ANALYSIS OF STRESS AROUND A HOLE SUBJECTED TO BIAXIAL LOADING CONDITION

تحليل الأجهادات حول الثقوب تحت تأثير الحالات المختلفه للأحمال المزدوجة

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ملخص البحث

يهتم هذا البحث بتحليل الاجهادات المركزه حول محيط الثقوب التي لاغنىعنها في المكونات الهندسيه و على سبيل المثال الالواح المعدنيه المحددة الأبعاد او الغير محددة الابعاد.

و على ذلك فقد تم تقديم طريقه موسعه لتحليل الاجهادات حول الثقوب باقتراح وتطوير معادلات تحليليه لتقدير الاجهادات وكذلك معامل تركيز هذه الاجهادات حول محيط الثقوب تحت تأثير الاجهادات الخارجيه الثنائية الاتجاهات وكذلك المنفردة في احد الاتجاهات الرئيسيه . بالإضافه الى ذلك فقد لوحظ ان وجود الإجهادات المتبقيه حول الثقوب بعد رفع الاحمال الخارجيه لها تأثير يؤدي الى زيادة معاناة معدن المكون الهندسي وتقليل مقاومته للإجهادات الخارجيه. وقد تم استنتاج معادلات تسمح بدراسة مقدار الإجهادات المتبقيه وتحديد معامل تركيزها حول محيط الثقوب حتى يمكن تقدير مدى مقاومة معدن المكون الهندسي للإجهادات الخارجيه الواقعه عليها اثناء التشغيل والتحميل .

وقد استخدمت المعادلات المقترحه في تقدير معامل تركيز الإجهادات بيانيا على مدى يسمح بتغيير مقدار معامل الاجهادات الخارجيه الثنائية في الاتجاهات المتعامده السالبه او الموجبه وكذلك تغيير معامل ابعاد الالواح المعدنيه المحتويه على الثقوب.

ولتأكيد فعالية هذه الطريقه التحليليه المقترحه تم معمليا استخدام طريقة التحليل الضوئي الانعكاسي لتقدير معامل تركيز الإجهادات المتبقيه عمليا ومقارنة النتائج المعمليه بالنتائج البيانيه المقترحه واظهرت المقارنه توافق مقبول.

ABSTRACT

This work deals with the development of an analytical formula that may be useful to characterize the stress distribution around discontinuities. Hence to satisfy the original problem of stress distribution as well as residual stress that remained around the boundary of holes in plates.

The method of analysis presented here involves with perturbation in stress concentration factor that defines the stress distribution concerning applied stress. In addition the perturbation in the stress distribution appears due to the presence of residual stress, that remaines after creating the hole and removing all the external forces.

A numerical analysis first presented to satisfy the general solution of stress distribution around a hole in plates for a variety of biaxial loading conditions. The stress distributions are then determine for variable values of rectangular coordinates representing the width to length ratio of the finite plate b_x/a_y (b_x nondimensionalized concerning a_y).

Furthermore, the proposed formula successfully employed to solve the problem of stress distribution around a hole in infinite plates under biaxial state of stress. Moreover, numerical results presented over a range of biaxial loading conditions for a variety of nondimensionalized factor, r/a_r (a_r concerning the hole radius).

Comparison of the results indicates excellent agreement with the known solution.

KEYWORDS

Stress distribution, biaxial loading, residual stress, boundary of the hole, stress concentration factor .

INTRODUCTION

Interaction between such hole and the loading condition strongly effects the gradient of stress distribution and related phenomena. The account of this phenomenon is of prime importance for accurate evaluation of stress resistance of the material of a structure containing a circular hole. While a number of techniques attempted to predict the stress distribution field for uniaxial loading and symmetric biaxial loading conditions [1-4], the most commonly used technique for data analysis in plates containing a hole based on the finite element technique[5-7]

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Analytical solution for stress distributions around the boundary of a hole subjected to a variety of biaxial loading conditions in plates is a problem that has remained unsolved yet. At present, the only way out is by stress function reported in [8] which can prove stress distribution in a thin, infinite plate with a circular hole subjected to uniaxial tensile loading condition only.

Furthermore, stress distribution around the boundary of a hole in finite plates subjected to a variety of biaxial loading conditions turns out to be much complicated than expected. However, under this type of loading conditions the stress functions reported in the available literature is insufficient to solve the problem[9].

Considering the shearing operations, such as that in piercing or drilling, lead to residual stresses that remain with the manufactured part after it deformed [10]. Moreover, residual stress may remain around the boundary of a hole subjected to a state of loading conditions after removing all external forces.

The residual stresses on the surface and in interior of a metal part considered to be undesirable because they lower the fracture strength of the part. The residual stress can also lead to stress cracking or stress corrosion cracking over a time [11-14]. Most of the available literature concerned with the measurement of residual stress by the hole-drilling technique [15 16].

Therefore, it is worthwhile to formulate a stress equation that may satisfy the rule of this type of loading conditions. This formula may take into consideration the effects of residual stress remained after removing all the external forces

FORMULATION OF THE PROBLEM

It was essentially evident that the selection of a stress component for this particular problem is difficult since non of the available stress functions is satisfactory. To overcome this difficulty, a method of superposition has commonly been used which employs two different stress components.

The stress equation may be adapted to represent the stress distribution when the plate is in a condition of uniform biaxial loading, considering the effect of residual stress that remained after removing all the external forces.

1-Several alternatives have proposed to select a mathematical equation to satisfy the condition of stress distribution around the boundary of a circular hole subjected to biaxial loading conditions Therefor, the first stress component may be selected as follows

$$\sigma_{H} = \sigma_{app}\left\{\left(1+\frac{k-2a_{xy}}{3}\right)+2\left(1-\frac{k+a_{xy}}{2}\right)\cos 2\theta\right\} \quad (1)$$

A derivation of this equation included in Appendix-I

Where k biaxial loading ratio ($k=\sigma_2/\sigma_1$).

σ_H stress distribution around a hole

σ_{app} the applied stress

a_{xy} finite dimension ratio ($a_{xy}=b_y/a_x$)

2- The second stress component must have associated stress that satisfies the stress remaining within the body around the boundary of the hole after removing the external forces Thus, stress component reported in appendix-II, can effectively be employed to estimate the residual stress as follows:

$$\sigma_{RH} = \sigma_{app}\left(\frac{(1-\mu k)}{(1+\mu)}\right)\left(\frac{a_r - r}{a_r}\right)\cos 2\theta \quad (2)$$

A derivation of this equation included in Appendix-II.

Where $r\ \theta$ polar coordinates system.

a_r radius of the hole.

r radial distance from the edge of the hole.

μ material Poisson's ratio.

k biaxial loading ratio.

Thus, the required equation for the original problem obtained by superposition of the first and second stress component. Towards this end one may assume the follow.

$$\sigma_\theta = \sigma_{app}\left\{\left(1+\frac{k-2a_{xx}}{3}\right)+\left(2\left(1-\frac{k+a_{xx}}{2}\right)+\left(\frac{(1-\mu k)}{(1+\mu)}\right)\left(\frac{a_r-r}{a_r}\right)\right)\cos 2\theta\right\} \quad (3)$$

This equation gives the stress distribution satisfying the biaxial loading conditions when the plates subjected to tension-tension, tension-compression or compression-compression, taking into account the corresponding residual stress values remained after removing all the external forces.

Moreover, the required equation can be written as follows to satisfy the original problem of determination of stress distribution around a hole in an infinite plate.

$$\sigma_\theta = \sigma_{app}\left\{\left(1+\frac{k}{3}\right)+\left(2\left(1-\frac{k}{2}\right)+\left(\frac{(1-\mu k)}{(1+\mu)}\right)\left(\frac{a_r-r}{a_r}\right)\right)\cos 2\theta\right\} \quad (4)$$

Consequently, the full-field stress distribution around the boundary of a hole for different biaxial loading conditions can be computed by employing the proposed equations which satisfy the original problem in finite as well as infinite plates.

EXPERIMENTAL VERIFICATIONS

To examine the implication of the proposed equation, one of existing experimental method of analysis would consider. Thus, photoelastic coating technique can apply to verify the analytical results [17].

Specimens photographed to record the fringe patterns that remained after removing all the external forces. Fig. 8 shows isochromatic fringe patterns that remained after creating the hole and removing all the external forces. These fringe patterns corresponding to the residual stress remained around the boundary of the hole.

The residual stress distributions around the circumference of the holes were determine experimentally by photoelastic coating technique. Stresses are related to birefringence by stress optic law [17]

$$\sigma_1 - \sigma_2 = \left(\frac{N}{2h}\right)\left(\frac{E}{(1+\mu)}\right)\left(\frac{\lambda}{C}\right) \quad (5)$$

Isochromatic fringe data corresponding to non-loading conditions were then substitute into equation (5) to experimentally determine the residual stress around the boundary of the holes

NUMERICAL RESULTS AND DISCUSSION

Although the analytical results are obtained for a general combined loading conditions, for the sake of simplicity of presentation of numerical results, we consider some separate loading conditions corresponding to residual stress remained around the holes, distribution of stress in finite plates and distribution of stress around a hole in infinite plates respectively. The results are presented in the form of plots of stress concentration factors over the complete range of circular angle θ (0 to $\pi/2$).

The proposed equations (1-4) were employed to compute the stress distribution around circular holes subjected to a variety of biaxial as well as uniaxial loading conditions. Normalized stress distribution in finite plates, for different biaxial stress ratio k such as k = + also for k = - indicated in Fig. 1.

Analytical results indicate that, the change in the stress ratio k has not only affected the distribution pattern of stresses, but also has the effect of changing the amplitude and direction of the stresses.

By using the proposed equations the normalized residual stress concentration around a hole can be represented within the distance which is in the range of $a_r/2 \subseteq r \subseteq a_r$ as shown in Fig. 2. These stresses constitute the total stress distribution and the corresponding residual stresses remained around the boundary of a hole in finite plate as indicated in Figs. 3 and 4. Changes in circumference distributed stress corresponding to changes in finite dimension ratio ($a_{xy} = b_x/a_x$) and nondimensionalized factor r/a_r clearly illustrated in Fig. 4.

It is of practical interest to compare the results obtained by using the proposed equation with the stress equation solution for stress distribution in finite plate reported in [9]

$$\sigma_\theta = \sigma_{app} [(1 - 2 \cos 2\theta) - (1-2 \cos (2\theta - \pi)] \quad (6)$$

The comparison of the obtained results indicate identical result for uniaxial loading condition. On the other hand, a reveres trend seen in the results obtained by equation (6) as shown in Fig. 1.

The next step is to examine the applicability of the developed equation in solving the problem of stress distribution in infinite plates. Thereafter compare the present results with the results obtained by using the available equation in literature[8]

$$\sigma_{\theta_i} = \sigma_{app}\{ 1+ 2 \cos 2\theta \} \quad (7)$$

The application of this equation is limited to solve the problem of an infinite large plate subjected to uniaxial loading only. Comparison of the results shown in Figs. 5, 6 and 7 indicate excellent agreement for uniaxial loading condition.

Thus the results of the presented method of analysis is more general, agree with these results as reported in references [8,9] for finite as well as infinite plates. Moreover, the developed equations make it possible to predict the stress distribution in finite as well as infinite plates under friable values of biaxial stress system or uniaxial loading condition.

The results obtained clearly indicate that the presence residual stress around a hole in plates subjected to biaxial loading state of stress increases the stress by a factor of stress concentration. The primary advantages of the proposed equation is that, it provides nearly whole-field estimation of stress distribution around a hole within the distance at the neighborhood of the hole and distance that is equal to he radius of the hole.

CONCLUSIONS

1-A numerical method of estimation of the stress distribution around a circular hole in finite plates under variable biaxial state of stress presented in this investigation. The present method of analysis is being extended for more general practical problems involving stress distribution around holes in infinite plates.

2-Using the developed equations the stresses distribution around a hole can easily compute at different values of biaxial as well as uniaxial loading conditions. Furthermore, it is possible to obtain the residual stress around the boundary of a hole, however characterize the state of stress around the boundary of a hole after removing all the external forces.

3-The proposed equations provide nearly whole-field estimation of stress distribution around a hole in finite as well as infinite plates subjected to a variety of biaxial stress systems. Hence facilitate the determination of stress distribution in a simple way as being sufficiently accurate for biaxial stress system that usually exist in actual practice.

4-While the proposed equations facilitate the determination of stress distribution around the boundary of circular holes subjected to a variety of biaxial loading conditions. Yet, equations that reported in literature restricted to some limited loading conditions.

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APPENDIX-I
EQUATION FOR STRESS DISTRIBUTION SATISFYING THE BIAXIAL LOADING CONDITIONS

If a small circular hole made in the middle of a plate, the stress distribution around the hole will change. Therefore, solution may adapt to represent the stress distribution around the hole submitted to uniform biaxial loading conditions. Thus formulation procedure will depend upon a stress function expressed as the following

$\Phi^{II} = f(r)\cos 2\theta$

The general solution is:

$f(r) = (Ar^2 + Br^4 + C.1/r^2 + D)$

The stress function is therefore,

$\Phi^{II} = (Ar^2 + Br^4 + C\ 1/r^2 + D)\ .\cos 2\theta$

The corresponding stress components, have given as the following

$\sigma_r = 1/r\ \partial\Phi/\partial r + 1/r^2\ \partial^2\Phi/\partial\theta^2$

$= -(2A + 6C/r^4 + 4D/r^2) \cos 2\theta$

$\sigma_\theta = \partial^2\Phi/\partial r^2$

$= (2A + 12B\ r^2 + 6C/r^4) \cos 2\theta$ (a)

$\tau_{r\theta} = -\partial/\partial r\ (1/r\ \partial\Phi/\partial\theta)$

$= (2A + 6B\ r^2 - 6C/r^4 - 2D/r^2) \sin 2\theta$

The constants of integration are now to be determine from the condition that the edge of the hole is free from external forces

$2A + 6C/b^4 + 4D/b^2 = \sigma_{app}$
$r \subseteq b$

$2A + 6C/a^4 + 4D/a^2 = 0$
$r \supseteq a$

$2A + 6B\ b^2 - 6C/b^4 - 2D/b^2 = \sigma_{app}$
$r \subseteq b$

$2A + 6B\ a^2 - 6C/a^4 - 2D/a^2 = 0$
$r \supseteq a$

where r variable radial distance ($a \subseteq r \subseteq b$), and b radius of concentric circle.

Solving the above equations, we obtain:

$A = \sigma_{app}$ $B = 0$ $C = a^4\ \sigma_{app}$. $D = -a^2\sigma_{app}$

Substituting these constant into equation (a), we fined:

$\sigma^{II}_\theta = 2\sigma_{app}\ [\ 1 + 3(a_0/r_0)^4\ ] \cos 2\theta$ (b)

Furthermore, the stress component produced by the uniform applied stress σ_{app} on the outer boundary of a hole made in the plate can calculate as follows:

$\sigma^{I}_\theta = \sigma_{app}\ [\ 1 + (a_0/r_0)^2]$ (c)

The required solution for the original problem obtained by superposition as follows

$\sigma_{\theta\theta} = \sigma^{I}_\theta + \sigma^{II}_\theta$

$= \sigma_{app}\ \{[1 + (a_0/r_0)^2] + 2[1 + 3(a_0/r_0)^4\ ] \cos 2\theta\ \}$ (d)

The state of stress at the boundary of the hole can define by a set of limits such as k, r_I, r_{II}, a_I, a_{II} and a_{xy}. They related to each other by the following relationships:

$$(k-2a_{xy})\, r_I^2 - 3\, a_I^2 = 0$$

$$(k+a_{xy})\, r_{II}^2 + 6\, a_{II}^2 = 0 \tag{c}$$

Where k is the factor that represent the biaxial state of stress ($k=\sigma_2/\sigma_1$) that satisfy the loading condition in the valid distance near the hole. Such as that the factor a_{xy} ($=b_x/a_y$) represents the finite width ratio in finite plate.

Substituting equations(c) into equation(d), thus the required solution for the original problem obtained as follows:

$$\sigma_{\theta} = \sigma_{app}\left\{\left(1+\frac{k-2a_{xy}}{3}\right)+2\left(1-\frac{k+a_{xy}}{2}\right)\cos 2\theta\right\} \tag{1-1}$$

This equation represents the tangential stresses at the boundary of a hole made in the middle of a plate subjected to uniaxial as well as biaxial loading conditions.

APPENDIX-II

STRESS EQUATION REPRESENT THE RESIDUAL STRESS REMAINED AFTER REMOVING ALL THE EXTERNAL FORCES.

Evidently selection of a stress function that can employ to compute the residual stress. However, to satisfy the stress condition around a hole it must be depends upon the stress-strain relation for the two-dimensional state of stress.

Consider the strain in term of stress and the constant associated with it. Thus, the stress-strain relations for two-dimensional or plane state of stress expressed as.

$$\varepsilon_{x} = \left(\frac{1}{E}\right)(\sigma_{xx} - \mu\,\sigma_{yy})$$

$$\varepsilon_{y} = \left(\frac{1}{E}\right)(\sigma_{yy} - \mu\,\sigma_{xx})$$

$$\varepsilon_{x} - \varepsilon_{y} = \left(\frac{1+\mu}{E}\right)(\sigma_{xx} - \sigma_{yy})$$

$$\dot{\varepsilon}_{v} = \varepsilon_{x} - \varepsilon_{y}$$

and

$$\dot{\sigma}_{v} = \sigma_{xx} - \sigma_{yy}$$

$$\dot{\varepsilon}_{v} = \left(\frac{1+\mu}{E}\right)\dot{\sigma}_{v} \tag{a}$$

And the stress in principal direction given as

$$\varepsilon_{1} = \left(\frac{1}{E}\right)(\sigma_{1} - \mu\,\sigma_{2}) \tag{b}$$

The normal stresses σ_{xx} and σ_{yy} can establish by using the following equation, which is apparent from an examination of Mohr's circle:

$$(\sigma_{xx} - \sigma_{yy}) = (\sigma_1 - \sigma_2)\cos 2\theta \tag{c}$$

$$\sigma_{xx}\ \alpha\ \sigma_{12}\cos 2\theta$$

Similarly

$$\varepsilon_x^{*} \;\alpha\; \varepsilon_1 \cos 2\theta \qquad \text{(d)}$$

By substituting equations(a)and(b) into equation(d), the principal strain associated with this principal strain function simplified as the following:

$$\left(\frac{1+\mu}{E}\right)\sigma_x^{*} \;\alpha\; \left(\frac{1}{E}\right)(\sigma_1 - \mu\,\sigma_2)\cos 2\theta$$

$$\left(\frac{1+\mu}{E}\right)\sigma_x^{*} = Q\left(\frac{1}{E}\right)(\sigma_1 - \mu\sigma_2)\cos 2\theta$$

$$\sigma_x^{*} = Q\left(\frac{1}{1+\mu}\right)\left(\sigma_1\left(1-\mu\frac{\sigma_2}{\sigma_1}\right)\right)\cos 2\theta$$

$$= Q\left(\frac{1}{1+\mu}\right)\sigma_1(1-\mu k)\cos 2\theta$$

$$= Q\left(\frac{1-\mu k}{1+\mu}\right)\sigma_1 \cos 2\theta \qquad \text{(e)}$$

The term Q is a constant represents correction factor. This factor Q must applied to the value of principal stresses difference acting on the plate. Hence to estimate the true value of principal strain difference that will effect magnitude of the stress produces at the neighborhood of the hole boundary. This factor may express as:

$$Q = \left(\frac{a_r - r}{a_r}\right) \qquad \text{(f)}$$

It is evident from an inspection of equation(f) and equation(e) that the effective stress may express as.

$$\sigma_x^{*} = \left(\frac{a_r - r}{a_r}\right)\left(\frac{1-\mu k}{1+\mu}\right)\sigma_1 \cos 2\theta \qquad \text{(j)}$$

where, σ_1 consider to be the applied stress in the principle direction. Thus $\sigma_1 = \sigma_{app}$.

When the stress difference in the stress field reaches the material elastic limit of the plate the residual-stress initially introduces resulting during the formation of the hole. Hence the remained stress redistributed after removing all the external forces.

In general the required solution for a continuous distribution of the residual stresses at a considerable distance which is in the range of $a_r/2 \subseteq r \subseteq a_r$ can be approximately determine as.

$$\sigma_{eff} = \sigma_{app}\left(\frac{a_r - r}{a_r}\right)\left(\frac{1-\mu k}{1+\mu}\right)\cos 2\theta \qquad \text{(II-1)}$$

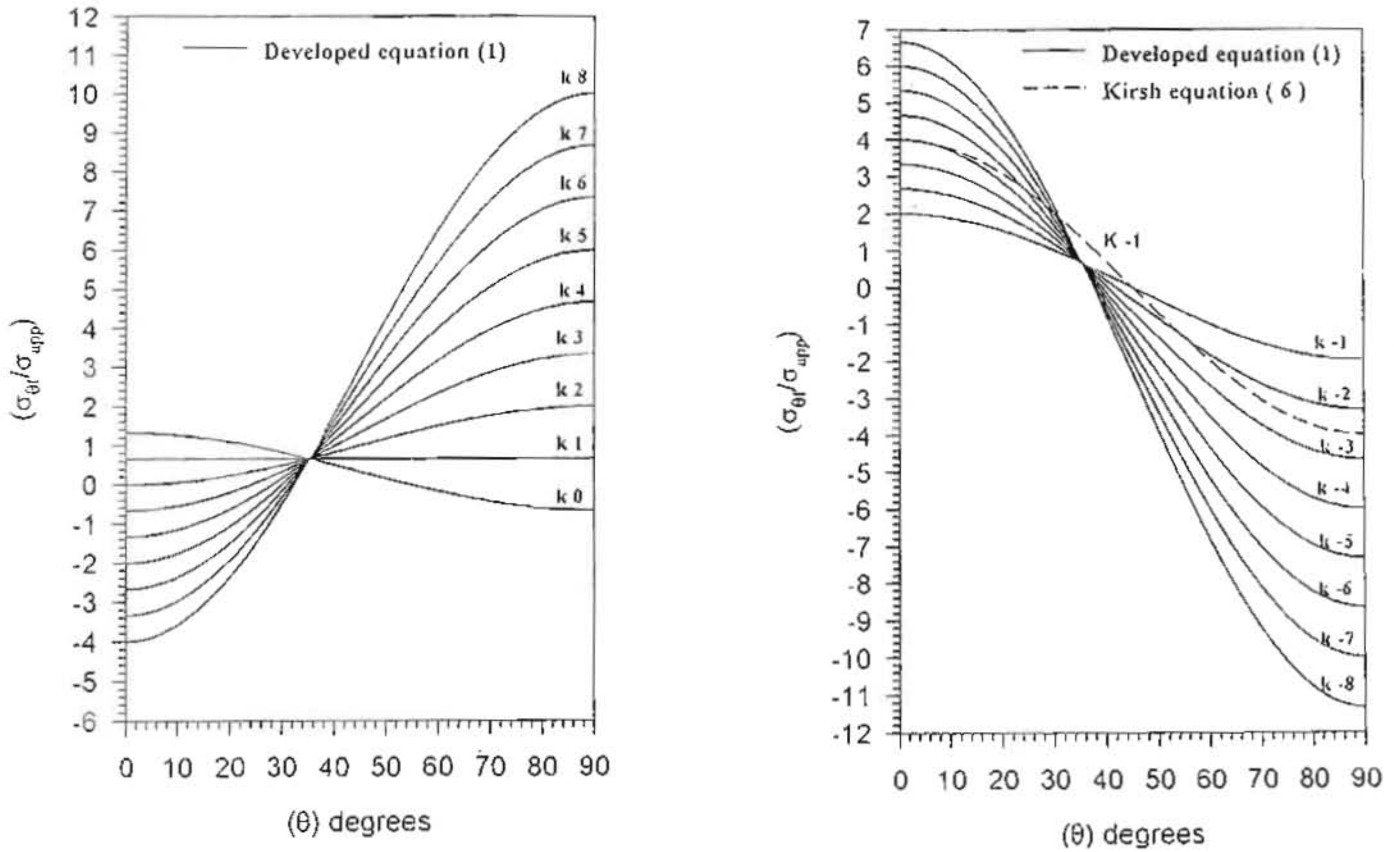


Fig. 1. Normalized stress distribution around the boundary of a hole in finite plate for different biaxial ratio k.

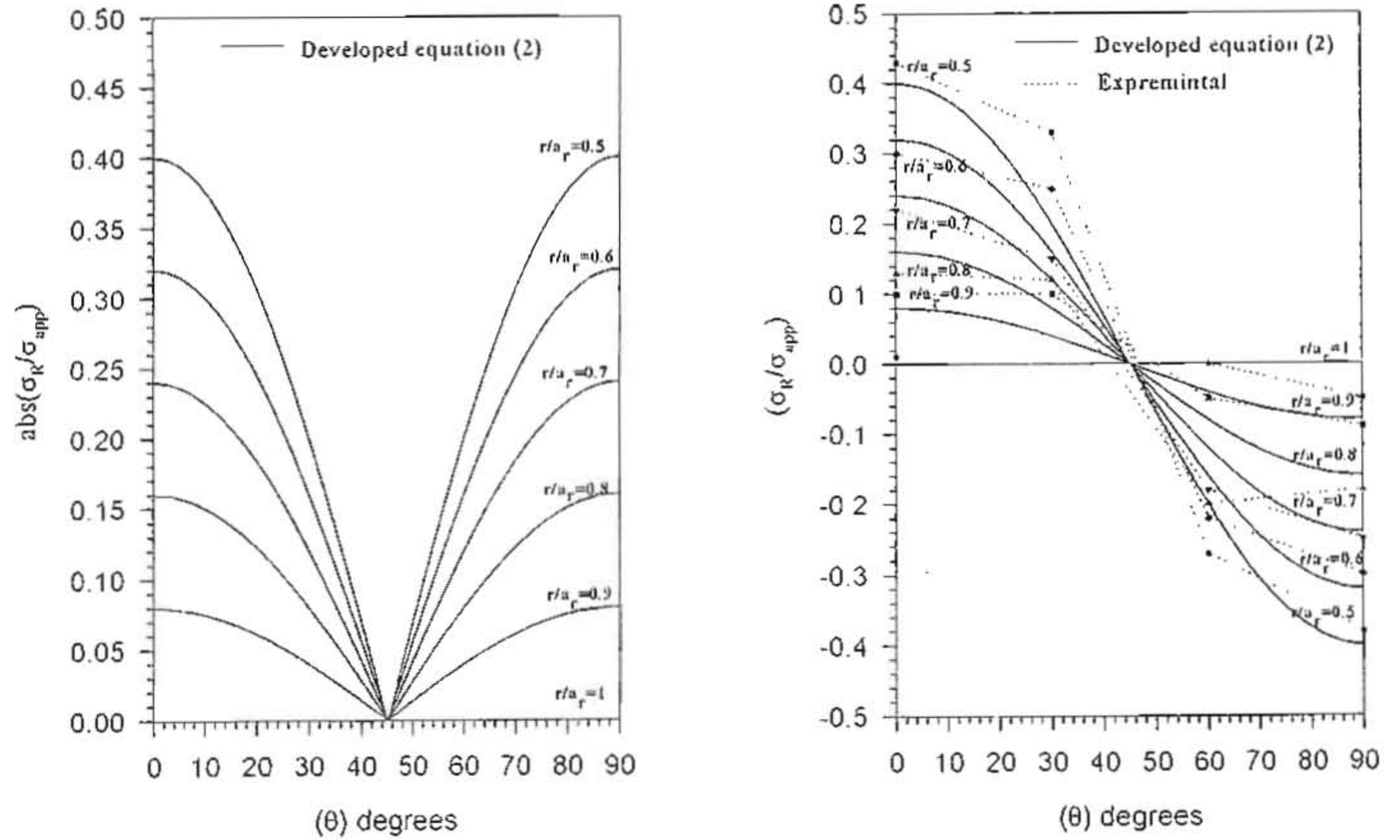


Fig. 2. Distribution of stress concentration factor that represent the residual stress remained around a hole after removing all the external forces.

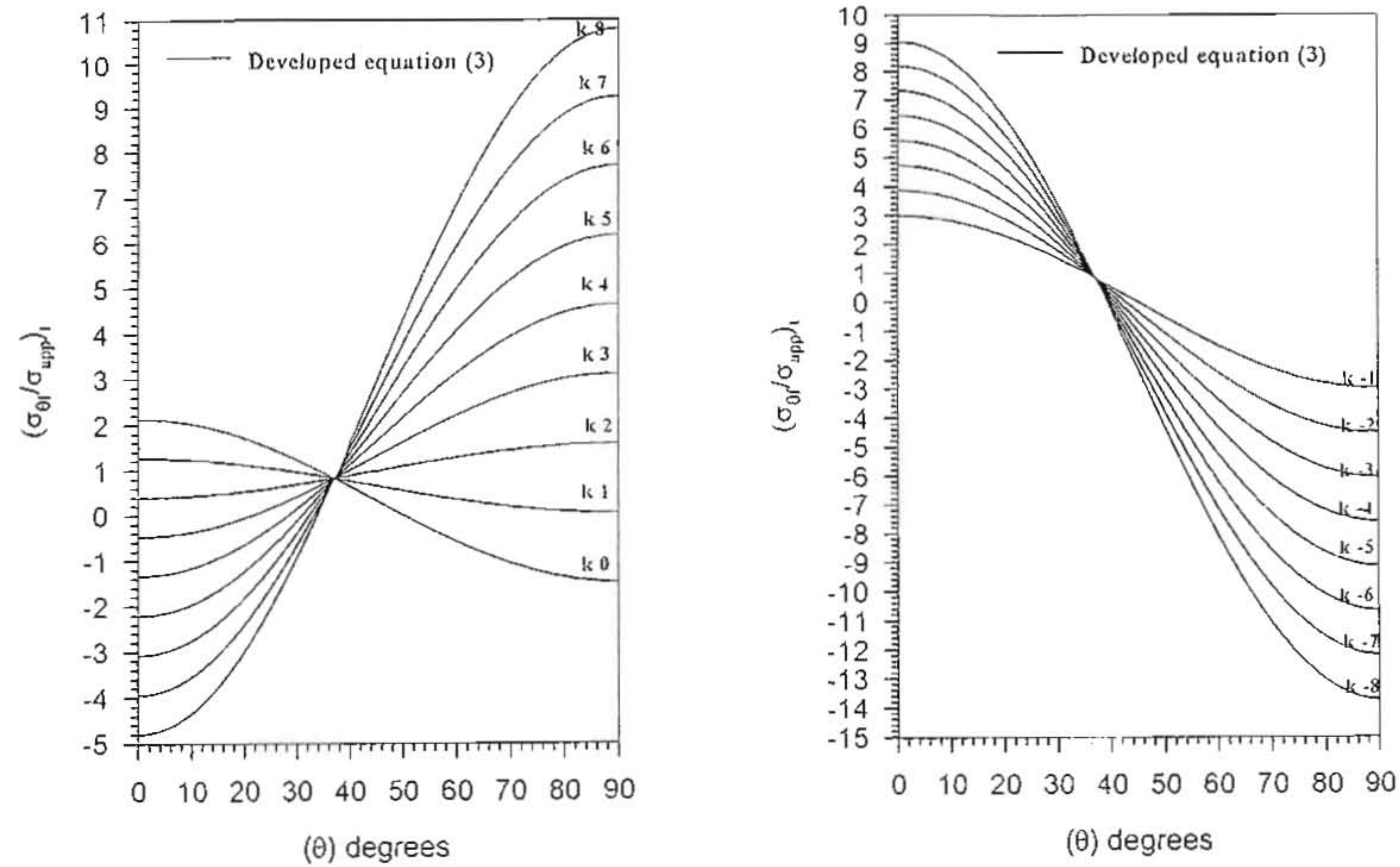
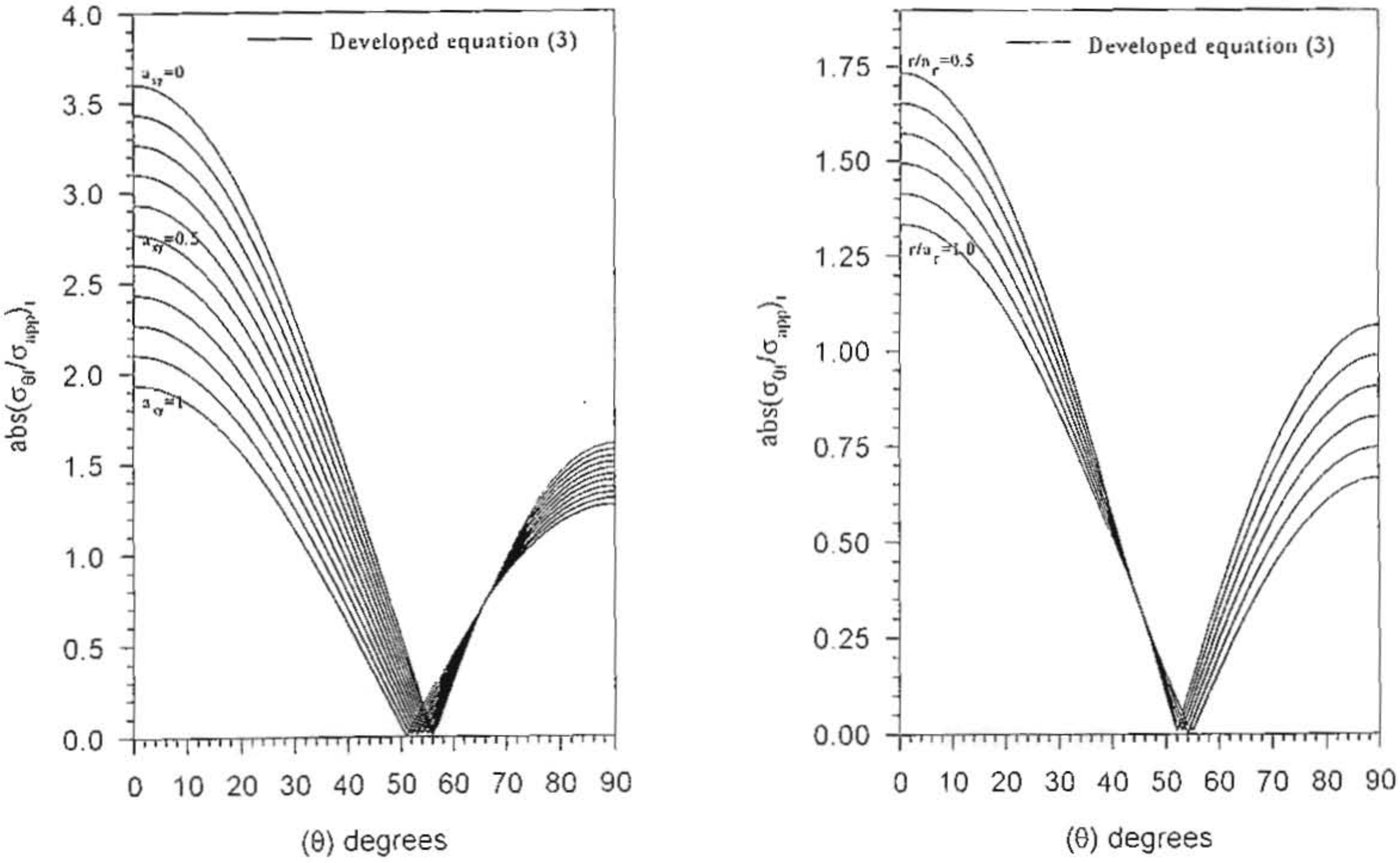


Fig.3. Distribution of the total stress and the corresponding residual stress remained around a hole in finite plate for different stress ratio k.



(a) (b)

Fig. 4. Changes in circumferential distributed stress corresponding to (a) changes in finite dimension ratio $a_{\sigma y}$ and (b) changes in nondimensionalized factor r/a_r.

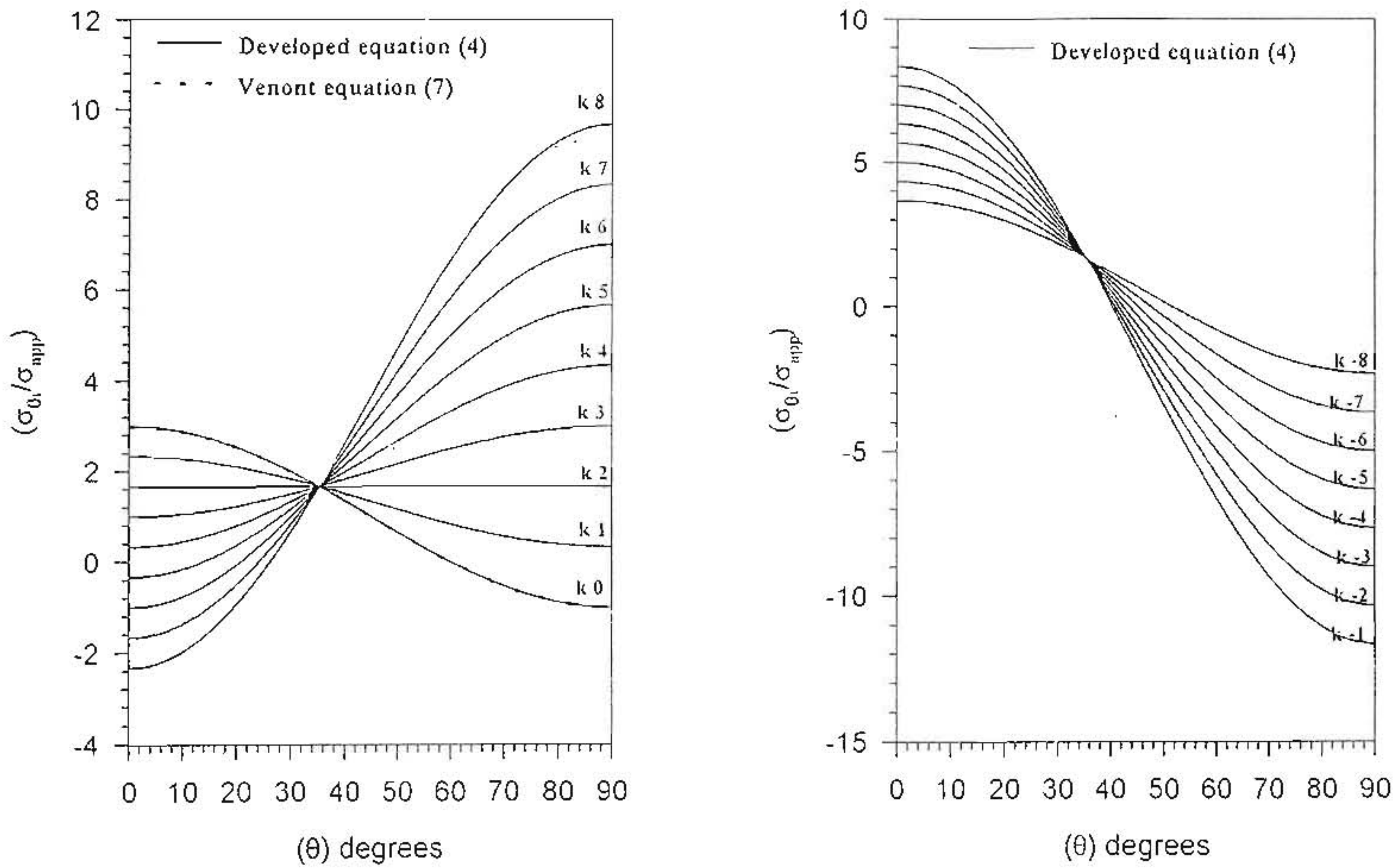


Fig. 5. Changes in stress distribution around a hole in an infinite plate for different biaxial ratio k.

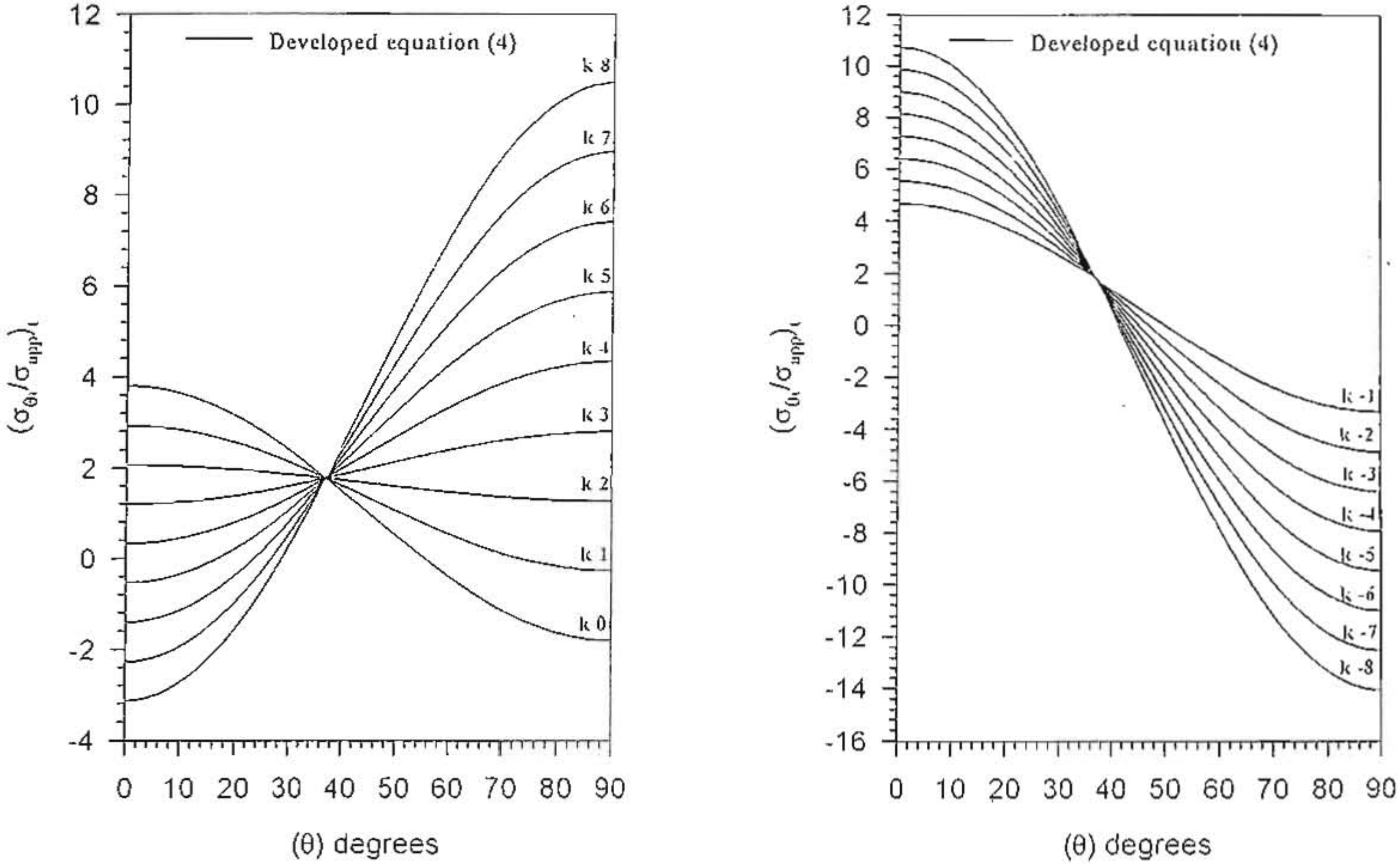


Fig. 6. Normalized stress distribution in an infinite plate for different biaxial ratio k including the corresponding residual stress.

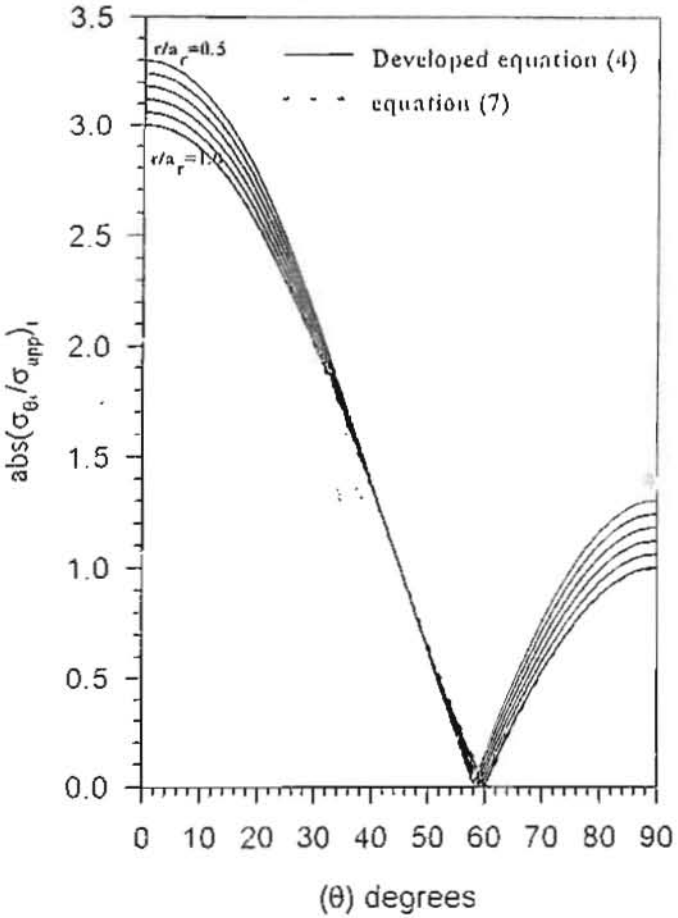


Fig. 7. Changes in circumferential distributed stress corresponding to the changes in the residual stress concentration in the range of $a_r/2 \subseteq r \subseteq a_r$.

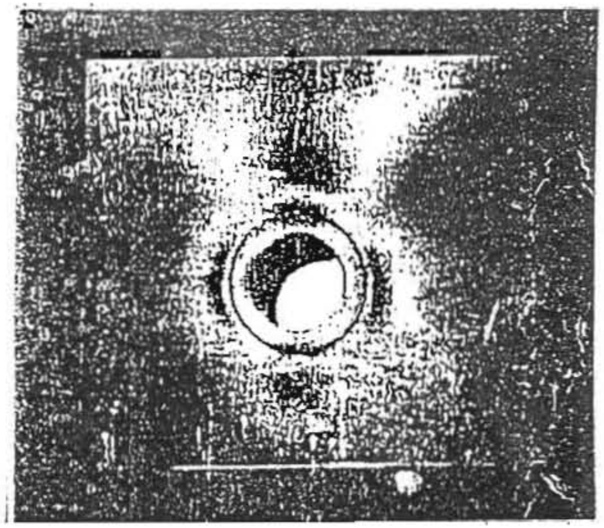


Fig. 8. Isochromatic fringe patterns that remained around a hole after removing all the external forces.