f_{3,1} & f_{3,2} & \cdots & f_{3,m} \\ \cdots & \cdots & \cdots & \cdots \\ f_{m,1} & f_{m,2} & \cdots & f_{m,m} \end{pmatrix}_1 \Delta p_1 + \begin{pmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,m} \\ f_{2,1} & f_{2,2} & \cdots & f_{2,m} \\ f_{3,1} & f_{3,2} & \cdots & f_{3,m} \\ \cdots & \cdots & \cdots & \cdots \\ f_{m,1} & f_{m,2} & \cdots & f_{m,m} \end{pmatrix}_2 \Delta p_2 \cdot + \begin{pmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,m} \\ f_{2,1} & f_{2,2} & \cdots & f_{2,m} \\ f_{3,1} & f_{3,2} & \cdots & f_{3,m} \\ \cdots & \cdots & \cdots & \cdots \\ f_{m,1} & f_{m,2} & \cdots & f_{m,m} \end{pmatrix}_n \Delta p_n \qquad (48)$$

To solve this system of equations we can write them as:

$$\left.\begin{aligned} q_{1,1} &= (f_{1,1})_1\Delta p_1+(f_{1,1})_2.\Delta p_2 + (f_{1,1})_3.\Delta p_3 +....+(f_{1,1})_n\Delta p_n. \\ q_{2,1} &= (f_{2,1})_1\Delta p_1+ (f_{2,1})_2.\Delta p_2 + (f_{2,1})_3.\Delta p_3 +....+(f_{2,1})_n\Delta p_n. \\ q_{3,1} &= (f_{3,1})_1.\Delta p_1+(f_{3,1})_2.\Delta p_2 + (f_{3,1})_3.\Delta p_3 +.... +(f_{3,1})_n\Delta p_n. \\ ... &= \quad ..... \quad + \quad ..... \quad + \quad ..... \quad +....+ .... \\ q_{m,m} &=(f_{m,m})_1\Delta p_1 +(f_{m,m})_2\Delta p_2 +(f_{m,m})_3\Delta p_3 +....+(f_{m,m})_n\Delta p_n. \end{aligned}\right\} \quad (49)$$

then:

$$q_{m^2,1} = F_{m^2,m} * \Delta P_{m,1} \quad (50)$$

where:

$$q = vec\ (Q_S - Q_X^o) \quad (51)$$

$$F_{m^2,m} = \left[vec(\frac{\partial Q_X}{\partial P_1}) \ vec(\frac{\partial Q_X}{\partial P_2}) \ ...... vec(\frac{\partial Q_X}{\partial P_n}) \right] \quad (52)$$

Note that, the operator "vec" produces a vector by staking the columns of a quadratic matrix one under another in a single column. Figure (1) shows a diagram for the solution steps of the case study, which will be solved using the SOD problem with the non-linear programming.

Here, ΔP_i (i = 1, 2, 3, ..., n) are understood as the improvements, which are to be optimally solved for, to the initially adopted approximate weights P^o. In the current study, the solution is completed by using a linear complementar algorithm. Then, the mathematical model can be reformulated in matrix and vector form as:

$$\Delta P_{m,1} = q_{m^2,1} * (F_{m^2,m})^{-1} \quad (53)$$

subject to: $\Delta P_i = 0$

Vecdiag (Q_X) = vecdiag (Q_S).

where: Q_S : The required criterion matrix,

and:

$$Q_X = Q_X^o + \sum_{i=1}^{n} \frac{\partial Q_X}{\partial p_i} \Delta p_i \quad (54)$$

$$Q_X^o = \left\{ (A^T P A + D D^T)^{-1} - G (G^T D D^T G)^{-1} \right\}_{p^o} \quad (55)$$

Finally, the optimal values of the observational weights will be: $P_i = P^o_i + \Delta P_i$ (56)

4. NUMERICAL EXAMPLE

The schematic two dimensional trilateration network is used to evaluate the two different solution methods. This network comprises of seven points P1 – P7 with approximate coordinates as shown in Figure 1. The approximate coordinates of the net points are listed in Table 1.

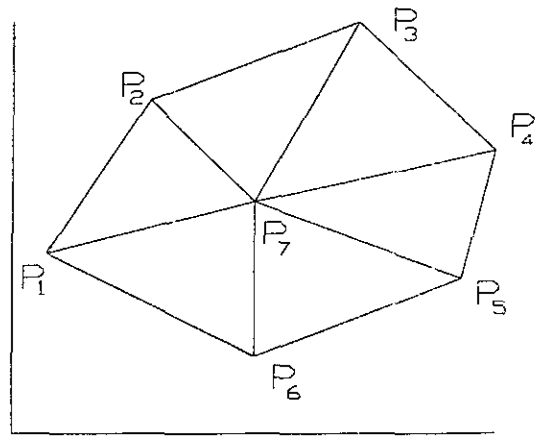

Fig. 1 Hexagon Figure (horizontal network) with a central point

Table 1 The approximate coordinates of the netpoints

Point	X (m)	Y (m)
1	100.00	700.00
2	400.00	1300.0
3	1000.0	1600.0
4	1400.0	1100.0
5	1300.0	600.00
6	700.00	300.00
7	700.00	900.00

5. RESULTS AND DISCUSSIONS

In this research, the simulated case study is solved for the free network concept. The criteria cofactor matrix for both solution methods will be the unit matrix, which ensures the properties of homogeneity and isotropy. The realization of the results will be performed for a two dimensional trilateration network (case study).

In this study, the SOD problem are solved using two different methods, namely: the least squares technique and the non- linear programming method.

The obtained variance covariance matrix of the estimated coordinates resulted from the SOD problem solution using the least squares technique, that ensures the given criterion matrix was computed as follows:

$\hat{\sigma}_{\circ} = 10^{-5} *$

Columns 1 through 7

0.1233	0.0139	0.0149	-0.0056	-0.0206	-0.0174	-0.0504
0.0139	0.1015	0.0130	-0.0352	-0.0551	-0.0250	-0.0178
0.0149	0.0130	0.1190	-0.0360	-0.0295	-0.0158	-0.0551
-0.0056	-0.0352	-0.0360	0.1089	0.0025	0.0123	0.0056
-0.0206	-0.0551	-0.0295	0.0025	0.1008	0.0045	-0.0120
-0.0174	-0.0250	-0.0158	0.0123	0.0045	0.1321	0.0309
-0.0504	-0.0178	-0.0551	0.0056	-0.0120	0.0309	0.1330
-0.0214	0.0119	0.0193	-0.0108	0.0043	0.0022	0.0203
-0.0528	0.0219	-0.0333	0.0189	-0.0167	0.0079	0.0202
0.0008	-0.0015	0.0292	-0.0235	0.0354	-0.0595	-0.0231
-0.0016	0.0069	-0.0105	0.0224	0.0126	-0.0211	-0.0308
0.0231	-0.0123	0.0078	-0.0464	-0.0048	-0.0530	-0.0188
-0.0129	0.0171	-0.0055	-0.0078	-0.0347	0.0110	-0.0049
0.0066	-0.0394	-0.0174	-0.0054	0.0130	-0.0091	0.0029

Columns 8 through 14

-0.0214	-0.0528	0.0008	-0.0016	0.0231	-0.0129	0.0066
0.0119	0.0219	-0.0015	0.0069	-0.0123	0.0171	-0.0394
0.0193	-0.0333	0.0292	-0.0105	0.0078	-0.0055	-0.0174
-0.0108	0.0189	-0.0235	0.0224	-0.0464	-0.0078	-0.0054
0.0043	-0.0167	0.0354	0.0126	-0.0048	-0.0347	0.0130
0.0022	0.0079	-0.0595	-0.0211	-0.0530	0.0110	-0.0091
0.0203	0.0202	-0.0231	-0.0308	-0.0188	-0.0049	0.0029
0.0893	0.0066	-0.0289	-0.0412	-0.0373	0.0122	-0.0263
0.0066	0.1154	-0.0279	-0.0274	-0.0183	-0.0054	-0.0090
-0.0289	-0.0279	0.1136	0.0034	0.0126	-0.0179	-0.0128
-0.0412	-0.0274	0.0034	0.0851	0.0167	-0.0274	0.0129
-0.0373	-0.0183	0.0126	0.0167	0.1373	-0.0056	-0.0008
0.0122	-0.0054	-0.0179	-0.0274	-0.0056	0.0909	-0.0090
-0.0263	-0.0090	-0.0128	0.0129	-0.0008	-0.0090	0.0938

σ

in which

Trace (Tr_1) = $1.5439 * 10^{-5}$

The obtained variance covariance matrix of the estimated coordinates resulted from the SOD problem solution using the non linear programming method, that ensures the given criterion matrix was computed also as follows:

$\hat{\sigma}_{\circ} = 10^{6}$

Columns 1 through 7

0.7634	0.1360	0.0494	-0.1256	-0.1530	-0.1353	-0.2429
0.1360	0.3672	0.1271	-0.1750	-0.2411	-0.1438	-0.0886
0.0494	0.1271	0.4282	-0.1512	-0.1853	-0.2209	-0.2241
-0.1256	-0.1750	-0.1512	0.5201	0.0126	0.0470	0.0489
-0.1530	-0.2411	-0.1853	0.0126	0.5332	0.0702	-0.0841
-0.1353	-0.1438	-0.2209	0.0470	0.0702	0.6842	0.2795
-0.2429	-0.0886	-0.2241	0.0489	-0.0841	0.2795	0.5895
-0.1081	0.0654	0.0915	-0.0422	0.0406	-0.0633	0.0697
-0.3080	0.0734	-0.1033	0.1114	-0.0623	0.0613	0.0988
0.0077	0.0020	0.1113	-0.0784	0.1631	-0.2885	-0.1885
-0.0678	-0.0347	-0.0264	0.1056	0.0722	-0.0578	-0.1162
0.2076	-0.0854	0.0696	-0.2444	-0.0440	-0.2419	-0.1229
-0.0411	0.0278	0.0615	-0.0016	-0.1206	0.0030	-0.0209
0.0175	-0.0305	-0.0275	-0.0272	-0.0014	0.0063	0.0019

Columns 8 through 14

-0.1081	-0.3080	0.0077	-0.0678	0.2076	-0.0411	0.0175
0.0654	0.0734	0.0020	-0.0347	-0.0854	0.0278	-0.0305
0.0915	-0.1033	0.1113	-0.0264	0.0696	0.0615	-0.0275
-0.0422	0.1114	-0.0784	0.1056	-0.2444	-0.0016	-0.0272
0.0406	-0.0623	0.1631	0.0722	-0.0440	-0.1206	-0.0014
-0.0633	0.0613	-0.2885	-0.0578	-0.2419	0.0030	0.0063
0.0697	0.0988	-0.1885	-0.1162	-0.1229	-0.0209	0.0019
0.4668	0.0605	-0.1592	-0.1721	-0.1859	0.0179	-0.0816
0.0605	0.5993	-0.0993	-0.1784	-0.1819	-0.0461	-0.0253
-0.1592	-0.0993	0.4864	0.0552	0.0535	-0.0496	-0.0158
-0.1721	-0.1784	0.0552	0.3324	0.0527	-0.0158	0.0513
-0.1859	-0.1819	0.0535	0.0527	0.7353	0.0189	-0.0313
0.0179	-0.0461	-0.0496	-0.0158	0.0189	0.1830	-0.0165
-0.0816	-0.0253	-0.0158	0.0513	-0.0313	-0.0165	0.1801

σ

in which

Trace (Tr_2) = $6.8693 * 10^{-6}$

An insight into both obtained variance covariance matrices of the estimated coordinates resulting from the SOD problem solution using non-linear programming method and least squares technique shows that the non-linear programming method yields the best results of precision ($Tr_1 = 6.8693 * 10^{-6}$), while the least squares technique give the worst results ($Tr_2 = 1.5439 * 10^{-5}$)

The maximum average improvement in precision is about 125 percent when the non-linear programming method is used for solving the SOD problem instead of the least squares technique.

The reproduced cofactor matrix, which was obtained from the SOD problem solution using the non-linear programming, satisfies the required criteria, while the reproduced cofactor matrix, which was obtained from the SOD problem solution using the least square technique, could not satisfy the required criteria.

6.CONCLUSIONS:

The main objective of the present research is to *investigate the statistical and practical validity* of the non linear programming method, as applied for solving the SOD problem.

For the simulated case study, both least squares technique, as a conventional method, and non linear programming method have been applied.

Based on the obtained results, the following conclusions can be drawn:

- The obtained results show the validity of the non-linear programming in solving the optimization problems of geodetic networks.
- The obtained variance covariance matrix of the estimated coordinates resulting from the solution using the non-linear programming method, ensures the requirements.
- The obtained results showed a significant improvement in the precision when the non-linear programming method is used for solving the SOD problem instead of the least squares technique. Therefore, the non-linear programming method is recommended to be used in the solution of the optimization problems.

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