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VISCOELASTIC BEHAVIOUR OF PVC PIPES UNDER INTERNAL PRESSURE

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ABSTRACT

This paper is interested in introducing an analysis of the behavior of the PVC pipes under internal pressure, tension and bending. Analytical methods are given in the form of equations to provide a method for predicting the radial displacement of the pipe by using the elastic-viscoelastic correspondence principle. Moreover, results for radial displacement and strain ratio of pipes under combined loads are presented and compared.

يهتم هذا البحث بدراسة السلوك المرن اللزج لمواسير من كلوريد الفينيل المتعدد تحت تأثير الضغط و كذلك الشد والثني. ويتضمن طريقة تحليلية بالمعادلات للحصول على طريقة للتنبؤ بالإزاحة النصف قطرية للإسطوانات باستخدام مبدأ المرونة-المرونة اللزوجة المتوافق. النتائج التي تم الحصول عليها للإزاحة النصف قطرية و نسب الإنفعال للمواسير تحت تأثير الإجهادات المختلفة تم ادراجها خلال البحث.

Keywords: bending, pipe, pressure, radial displacement, tension, viscoelastic

1. INTRODUCTION

Viscoelastic response is often used as a probe in polymer science, since it is sensitive to the material's chemistry and microstructure. The linear viscoelasticity can be incorporated into the general theory of mechanics of materials, so that structures containing viscoelastic components can be designed and analyzed. On the other hand, the viscoelastic behaviour is similarly found in other materials such as wood, human tissue and solid rocket propellants, to name a few. In these cases, the material's viscoelasticity must be taken into account in the simulation.

Drozdov [1] derived a constitute model for the nonlinear viscoelastic behaviour of polymers under isothermal loading. The model is utilized to calculate stresses and displacements built-up in a conic pipe under the action of torques applied to its edges. The effect of geometrical parameters of the pipe and the loading history on stresses and displacements is studied numerically. Stress relaxation tests on a pipe grade PE80 medium density polyethylene (BP Chemicals) have been carried out to provide a thermo-viscoelastic model for use in calculating thermal stress development and relaxation during pipe manufacture [2]. Whereas, the three linear viscoelastic properties of an Ashland neat urethane adhesive were measured [3].

It is proved [4] that two different decompositions of strain may be assigned to every linear viscoelastic solid, especially for the so-called three-parameter solids. Further, a constitutive law has been proposed [5] for the response of a porous viscoelastic solid under 3-D triaxial stress states.

An analytical solution has been obtained for the multilayered viscoelastic thick cylindrical shell for internal pressure and thermal loads, Renganathan et al [6]. The design hoop strain of a pipe loaded by internal pressure and bending moment under creep has been determined [7]. Ozkan and Mohareb [8] have studied pipes subjected to internal pressure and axial force. A comparison of experimental and finite element results has been done.

In addition, elements of steam and gas turbines, jet engines, steam boilers, rockets, oil and gas processing plants are widely used of thin-walled pipes. Koltunov and Troyanovskii [9] had studied the state of stress of a long thick viscoelastic cylinder enclosed in an elastic shell; as well a numerical example has been examined. While, [10] had proposed the stress distribution in thick-walled tube (viscoelastic body) under external pressure and axial tension. Chao et al. [11] had used analytic continuation associated with the successive alternative technique and Laplace transformation, a thermoviscoelastic solution of a three-phase cylinder has been found. Also some typical examples have been discussed and the results have been found to agree well with the thermoelastic solution. Lai et al. [12] tried to find the radial displacement of a freezing tunnel in cold region which has a viscoelastic domain. The formulae of Laplace transformation with respect to time for frost forces of the lining-frozen surrounding rock-unfrozen surrounding rock system is presented from the elastic-viscoelastic corresponding principle.

The aim of this paper is to present a theoretical analysis for the PVC pipe under internal pressure as well as bending and tension throughout the elastic-viscoelastic correspondence principle.

2. THEORETICAL ANALYSIS

Most polymers do not exhibit the unrestricted flow permitted by the Maxwell model, although it would be a reasonable model for bouncing putty or warm tar. The Voigt model, a spring-dashpot parallel arrangement, is placed in series with a single spring, the overall model is called the standard linear solid (S.L.S) or also the "Zener solid"[13, 14]. This model is convincing for describing creep, the time-dependent strain under a given constant stress. The spring compliance (C) is the reciprocal of the spring stiffness. The compliance of the spring in the Voigt part of the model is denoted by ($C_v = 1/K_v$) (see Fig. 1) and that of the other spring is ($C_g = 1/K_g$). The viscosity of the dashpot is (η).

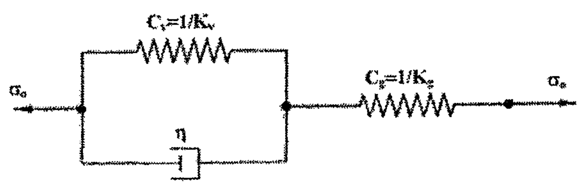

Fig. 1 The standard linear solid.

In the Voigt part of the model, the strain (ε_v) in both the spring and the dashpot is identical, while the total transmitted stress is the sum of the stress in the spring plus that in the dashpot.[15-17]

2.1 The Constitutive Equation

The constitutive equation of the standard linear solid Fig. 1 is used to be given as [15, 17]:

$$\sigma + \frac{\eta}{K_g + K_v}\dot{\sigma} = \frac{K_g K_v}{K_g + K_v}\varepsilon + \frac{\eta K_g}{K_g + K_v}\dot{\varepsilon} \qquad (1)$$

And for a constant stress (i.e. $\dot{\sigma} = 0$)

$$\sigma = \frac{K_g K_v}{K_g + K_v}\varepsilon + \frac{\eta K_g}{K_g + K_v}\dot{\varepsilon} \qquad (2)$$

Which give a creep function $\varepsilon(t)$,

$$\left(at\ t = 0\ \varepsilon_g = \frac{\sigma_o}{K_g}; \varepsilon_v = 0 \right)$$

Take $C_g = \frac{1}{k_g}$, and $C_v = \frac{1}{k_v}$ (3)

Then;

$$\frac{1}{H} = \frac{1}{K_g} + \frac{1}{K_v} = C_g + C_v \qquad (4)$$

And

$$n = \frac{\eta}{K_g + K_v}, \qquad \tau = \eta C_v \qquad (5)$$

The unit of τ is time, and it will be seen that this ratio is a useful measure of the response time of the material's viscoelastic response. Therefore,

$$\varepsilon(t) = \frac{\sigma_o}{K_g}\left[\frac{K_g}{H} + \left(1 - \frac{K_g}{H}\right)e^{-\frac{H}{nK_g}t}\right] \qquad (6)$$

And the compliance C, will be

$$\frac{\varepsilon(t)}{\sigma_o} = C_{crp}(t) = \left[\frac{1}{H} + \left(\frac{1}{K_g} - \frac{1}{H}\right)e^{-\frac{H}{nK_g}t}\right] \qquad (7)$$

Or Eq. 7 can be written as

$$C_{crp}(t) = C_g + C_v\left(1 - e^{-\frac{t}{\tau}}\right) \qquad (8)$$

For the standard linear solid, it is found that $C_r = C_g + C_v$; is an asymptotic curve, then [18]

$$C_{crp}(t) = C_g + \left(C_r - C_g\right)\left(1 - e^{-\frac{t}{\tau}}\right) \qquad (9)$$

Or

$$C_{crp}(t) = C_r + \left(C_g - C_r\right)e^{-\frac{t}{\tau}} \qquad (10)$$

2.2 The Viscoelastic Pipe

Consider a long rectilinear thin-walled pipe of circular cross section under internal (p) pressure. Denote the radius of the median surface by (r) and wall thickness, which is constant, by (b), the hoop strain (ε_θ) can be given as;

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_z\right) \qquad (11)$$

$$= \frac{\Pr}{bE}\left(1 - \frac{\nu}{2}\right) \qquad (12)$$

Therefore the redial expansion (δ_r) is then

$$\delta_r = r\varepsilon_\theta = \frac{\Pr^2}{bE}\left(1 - \frac{\nu}{2}\right) \qquad (13)$$

For isotopic materials the shear modulus G and the Bulk modulus K are related to Young's modulus E and Poisson ratio ν as,

$$E = \frac{9GK}{3K + G}, \qquad \nu = \frac{3K - 2G}{6K + 2G} \qquad (14)$$

Or in s-domain as

$$E(s) = \frac{9G(s)K(s)}{3K(s) + G(s)} \qquad (15)$$

$$\nu(s) = \frac{3K(s) - 2G(s)}{6K(s) + 2G(s)} \qquad (16)$$

The associated solution to the radial expansion of the closed end cylinder, in the Laplace plane will be, using Eq. 13:

$$\bar{\delta}_r = \frac{\bar{P}r^2}{b}\left(\frac{1}{E(s)} - \frac{\nu(s)}{2E(s)}\right) \tag{17}$$

Substituting with the values of E and ν (Eqs. (15) and (16)) in the pervious equation yield the following equation for the radial displacement

$$\bar{\delta}_r = \frac{\bar{P}r^2}{b}\left(\frac{1}{4}\frac{1}{G} + \frac{1}{6}\frac{1}{K}\right) \tag{18}$$

As an approximation, the shear operator for the standard linear solid model is [18]

$$G = G_r + \frac{(G_g - G_r)s}{s + \frac{1}{\tau}} \tag{19}$$

Or in s-domain is:

$$G = G_r + \frac{(G_g s - G_r s)}{s + \frac{1}{\tau}} \tag{20}$$

The bulk modulus is assumed to be a constant to a good approximation ($K=K_e$) [18]

Therefore; the radial displacement in s-domain (Eq. 18) could rewrite as

$$\bar{\delta}_r = \frac{\bar{P}r^2}{b}\left(\frac{s + \frac{1}{\tau}}{4G_r\frac{1}{\tau} + 4G_g s} \times \left(\frac{1}{s}\right) + \frac{1}{6}\frac{1}{K_e}\left(\frac{1}{s}\right)\right) \tag{21}$$

For a constant internal pressure $P(t) = P_o$,

$$\bar{P} = \left(\frac{P_o}{s}\right) \tag{22}$$

After taking the inverse of Laplace transformation, of equation Eq.(21) gives

$$\delta_r = \frac{P_o r^2}{b}\left(\frac{1}{4G_r} - \left(\frac{1}{4G_r} - \frac{1}{4G_g}\right)e^{-t/\tau_c} + \frac{1}{6}\frac{1}{K_e}\right)$$

$$\text{Where} \quad \tau_c = \tau\frac{G_g}{G_r} \tag{23}$$

Therefore the radial displacement $\delta_r(t)$, for a viscoelastic pipe under constant pressure can be written in the form

$$\delta_r(t) = \frac{r^2 P_o}{b}\left[\left(\frac{1}{4G_r} + \frac{1}{6K_e}\right) - \left(\frac{1}{4G_r} - \frac{1}{4G_g}\right)e^{-t/\tau_c}\right] \tag{24}$$

2.3 Thin-Walled Pipe under Internal Pressure and Axial Load

Consider a rectilinear thin-walled pipe with its end plates under internal pressure p and axial tensile force N. As in the previous case, the plane stress state in the median surface of the pipe is membrane and statically determinate. The stress state for the pipe yield, (Golub et al. [19])

$$\sigma_\theta = \frac{pr}{b}, \quad \sigma_z = \frac{pr}{2b} + \frac{N}{2\pi rb}, \quad \sigma_r = 0 \tag{25}$$

And the hoop strain (ε_θ) can be given as;

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{pr}{b} - \nu\left(\frac{pr}{2b} + \frac{N}{2\pi rb}\right)\right) \tag{26}$$

And the radial displacement is

$$\delta_r = \frac{pr^2}{b}\left(\frac{1}{E} - \frac{\nu}{2E}(1+n)\right) \tag{27}$$

Where

$$n = \frac{N}{\pi r^2 p} \tag{28}$$

n is a dimensionless parameter, a measure of variation in the stress state of the thin-walled pipe under internal pressure and axial load. Substituting in Eq. 27 with the values of E and ν (i.e. Eqs. (15) and (16)) yields:

$$\delta_r = \frac{pr^2}{b}\left(\frac{3+n}{18k} + \frac{3-n}{12G}\right) \tag{29}$$

The radial displacement $\delta_r(t)$, for a viscoelastic pipe under constant pressure and axial load N can be written in the form, taken in consideration Eqs. (20)-(23)

$$\delta_r(t) = \frac{r^2 P_o}{3b}\left[\left(\frac{(3-n)}{4G_r} + \frac{(3+n)}{6K_e}\right) - (3-n)\left(\frac{1}{4G_r} - \frac{1}{4G_g}\right)e^{-t/\tau_c}\right] \tag{30}$$

Where the notation is the same as in Eqs. (24) and (28)

While the longitudinal strain is

$$\varepsilon_z = \frac{1}{E}\left(\left(\frac{pr}{2b} + \frac{N}{2\pi rb}\right) - \nu\frac{pr}{b}\right) \tag{31}$$

$$\varepsilon_z = \frac{pr}{b}\left(\frac{1}{2E}(1+n) - \frac{\nu}{E}\right) \tag{32}$$

Substituting in the pervious equation with the values of E and ν (i.e. Eqs. (15) and (16)) yields:

$$\varepsilon_z = \frac{pr}{b}\left(\frac{n+3}{18k} + \frac{n}{6G}\right) \tag{33}$$

The longitudinal strain $\varepsilon_z(t)$, for a viscoelastic pipe under constant pressure and axial load N can be written in the form, taken in consideration Eqs. (20)-(23)

$$\varepsilon_z(t) = \frac{rP_o}{6b}\left[\left(\frac{n}{G_r} + \frac{(n+3)}{3K_e}\right) - n\left(\frac{1}{G_r} - \frac{1}{G_g}\right)e^{-t/\tau_c}\right] \tag{34}$$

2.4 Pressure Vessel under Internal Pressure and Bending

Consider a rectilinear thin-walled pipe with end plates under internal pressure p and bending moment M_B. It is assumed that the pipe is long and is subjected to pure bending. The plane stress state in the median surface of the pipe is membrane and statically determinate. The stress state for the pipe yield [19]

$$\sigma_\theta = \frac{pr}{b}, \quad \sigma_z = \frac{pr}{2b} + \frac{M_B}{2\pi r^2 b}, \quad \sigma_r = 0 \qquad (35)$$

And the hoop strain (ε_θ) can be given as;

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{pr}{b} - \nu\left(\frac{pr}{2b} + \frac{M_B}{2\pi r^2 b}\right)\right) \qquad (36)$$

And the radial displacement is

$$\delta_r = \frac{pr^2}{b}\left(\frac{1}{E} - \frac{\nu}{2E}(1+m)\right) \qquad (37)$$

Where

$$m = \frac{M_B}{\pi r^3 p} \qquad (38)$$

m is a dimensionless parameter, a measure of variation in the stress state of the thin-walled pipe under internal pressure and bending moment. Substituting in Eq. 33 with the values of E and ν (i.e. Eqs. (15) and (16)) yields:

$$\delta_r = \frac{pr^2}{b}\left(\frac{3+m}{18k} + \frac{3-m}{12G}\right) \qquad (39)$$

The radial displacement $\delta_r(t)$, for a viscoelastic pipe under constant pressure and bending moment M_B can be written in the form, taken in consideration Eqs. (20)-(23)

$$\delta_r(t) = \frac{r^2 P_o}{3b}\left[\left(\frac{(3-m)}{4G_r} + \frac{(3+m)}{6K_e}\right) - (3-m)\left(\frac{1}{4G_r} - \frac{1}{4G_g}\right)e^{-t/\tau_c}\right] \qquad (40)$$

Where the notation is the same as in Eqs. (24) and (28)

On the other hand the longitudinal strain is

$$\varepsilon_z = \frac{1}{E}\left(\left(\frac{pr}{2b} + \frac{M_B}{2\pi r^2 b}\right) - \nu\frac{pr}{b}\right) \qquad (41)$$

$$\varepsilon_z = \frac{pr}{b}\left(\frac{1}{2E}(1+m) - \frac{\nu}{E}\right) \qquad (42)$$

Substituting in the pervious equation with the values of E and ν (i.e. Eqs. (15) and (16)) yields:

$$\varepsilon_z = \frac{pr}{b}\left(\frac{m+3}{18k} + \frac{m}{6G}\right) \qquad (43)$$

The longitudinal strain $\varepsilon_z(t)$, for a viscoelastic pipe under constant pressure and bending moment M_B can be written in the form, taken in consideration Eqs. (20)-(23)

$$\varepsilon_z(t) = \frac{rP_o}{6b}\left[\left(\frac{m}{G_r} + \frac{(m+3)}{3K_e}\right) - m\left(\frac{1}{G_r} - \frac{1}{G_g}\right)e^{-t/\tau_c}\right] \qquad (44)$$

2.5 Pressure Vessel under Internal Pressure, Axial Load and Bending

Consider a rectilinear thin-walled pipe with end plates under internal pressure p as well as axial load N and bending moment M_B. It is assumed that the pipe is long and is subjected to pure bending. The plane stress state in the median surface of the pipe is membrane and statically determinate. The stress state for the pipe yield

$$\sigma_\theta = \frac{pr}{b}, \quad \sigma_z = \frac{pr}{2b} + \frac{N}{2\pi rb} + \frac{M_B}{2\pi r^2 b}, \quad \sigma_r = 0 \qquad (45)$$

And the hoop strain (ε_θ) can be given as;

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{pr}{b} - \nu\left(\frac{pr}{2b} + \frac{N}{2\pi rb} + \frac{M_B}{2\pi r^2 b}\right)\right) \qquad (46)$$

And the radial displacement is

$$\delta_r = \frac{pr^2}{b}\left(\frac{1}{E} - \frac{\nu}{2E}(1+n+m)\right) \qquad (47)$$

Where (n) and (m) are dimensionless parameters, a measure of variation in the stress state of the thin-walled pipe under internal pressure as well as tension and bending moment. Substituting in Eq. 47 with the values of E and ν (i.e. Eqs. (15) and (16)) yields:

$$\delta_r = \frac{pr^2}{b}\left(\frac{3+n+m}{18k} + \frac{3-n-m}{12G}\right) \qquad (48)$$

The radial displacement $\delta_r(t)$, for a viscoelastic pipe under constant pressure and axial load N can be written in the form, taken in consideration Eqs. (20)-(23)

$$\delta_r(t) = \frac{r^2 P_o}{3b}\left[\begin{array}{l}\left(\frac{(3-n-m)}{4G_r} + \frac{(3+n+m)}{6K_e}\right) \\ -(3-n-m)\left(\frac{1}{4G_r} - \frac{1}{4G_g}\right)e^{-t/\tau_c}\end{array}\right] \qquad (49)$$

Where the notation is the same as in Eqs. (24) and (28)

On the other hand the longitudinal strain is

$$\varepsilon_z = \frac{1}{E}\left(\left(\frac{pr}{2b} + \frac{N}{2\pi rb} + \frac{M_B}{2\pi r^2 b}\right) - \nu\frac{pr}{b}\right) \qquad (50)$$

or

$$\varepsilon_z = \frac{pr}{b}\left(\frac{1}{2E}(1+n+m) - \frac{\nu}{E}\right) \qquad (51)$$

Substituting in the pervious equation with the values of E and v (i.e. Eqs. (15) and (16)) yields:

$$\varepsilon_z = \frac{pr}{b}\left(\frac{m+n+3}{18k}+\frac{n+m}{6G}\right) \tag{52}$$

The longitudinal strain $\varepsilon_z(t)$, for a viscoelastic pipe under constant pressure and axial load N can be written in the form, taken in consideration Eqs. (20)-(23)

$$\varepsilon_z(t) = \frac{rP_o}{6b}\left[\begin{array}{l}\left(\frac{n+m}{G_r}+\frac{(n+m+3)}{3K_e}\right)\\ -(m+n)\left(\frac{1}{G_r}-\frac{1}{G_g}\right)e^{-t/\tau_c}\end{array}\right] \tag{53}$$

3. RESULTS AND DISCUSSION

The displacement of the thin-walled pipe is introduced in the pervious section as well as the longitudinal strain. These analyses are integrated with the material model (see Fig. 1 and Table 1) in order to predict the relationship between the process parameters and the internal pressure as well as the radial displacement.

Table 1 The material used [18]

Material	G_g MPa	G_r MPa	K_e GPa	τ sec.	°C
Polyvinyl Chloride (PVC)	800	1.67	1.33	100	75

Fig. 2 shows the results obtained for the displacement versus the time with different pressure values. These results correspond to pure internal pressure with r = 0.05 m and b= 0.005m. As expected increasing the pressure increase the displacement. Generally at the beginning the displacement is sharply increases and then almost slightly increasing in displacement with time. It was noticed that the displacement approaches a plateau within a period of time about $25\text{x}10^4$ seconds. This saturated value of displacement is important for fracture analysis.

The displacement versus time for pipe (p=5kPa and r=0.05 m) under pure internal pressure, and corresponding to different (r/b) ratio is shown in Fig. 3. As the (r/b) ratio increase, the displacement is also increased. The wall thickness of the pipe will decrease with increasing this ratio (r/b) with the same value of the pipe radius As well as, the increasing in the displacement is started sharply and then slightly increasing is observed and is reached a plateau.

In Fig. 4, the calculated displacement (Eq. 30) with time is displayed for different n-values and corresponding to internal pressure with axial load. These results correspond to the pipe with p=5kPa, r=0.05 m and b=.005m. The radial displacement of the pipe is obtained from Eq. 30. The results demonstrate similar general trends between Figs. 2 and 4; except that the values are different due to the difference in stress state. Fig. 4 further shows that as the ratio (n) increases, the radial displacement decreases significantly, which is expected. It may be noted that for pipes under tensile loading, the increasing of parameter n is contained the increasing of the axial load while the internal pressure is constant. On the other hand, the ratio of the hoop strain and the longitudinal strain for pipes under internal pressure and tension is shown in Fig. 5 with different values of n. Fig. 5 shows that as n increases, the strain ratio of the pipe decreases and reached a low constant value with n= 3.

The effect of bending moment on the radial displacement with time is displayed in Fig. 6. These results correspond to bending with internal pressure of pipe with p=10kPa, r=0.05m and b=.005m. Fig. 6 further show that as m increases, the radial displacement of the pipe decreases, increasing m from 1 to 2, will decrease the displacement to have of its initial value. Hence, it can be concluded that the applying bending moment with constant internal pressure can be employed for small radial displacement. The effect of the bending moment on the strain ratio is displayed in Fig. 7. It can be seen for pipe under internal pressure and bending moment, strain ratio decreases with increase in the m parameter. However, the results show that, the strain ratio increases strongly with increase in time. Increasing the value of m to 3 will result in constant value of the strain ratio which is depend on the amount of K_e.

The results obtained from the theoretical analysis for the PVC pipe under tension, bending and internal pressure are shown in Figs. 8-10. The radial displacements of the pipe versus time for constant n and constant m are shown in Figs. 8-9, respectively. Fig. 8 shows that the radial displacement of the pipe decreases with increase in the parameter n, until it reached a constant value with n =2 and m=1. Similar observation can be noticed from the results for the pipe with constant n and different values of m (Fig. 9). As the parameter m increased to 2 and n=1, the radial displacement has a constant value which is depend on the amount of K_e. Finally, it is important to notice from Figs. 2-10 that for the PVC pipe (Table 1) at time equal to 5 times the value of (τ_c), the corresponding strain level in the pipe is almost constant. This may be due to the highest value of (G_g) compared with the value of (G_r).

4. CONCLUSION

This work presents mathematical models that can be used to calculate the radial displacement, the longitudinal strain as well as the strain ratio for any PVC pipe under internal pressure, tension or bending. Results for radial displacement and strain ratio of pipe under combined loads are presented and compared. The present work could be applied to any PVC pipe with different dimensions which makes the analysis a basic step for computer aided piping process.

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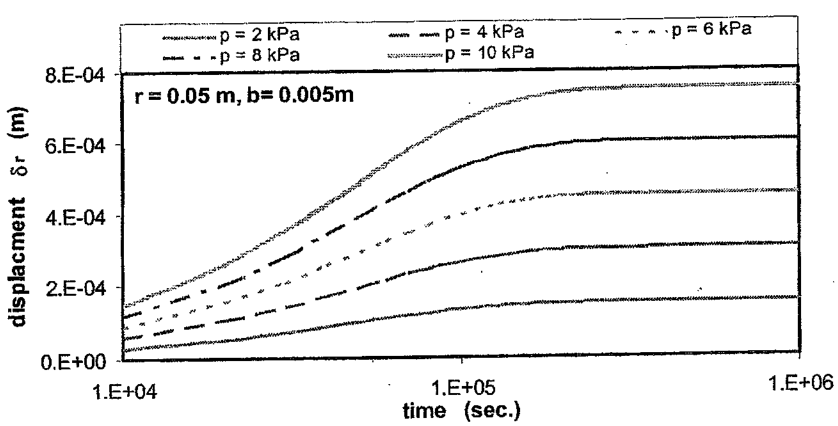

Fig. 2 The relationship between the displacement and time for pipe with different internal pressure

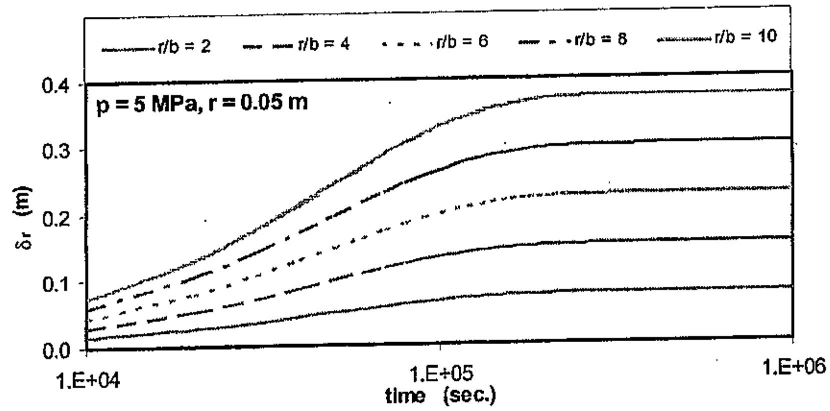

Fig. 3 The relationship between the displacement and time for pipe with different (r/b) ratio

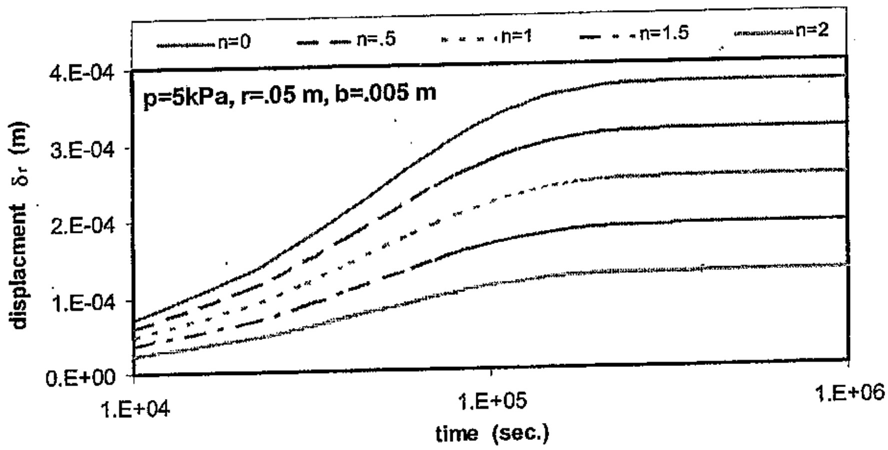

Fig. 4 The relationship between the displacement and time for pipe with different n ratio (Eq. 28)

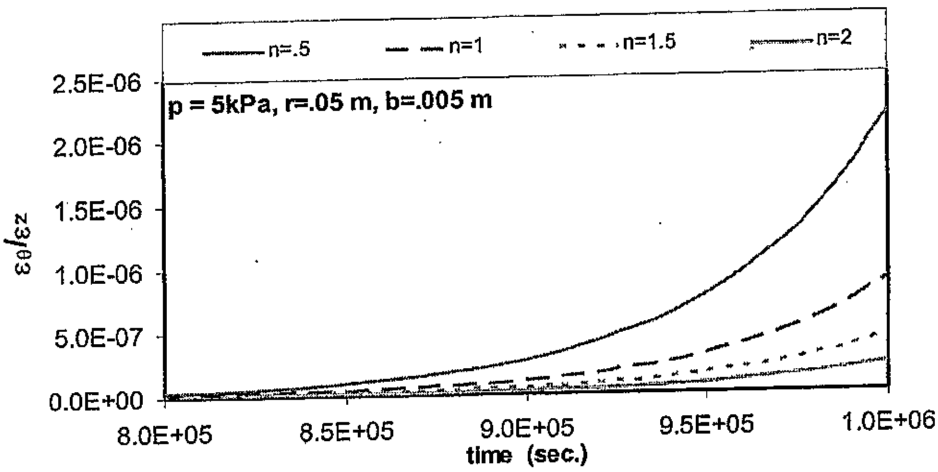

Fig. 5 The relationship between the strain ratio and time for pipe with different n ratio (Eq. 28)

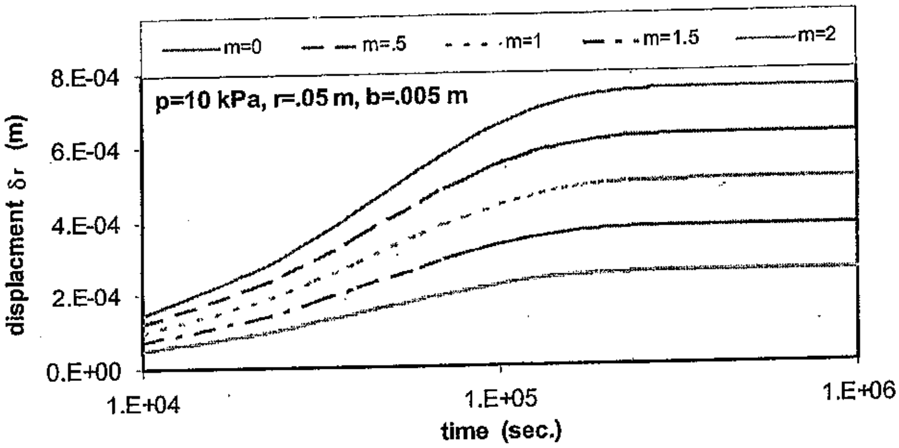

Fig. 6 The relationship between the displacement and time for pipe with different m ratio (Eq. 38)

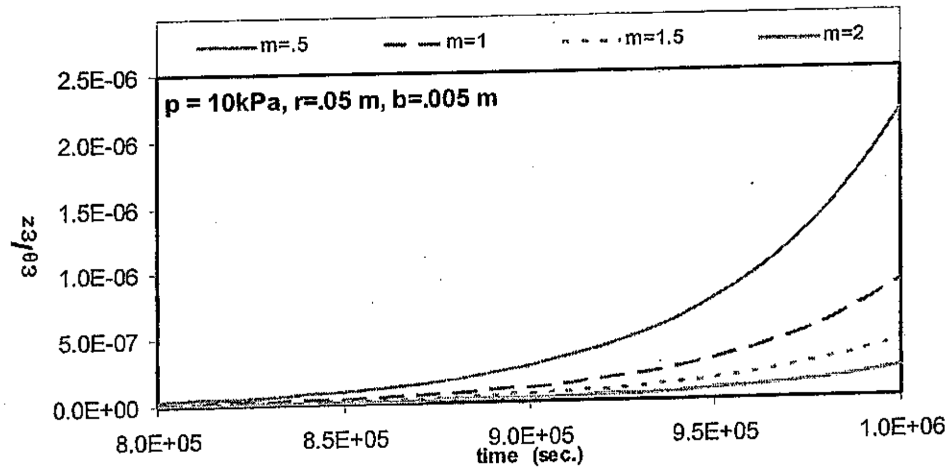

Fig. 7 The relationship between the strain ratio and time for pipe with different m ratio (Eq. 38)

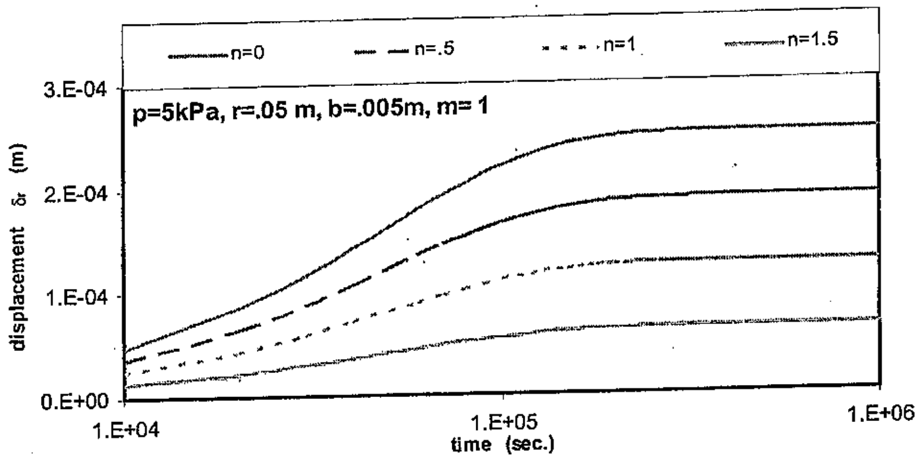

Fig. 8 The relationship between the displacement and time for pipe with different n ratio and constant m (Eq. 28, 38)

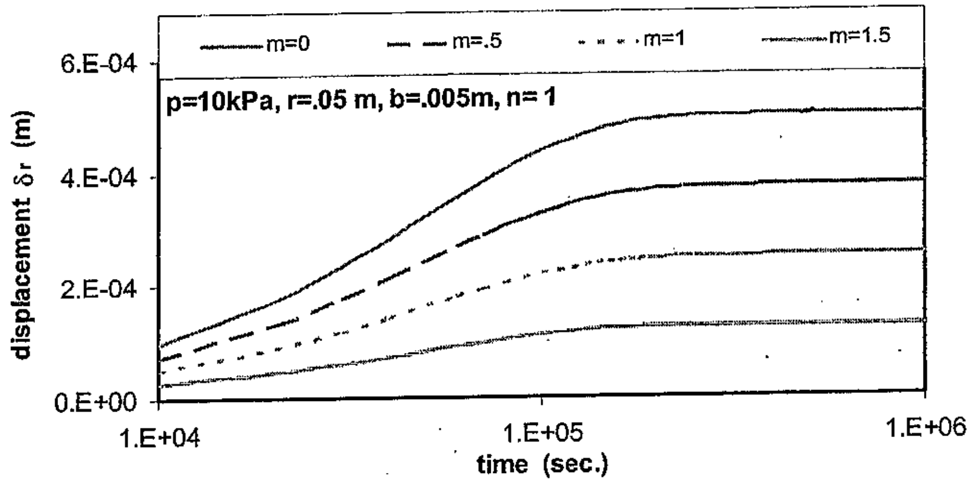

Fig. 9 The relationship between the displacement and time for pipe with different m ratio and constant n (Eq. 28, 38)

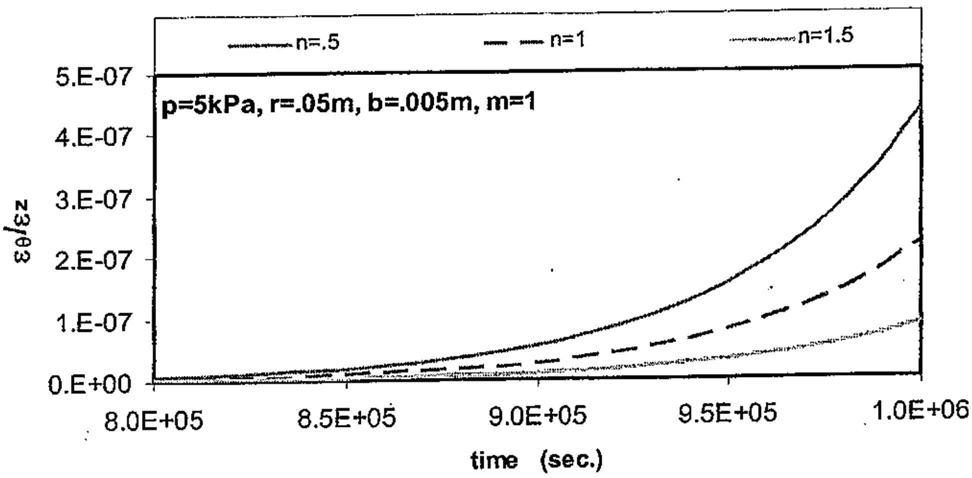

Fig. 10 The relationship between the strain ratio and time for pipe with different n ratio and constant m (Eq. 28, 38)