EXPERIMENTAL ANALYSIS OF MULTI-ITEM INVENTORY SYSTEMS

تحليل تجريبي لأنظمة المخزون المتعددة

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ملخص: تحتاج مشكلة المخزون إلى البرمجة الرياضية عندما يشتمل نظام المخزون على أكثر من نوع في ظل مصدر أو مصادر محدودة. و يصير حل هذه البرامج مملاً و أكثر تعقيداً خاصة كلما يزداد عدد الأنواع. لذلك يلجأ هذا التحليل إلى نموذج حاسبه تجريبي يختبر عدة حلول متاحة للبحث عن الحل الأنسب. وقد تم بناء صياغة مناسبة والتي تمكن من الوصول إلى حلول أقرب ما يمكن إلى الحلول الفردية و عند أقل تكلفة و هي قابلة لنماذج المخزون المختلفة و خاصة نموذج "كمية الطلب الاقتصادية". و يعتمد أسلوب الحل على معادلات و نسب بسيطة متكاملة للتبادل بين الأنواع الموجودة في النظام وكذلك يستعرض البدائل المناسبة. و قد أثبتت التجربة أن النموذج المقترح غير مقيد بعدد أو نوع المصادر او حتى عدد المتغيرات مما يجعله عملياً و بسيطاً.

ABSTRACT: When the inventory system accommodates more than one item under the effect of one or more constrained resources, the problem needs to some mathematical programs. The solution of these programs is more complicated and tedious specially when the number of items increased. Therefore, this analysis resorts to a new experimental model testing different feasible solutions searching for the best one. The problem formulation, which is mainly based on the economic order quantity (EOQ) model, is modified to ensure the minimum shift from the independent single-item order quantities and minimize the cost function of different models. Based on comprehensive equations and simple ratios, the solution procedure carries out the trade offs between all items in the system and exhibits the different alternatives if exist. No restrictions imposed on the number of items, the number/type of constraints, or the number of parameters in the system. Thus making the developed heuristic simple and practical in solving large problems.

(INVENTORY; CARRYING COSTS; EOQ; CONSTRAINTS, MULTIPLIER; NONLINEAR/ PROGRAMMING)

INTRODUCTION

Inventory is an idle resource for current or future purposes. An inventory system aims to maintain sufficient stocks of resources at the right time and minimizes the total inventory cost. It ensures a smooth production process or business activity. The nature of demand (deterministic or stochastic) and the number of products (single or multiple) are the main determinant of the system type; see Taha (1992).

Analysis of the single-item inventory systems, with no other constraints than the demand and system constructions, have received a far greater amount of study. Several efficient solution techniques were developed as shown in Buffa and Taubert (1972), Hax and Candea (1984), Elsayed and Boucher (1985), Riggs (1976) and Taha (1992). Also, Qualitative sensitivity analysis for inventory-production models of a single item was handled in Veinott (1964) . It is found that one of the most commonly used inventory systems, the order quantity-reorder level (Q,r) system, where a fixed order quantity Q is always placed when the inventory level decays to a fixed level r; see Zheng (1992) , and Brill and Chaouch (1995). In spite of insufficiency, the conception of single-item models is very important to construct a multi-item model. So, EOQ model is selected to exhibit an application for the model developed in this paper. EOQ model was discussed in the most of the cited literature. When the system accommodates ***multiple items*** with additional ***constraints***, the single-item techniques fail to solve the problem. Therefore, it needs to large-scale

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mathematical programs. The problem becomes more complicated when the number of items, constraints, or system parameters increases.

The existing literature is scarce on inventory systems from the type (Q,r) dealing with multi-items with constraints. The problem is how much the difference between the single-item and the constrained multi-item quantities. Sometimes the inventory carrying charge can be viewed as a policy variable; a higher value should be used to reduce the order quantities, or lower service levels should be used. These adjustments depend on the type of constraints violated, see Schroeder (1974) for a detailed discussion. But the policy of changing carrying charge is not an eventual solution because it may be imposed as an economical requirement. It seems that multiple items require relevant formulations to conduct the interaction between items under constrained environment. Some formulas have been presented in Peterson and Silver (1977).

Manne (1958) introduced, among the first approaches, an integer programming formulation for the multi-item capacitated inventory system and proposed to solve a relaxed linear programming problem. This problem was found large and difficult to solve when items share more than one resource. Dzielinski and Gomory (1965) adapted Manne's problem by using a decomposition procedure in their slow algorithm. Lasdon and Terjung (1971) proposed an alternate approach to Manne's LP problem. They solved the problem directly by using a revised simplex method and a generalized upper bounding procedure. Also, Kleindorfer and Newson (1975) treated Manee's problem by using Lagrangian dual problem of the original problem and established a relation between both problems.

The most popular two methods used to solve multi-item inventory problems are the Lagrangian method and the fixed-cycle (equal-order-interval) method. The former assumes that orders are received simultaneously without phasing orders for the different items. The latter adds constraint of having the same cycle for all items and allows the phasing of orders for the various items, which may not be required. Lagrangian method, which solving nonlinear programming model, happens to yield correct solution when the objective function is convex and the problem has a single linear constraint (convex space), see Taha (1992). Rosenblatt (1981) presents a detailed discussion for both methods. Parsons (1966) reported that all unconstrained quantities should be reduced by the same factor, the ratio between available and required resource values. Also, see Elsayed and Boucher (1985).

Hartley and Thomas (1982) examined two-item inventory system with a capacity constraint and distinguished between policies of fixed order quantities through numerical examples. The analysis involves the Lagrangian method and the fixed-cycle method and reported that the former method rarely produces the optimal policy. Bitran and Matsuo (1986) discussed relations between the original problem and Manne's problem. They presented an approximation scheme for the multi-item lot size problem through a linear convex combination of the optimal solution of the Manne's LP problem. They computed the error bounds for the combination and introduced the concept of relative infeasibility. Finally, they provided a bound on the duality gap of the Lagrangian dual problem which was found the same as that of Kleindorfer and Newson (1975).

Recently, Golany and Lev-Er (1992) have presented a comparative simulation analysis for several multi-item inventory models. A comparison to single-item models was included. An attempt was made to improve some existing models and introduce new ones. The results can be used as limited guidelines to practitioners by shedding the light on shortcomings of some models. But, no formulations appear to help in an extensive work based on this research. Hwang et al. (1993) have studied multi-item economic lot size models which attack setup reduction and quality improvement and developed a new procedure. Their work was reviewed and extended by Moon (1994) who introduced a complete formulation and used the Lagrangian method in his analysis. More recently, Davis (1995) has proposed a two-stage approach to solve the capacitated multi-item lot scheduling problem. The formulation, which is solved heuristically, is a combination of a nonlinear objective function and an integer program. An improvement to the scheduling of economic lot size production runs has been defined.

The aim here is to present a new simple procedure to solve the multi-item inventory problem. It differs, in structure, than those in the literature. It does not depend on the formulation of the problem. But it is applicable to a general formulation for (Q,r) system where r may be an excess, a shortage, or zero. It searches for the optimum solution and near alternatives amongst all feasible solutions An example problem is solved to illustrate the procedure.

PROBLEM FORMULATION

The multi-item problem is dealt with first by treating each item in an independent fashion, and the optimal order quantity and all related variables can be estimated using the single-item techniques presented in the literature cited before. If the solution does not violate all imposed constraints, the optimal order quantities are taken as found; otherwise the constraints will inversely affect the order quantity of each item. Of course, when at least one constraint is active (not redundant), the applicable order quantities will be less. In such case, for n items, this problem has been simply formulated as

$$\text{Minimize} \qquad TC(Q_1, Q_2, \ldots, Q_n) = \sum_{j=1}^{n} (O_j D_j / Q_j + iC_j Q_j / 2) \tag{1}$$

Subject to

$$\sum_{j=1}^{n} \rho_j Q_j \le R \tag{2}$$

$$Q_j \ge 0 \tag{3}$$

where

TC: total multi-item inventory cost,
Q_j : order quantity of item j,
O_j : order or setup cost of item j,
D_j : annual number of units demanded from item j,
C_j : purchase price or production cost per unit of item j,
i : annual inventory carrying cost rate for all items,
iC_j: inventory carrying cost ($/unit/year) of item j,
R : maximum resource allowed (investment, area, ..., etc.),
ρ_j: value of resource required per unit of item j.

This nonlinear programming problem, which is restricted to the EOQ model, has been solved by using: the Lagrangian and the fixed-cycle methods. Refer to Rosenblatt (1981), and Elsayed and Boucher (1985). Assume that items are received instantaneously without quantity discount. Further, the demand is well defined as an independent deterministic constant and the carrying cost associated with each item does not change due to interacting with other items. The setup cost associated with each order is item dependent but not time dependent. Given that the system is described by (Q,r), the objective function (1) can be stated in a form accommodates different models as

$$\text{Minimize} \qquad TC(Q_j, r) = \sum_{j=1}^{n} [O_j D_j / Q_j + iCf_1(Q_j, r) + f_2(Q_j, r) + \ldots + f_k(Q_j, r)] + b \tag{4}$$

where b is constant and r is the reorder level which may be constant or variable. It is very difficult to optimize this function using mathematical programs. But when using the proposed procedure, its role will be limited to substitution. It will be used only to assess the total inventory cost of each tried point, i.e. order quantities, after using the single-item components in the beginning of solution. The first component in right side always exists in all systems where one or more of the others may disappear according to simplification assumptions. Given that the available resources are wholly allocated to the system, constraint (2) will be relaxed to an equality as

$$\sum_{j=1}^{n} p_j Q_j = R \tag{5}$$

This to minimize the shift from the original independent (single-item) order quantities which actually lead to the minimum total inventory cost function at all. This represents zero residual constraint which will be used to make trade offs between all items in the system; therefore it is considered the controller of the procedure developed in this paper. This procedure does not need to solving mathematical programs, so it is capable of conducting the experiment to nonlinear constraints. Constraint (5) can be, therefore, stated as

$$\sum_{j=1}^{n} f(Q_j \setminus p_j) = R \tag{6}$$

The objective function (4) and constraint (5)/(6) beside constraint (3) represent the general formulation extracted for linear or nonlinear constrained problem. It does not affect the efficiency of the procedure which reduces the problem to just simple mathematical calculations.

SOLUTION PROCEDURE

To solve the problem under the former formulation, a solution procedure will be elaborated in comprehensive steps. Several notations and definitions will be used and explained internally through the procedure.

1-Obtain Q_j^*, the single-item optimal order quantity of item j which represents the maximum order quantity allowed when all items are integrated and violate the imposed constraint. For this purpose, use the EOQ model or any different (Q,r) model according to the imposed assumptions and nature of the system.

2-Check the control of the imposed constraints. If all constraints are not active, accept the current solution; otherwise, release the inactive constraints and continue.

3-Fix n-1 items at Q_j^* and solve constraint (5)/(6) for the remaining item; it may be negative. Repeat this for the n-1 items to get the minimum order quantity, Q_j^-, of item j. Intuitively, the optimum constrained order quantity of item j, Q_j^c, falls between these two quantities. To determine the latter quantity, an *experimental* quantity, Q_j^t, is assumed representing all feasible and infeasible solutions. Where $Q_j^- \le Q_j^t \le Q_j^*$ for item j.

4-Get an initial solution for Q_j^t by selecting arbitrarily any item (start item: s) at its Q_j^-, j=s. In turn, the remaining items reset at Q_j^t=Q_j^*, $j \neq s$. Note: an item may start infeasible (just negative) and switches after several iterations.

5-Substitute into the right hand side of objective function (1)/(4) to get TC of the current solution.

6-Increase Q_s^t by an arbitrary *increment* I units; if the start item reaches Q_s^*, go to step 8. This increase violates constraint (5)/(6) by Ip_s, assuming linear, which must be deducted to keep the mentioned constraint unviolated. If the constraint is nonlinear, I will take the order of the starting item. The deducted value $I\rho_s$ is partitioned between the remaining items as

$$I\rho_s = \sum q_j \rho_j, j \neq s \tag{7}$$

where q_j is the number of units that must be deducted from item j. To determine q_j for each item, a *marginal reduction multiplier* α_j will be heuristically proportional to a *partition ratio* Δ_j. This ratio indicates the relative effect or load of each item on constraint (5)/(6). The value of Δ_j is given as

$$\Delta_j = \rho_j Q_j' / \sum \rho_j Q_j' , j \neq s \tag{8}$$

Then Δ_j parts $I\rho_s$ as

$$I\rho_s = \sum \Delta_j (I\rho_s), j \neq s \tag{9}$$

Then, by substituting from Eq. (8) into Eq. (9) and comparing the components of right sides of the resulted Eq. and Eq. (7), the value of q_j is obtained as

$$q_j = I[\rho_s Q_j' / \sum \rho_j Q_j'], j \neq s \tag{10}$$

By dividing the two sides of Eq. (10) by I, the marginal reduction multiplier α_j is extracted as

$$\alpha_j = [\rho_s Q_j' / \sum \rho_j Q_j'], j \neq s \tag{11}$$

The value of α_j is not restricted to a fraction except in case of finding ρ_s near to ρ_j. It is a positive value varies depending on the experimental (or tried) order quantities. This value is an indicator for resource change of item j.

7-Compute the new experimental order quantities of the reaming items as

$$Q_j' \leftarrow Q_j' - q_j, j \neq s \tag{12}$$

Which represents a recursive equation. In addition to the current order quantity of the start item, we maintain a solution for the problem. Go to step 5.

8-Select Q_j^t for the n items which minimize the objective function as an experimental solution for Q_j^c. Note that the maximum accuracy will be reach when the value of the increment, I, is small as possible, otherwise we can resort to the graphical plotting and interpolation to estimate approximately the best order quantities.

The solution procedure is applicable for the systems subjected to more than one resource simultaneously. For simplicity, each of them can be satisfied separately and finally we select the order quantities which satisfy all of them together.

To facilitate the procedure, a computer program is constructed. It can be considered a computer aided procedure because the process is similar to what made in the discrete simulation processes. The program needs to few seconds to solve the problem. The program registers all solutions and alarms for those infeasible which may appear only during few iterations at the beginning.

COMPUTATIONAL RESULTS

The proposed procedure is demonstrated numerically by using a case problem presented in Elsayed and Boucher (1985). The system accommodates three items, their data are given in Table 1. The management has an upper limit on the investment of $16,000. The inventory carrying cost rate for each item is 0.18 and no shortages are allowed.

Table 1 An Inventory System Data.

		Item 1	Item 2	Item3
Annual demand	D_j	1500	1500	2500
Unit cost	C_j	$60	$30	$80
Setup cost	O_j	$60	$60	$60

The steps of the solution procedure will be conducted in the given sequence using the computer program. The single optimal order quantities are found using EOQ model such that

$$Q_j^* = \sqrt{2O_jD_j/IC_j} \qquad j=1,2,\ldots,n \tag{13}$$

Q_1^*=129.10, Q_2^*=182.57, and Q_3^*=144.34 units which represent the maximum quantities. The inventory investment corresponding to these quantities is $24,770 > $16,000. Use the equation $60Q_1' + 30Q_2' + 80Q_3' = \$16,000$. Then, the minimum quantities are $Q_1^{`}$ =-17.07, $Q_2^{`}$ =-109.77, and $Q_3^{`}$ =34.71 units. The negative values do not represent feasible solutions because they violate constraint (3). However, starting with a negative value does not affect the experimental results around the best solution. Table 2 shows a part of the output gained from the program taking l=2. Hence, Q_j^c=82.93, 118.23, and 93.47 units with TC=$4891.339 besides alternatives differ by few dollars.

Table 2 A part of the program output.

Experimental Quantities				
Q1	Q2	Q3	Cost $	Resource $
76.93	122.09	96.52	4901.161	15999.990
78.93	120.80	95.50	4895.926	15999.990
80.93	119.51	94.48	4892.682	15999.990
82.93	118.23	93.47	4891.339	15999.990
84.93	116.94	92.45	4891.821	15999.990
86.93	115.65	91.43	4894.059	15999.990

CONCLUSIONS

In production and business, most inventory situations involve multiple items. If single-item systems are used, the resulting order quantities could violate the available space, purchasing budgets or other economical and environmental constraints. Therefore, the problem requires some formulations sensitive to the interaction between items. The objective function is always nonlinear due to the procurement cost, thus making nonlinear programming the most suitable formulation for exact solution. Moreover, the solution of large scale programs which augment when the number of items and/or constraints increases, is computationally difficult specially in case of existing nonlinear constraints.

Here, the developed solution procedure does not resort to solving the nonlinear programming model, but it searches the best solution experimentally. So, the problem is formulated in a general fashion to accommodate several systems. It does not restrict the number/type of constraints or the shape of objective function/feasible area (convex , concave). Thus making it practical and reliable. It carries out the computations through simple equations, ratios, and multipliers; this needs CPU seconds using the developed program. Moreover, It can be concluded that the procedure solves an exact form heuristically. The procedure exhibits all feasible solutions including the best one. The solutions around the best one can be considered different *alternatives* because they slightly differ; this property adds to the advantages of the procedure. Furthermore, It can be extended to different inventory environments.

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