

# Comparative Methods for Analysis of Plane Raft Foundations

By  
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## Abstract :

The analysis of soil structure interaction is still a query and on the basis of different assumptions; several methods have been elaborated for such analysis. Different methods used for analysis of raft foundations are considered. A computer program has been developed for analysis of raft foundation considering the interaction between the raft and the underlying soil strata. In the analysis; the raft is divided into a certain number of elements and the soil is modeled by a series of an infinite number of linearly elastic springs. Each spring has a stiffness equal to the modulus of subgrade reaction which varies according to soil status.

Methods for analysis of raft foundation investigated in this study include: the finite difference method, the grid method, the raft analysis as individual strips on elastic supports, and the finite element method.

The different methods are considered along with the well known program SAP 80 and the results for the considered case gave close results. Thus the program can be effectively used for analysis of raft foundation using any of the previously mentioned methods. Finally, a suggested method for design of the raft foundation is presented.

## 1 - Introduction :-

Raft (or mat) is usually used to define a substructure in which loads are transmitted to the soil by means of a continuous slab covering the entire area of the bottom of a structure, like floor. Rafts are usually designed and analyzed as a rigid or flexible plates resting on an elastic foundation. Mats are usually used when the building loads are so heavy or the allowable soil pressure is so small that the individual footings would cover more than about half the building area. In addition to the advantage of distributing the building loads over the entire building area, mats are used to decrease differential settlements and total settlement. Mat foundations are also used to resist hydrostatic uplift and bridge over isolated pockets of soft soil.

The simplified methods or rigid methods of analysis are easy to use and do not need computer assistance. However, the accuracy of these methods is very poor and in many cases the results are diverged from the right solution. Moreover, they can handle only a certain geometry. On the other hand, in the flexible methods, the raft is assumed to be relatively flexible and its flexural rigidity is taken into account for the conclusion of the contact pressure between the soil and the raft. These methods take also the effect of the soil stiffness into consideration through the use of idealized soil model.

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## 2 - Rigid Methods ( Conventional Methods ) :-

In this method, the raft is assumed to be infinitely rigid and the contact pressure has planer distribution. The centroid of the contact pressure coincides with the line of action of the resultant of all vertical loads acting on the raft and the contact pressure  $q$  can be calculated from the equation:

$$q = R \left( \frac{1}{A} \pm \frac{e_y}{I_x} y \pm \frac{e_x}{I_y} x \right) \quad (1)$$

Where :

$R = \Sigma Q =$  Total loads acting on the raft.

$A =$  Total area of the raft.

$x, y =$  Coordinates of any given point on the mat with respect to the  $x$  and  $y$  axes passing through the centroid of the raft area.

$e_x, e_y =$  Coordinates of the resultant force.

$I_x, I_y =$  Moment of inertia of the raft area with respect to the  $x$  and  $y$  axes respectively.

The raft is analyzed as a whole in each of two perpendicular directions. Thus, the total shear force acting at any section across the entire mat is equal to the arithmetic sum of all forces (loads) and reactions (contact pressure) to the left or right of the section. The total bending moment acting on such a section is equal to the sum of all moments on either side of this section. This solution is considered a highly indeterminate problem. Therefore, an approximate procedure may be adopted as the raft is divided into perpendicular bands, each band carrying a row of columns, taking full loads in each direction.

The solution by rigid methods are very approximate as the contact pressure distribution is considered plane, varying linearly and its resultant coincides with the resultant of all external loads and moments acting on the raft. So, such methods neglect the increase of the contact pressure near the columns and divides the raft into separate strips neglecting the shear transfer due to continuity between adjacent strips. Moreover they consider full column loads on each strip in each direction, which is very conservative.

Good results can be obtained using methods when ACI- Committee assumptions for rigid footing are fulfilled.<sup>(1)</sup>

## 3 - Flexible Methods :-

In these methods, the raft is assumed to be relatively flexible and its flexural rigidity is taken into account for the conclusion of the contact pressure between the soil and the raft. These methods take also the effect of the soil stiffness into consideration through the use of idealized soil model. The resulting contact pressure distribution

generally has a curved surface, which is more realistic than the rigid methods. Among the flexible methods are the following discussed methods.

### 3-1- Beams on Elastic Supports :-

In this method the raft is analyzed in both directions as individual strips resting on soil. Each strip is analyzed under full column loads. The theoretical solution of a beam on elastic foundation was treated in considerable detail by Hetenyi(1946)<sup>(3)</sup>

#### 3-1-1 . The Differential Equation of the Elastic line :

The differential equation governing the behavior of a beam supported along its entire length by an elastic medium and subjected to vertical forces acting in the principal plane of the symmetrical cross section is given by :

$$EI = \frac{d^4 y}{dx^4} = -ky + q \quad (2)$$

where :

EI = flexural rigidity of the beam.

x,y = horizontal and vertical coordinates.

k = modulus of foundation.

q = distributed load acted on the beam.

Along the unloaded parts of the beam, where no distributed load is acting  $q = 0$ , and the equation above will take the form :

$$EI = \frac{d^4 y}{dx^4} = -ky \quad (3)$$

The general solution of the deflection line of a straight prismatic bar supported on an elastic foundation and subjected to transverse bending forces, but with no  $q$  loading takes the form :

$$y = e^{\lambda x} (C_1 \cos \lambda x + C_2 \sin \lambda x) + e^{-\lambda x} (C_3 \cos \lambda x + C_4 \sin \lambda x) \quad (4)$$

Here,  $\lambda$  includes the flexural rigidity of the beam as well as the elasticity of the supporting medium, and is an important factor influencing the shape of the elastic line and the Cs are constants which can be determined from the boundary conditions.

### 3-2- The Finite Difference Method :-

This method is based on the assumption that the subgrade can be substituted by a bed of uniformly distributed elastic springs with a spring constant (coefficient of subgrade reaction ks).

In the mechanics of solution, the mat is divided into a grid, with sufficient divisions taken so that all columns fall at the intersection of grid lines. The grid should be subdivided so that  $\Delta x = \Delta y$ , the difference expression then being in their simplest form; however, it is easy to derive finite difference expression for the case  $\Delta x \neq \Delta y$ .

From the foundation engineering standpoint, the plate problem is a concrete slab on an elastic medium. Timoshenko(1959) expanded the differential equation for deflection of such plate<sup>(7)</sup>:

$$\nabla^4 w = \frac{q}{D} + \frac{P}{D (\partial x \partial y)} \quad (5)$$

where :

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

q = Intensity of load.

p = Concentrated load at point of interest.

w = Deflection.

$$D = \text{Rigidity of the plate} = \frac{Et^3}{12(1-\mu^2)}$$

Making direct substitution of  $\frac{\partial^4 w}{\partial x^4}$  and  $\frac{\partial^4 w}{\partial y^4}$  and  $\frac{\partial^4 w}{\partial x^2 \partial y^2}$  and using  $\partial x = \partial y = h$ , the finite difference equation in terms of deflection at any point within a plate using a square grid will be : ( see Fig (1) )

$$20w_0 = 8(w_T + w_B + w_R + w_L) + 2(w_{TL} + w_{TR} + w_{BL} + w_{BR}) + (w_{TT} + w_{BB} + w_{LL} + w_{RR}) \\ = \frac{qh^4}{D} + \frac{ph^2}{D} \quad (6)$$

The sign convention is based on  $+q$  and  $+p$  in the downward direction. The  $q$ -term may be upward soil pressure or downward plate loading. The soil pressure is based on the concept of subgrade reaction

$$-q = kw \quad (7)$$

The equation(6) can be applied at any intermediate point in the plate, but when it is applied at a point within two nodes of an edge or at corner, some of the deflections will fall off the plate. One of two approaches may be utilized:

- (1) Use of backward or forward difference expression.
- (2) Consider fictitious points off the plate and use

$$\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{moment perpendicular to edge} = 0$$

$$\frac{\partial^3 w}{\partial x^3} + (2 - \mu) \frac{\partial^3 w}{\partial y^3} = 0 \quad \text{shear perpendicular to edge} = 0$$

After the deflections are known, the bending moment at any point in each direction can be determined from theory of elasticity:

$$M_x = M'_x + \mu M'_y \quad (8)$$

$M_x$  = Bending moment per unit strip in x-direction.

$M'_x$  = Bending moment in the x- direction not including the influence of bending moment in the y- direction.

$M'_y$  = Bending moment in the y- direction not including the influence of bending moment in the x - direction.

By using the finite difference operators, the total bending moment at any interior node can be expressed as <sup>(2)</sup>:

$$\begin{aligned} -M_x &= \frac{D}{\partial x^2} (w_L - 2w_o + w_R) + \mu \frac{D}{\partial y^2} (w_T - 2w_o + w_B) \\ -M_y &= \frac{D}{\partial y^2} (w_T - 2w_o + w_B) + \mu \frac{D}{\partial x^2} (w_L - 2w_o + w_R) \end{aligned} \quad (9)$$

### 3-3- Grid Method :-

In this method ; the mat is discretized into a number of beam-column elements with bending and torsional resistance . The torsional resistance is used to incorporate the plate twist using the shear modulus G. The finite grid method produces non - conforming elements as well as interelement compatibility is insured only at the nodes. A theoretical development of this method and its application on mats was introduced by Bowles <sup>(2)</sup>.

#### 3-3-1 General Equation In Solution :

For the following development refer to fig.(2) at any node ( Junction of two or more members) on the structure, one may write.

$$P_i = A_i F_i \quad (10)$$

Which states that the external nodal force P is equated to the internal member forces F using a bridging constant A. For the full set of nodes on any structure and deleting subscripts this becomes :

$$\{ P \} = [ A ] \{ F \} \quad (11)$$

An equation relating internal-member deformation  $e$  at any node to the external nodal displacement is :

$$\{ e \} = [ B ] \{ x \} \quad (12)$$

Where both  $e$  and  $x$  may be rotations or translations. Form reciprocal theorem in structural mechanics and the matrix  $[ B ]$  is exactly the transpose of the  $[ A ]$  matrix, thus :

$$\{ e \} = [ A ]^T \{ x \} \quad (13)$$

The internal- member forces  $\{ F \}$  are related to the internal - member displacements as :

$$\{ F \} = [ S ] \{ e \} \quad (14)$$

These three equations are the fundamental equations in the grid method of analysis. By some algebraic manipulation we can get the only unknowns in this system of equations namely  $\{ x \}$  as follows:

$$\{ x \} = ([ A ][ S ][ A ]^T)^{-1} \{ p \} \quad (15)$$

with  $x$ 's , the internal member forces which are necessary for design can be obtained. Referring to Fig. (3) and using the conjugate-beam principle and solving the equation, the forces can be found in terms of the end slopes  $e_1$  and  $e_2$  as follows :

$$F_1 = \frac{4EI}{L} e_1 + \frac{2EI}{L} e_2 \quad (16)$$

$$F_2 = \frac{2EI}{L} e_1 + \frac{4EI}{L} e_2$$

The forces  $F_4$  and  $F_5$  are obtained from the spring equation for force deflection as

$$F_4 = k_1 \cdot e_s \quad F_5 = k_2 \cdot e_s \quad (17)$$

The soil spring will be obtained (due to Winkler's model) from the modulus of subgrade reaction as :-

$$K_1 = \frac{L}{2} b k_s \quad \text{and} \quad K_2 = \frac{L}{2} b k_s$$

where :

$L$  = The length of the element.

$b$  = The width of which the element is occupied.

The torsion factor for  $F_3$  is also included in the matrix which is equal to  $\frac{GJ}{L}$

where :

$$G = \text{Shearing modulus} = \frac{E}{2(1+\mu)}$$

E = Young's modulus .

$\mu$  = Poisson's ratio .

j = The torsional rigidity of the grid element .

The matrix equation is written as :

$$\{F\} = [S] \{e\} \quad (18)$$

$$[S] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 & 0 & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 & 0 & 0 \\ 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & 0 & k_1 & 0 \\ 0 & 0 & 0 & 0 & k_2 \end{bmatrix} \quad (19)$$

### 3-3-2- The Solution Procedure :

First the element  $[S] [A]^T$  is component by multiplying the element  $[S]$  matrix by the transpose of the element  $[A]$  matrix. Then the element  $[A] [S] [A]^T$  matrix is obtained which is  $6 \times 6$  and is placed at the appropriate locations in the global matrix  $[A] [S] [A]^T$  matrix. Thus the displacement vector  $\{X\}$  can be obtained after inverting the  $[A] [S] [A]^T$  global matrix and the element forces matrix is solved for each element in turn to find the element forces as :

$$\{X\} = ([A] [S] [A]^T)^{-1} \{P\} \quad (20)$$

$$\text{and } \{F\} = [S] [A]^T \{x\} \quad (21)$$

### 4 - The Finite Element Method :-

The process of subdividing all systems into their individual components or "elements" whose behavior is readily understood, and then rebuilding the original system from such components to study its behavior is a natural way in which the engineer or the scientist can proceed to solve a problem.

#### 4-1- Finite Element Techniques for Raft Analysis :-

Using the displacement method (stiffness method) to formulate equilibrium equations, a rectangular element with twelve degrees of freedom (Three degrees of freedom at each node) is used to analyze the raft.

By following the finite element technique, the whole raft is analyzed as an integrated system of a number of finite plates in bending, the elements are connected at nodes and resting on a system of infinite number of kinematically consistent springs. The origin of the global system of axes will be lying on the middle surface of the raft and the local axes are parallel to the global and the origin of the local axes is located at the center of the rectangular element as shown in fig . (4).

The state of deformation of the raft can be described entirely by one quantity. This is the lateral displacement  $w$  of the middle plane of the plate.

The plate-bending element used in this study is called MZC rectangle and is shown in fig(5 ). It has only one generic displacement ( $w$ ) translation in  $z$ -direction and it produces convergent results. The nodal displacements are :

$$q_i = (q_{i1}, q_{i2}, q_{i3}) = \left( w_i, \frac{\partial w_i}{\partial y}, -\frac{\partial w_i}{\partial x} \right) \quad \text{for } (i = 1,2,3,4) \quad (22)$$

So every node of the rectangular element in bending has three degrees of freedom which are

(I) Vertical translation ( $w$ ) normal to the plane of raft in  $z$  - direction.

(ii) Angle of rotation about  $y$ -axis  $\left( \frac{\partial w}{\partial y} \right)$

(ii) Angle of rotation about  $x$ -axis  $\left( -\frac{\partial w}{\partial x} \right)$

Thus the result is twelve degrees of freedom and the local stiffness matrix of plate element will be of dimension (12 x 12). Therefore the global system of equations are :

$$[k] \{q\} = \{P\} \quad (23)$$

$[K]$  = The global stiffness matrix which is the assembly of the local stiffness matrices of the plate elements and soil stiffness matrices each to their corresponding degrees of freedom. It is a symmetrical matrix of order  $(3n \times 3n)$  where  $n$  is the total number of nodes.

$\{q\}$  = The displacement vector of the whole system and its dimension is  $(3n)$  and it is the assembly of nodal actions which is :

$$P_i = \{ P_{i1}, P_{i2}, P_{i3} \} = \{ P_{zi}, M_{xi}, M_{yi} \} \quad \text{for } (i=1,2,3) \quad (24)$$



The symbol  $P_{zi}$  denotes a force in the z - direction but  $M_{xi}$  and  $M_{yi}$  are moments in the x and y axes.

From the previous discussion, we note that to get the displacements at the nodes with discretize the raft we must get the stiffness matrix for every element and assemble them to obtain the global stiffness matrix and get its inverse as

$$\{q\} = [K]^{-1} \{P\} \quad (25)$$

#### 4-2- Soil Stiffness Technique :

The soil element stiffness matrix is derived by replacing the springs over the entire element with four springs at the nodes. This can be achieved by dividing the soil into a finite element mesh identical to the mesh of the mat. Furthermore, it is assumed that the rectangular areas surrounding a given node, defined by the center lines of adjacent elements, undergo uniform deflection. Thus the soil is idealized as a set of isolated springs (Fig. 6) capable of resisting compression only :

$$k_i = \sum_e \frac{A_e}{4} k_s \quad (26)$$

where :

$k_i$  = Soil stiffness at a node .

$A_e$  = Area of element surrounding a node.

$k_s$  = Coefficient of subgrade for the element under consideration.

#### 4-3- Stresses In The Raft Elements : -

After the element have been assembled and the structure has been analyzed for nodal displacements, the generalized stresses of selected points in each element may be obtained as follows :

$$\{M\} = \{M_{xx} , M_{yy} , M_{xy}\} = [E] [B] \{q_e\} \quad (27)$$

where :

$M_{xx} , M_{yy}, M_{xy}$  = The internal bending moment about x and y axes and the twisting moment

$[E]$  = Matrix relating stresses to strains.

$[B]$  = Matrix gives strains at any point within the element due to unit values of nodal displacements.

$\{q_e\}$  = Nodal displacements array for the element.

Then the flexural stresses can be found with the aid of the following equation :

$$\{ \sigma \} = \{ \sigma_{x.}, \sigma_y , \sigma_{xy} \} = -\frac{12z}{t^3} \{ M \} \quad (28)$$

Thus the stresses within the raft foundation, soil pressure and displacements can be determined.

### **5 - The Computer Program**

A computer program has been developed to analyze the raft foundation in different numerical methods discussed before. This analysis is realized by dividing the raft into rectangular pieces creating a slab mesh consisting of elements and nodes. This procedure is called discretization, so instead of solving the problem for the entire body one operation, the solutions are formulated for each constituent unit and then the discrete equations are combined to obtain the solution of the original body four methods are used in this analysis namely :

- 1 - The finite difference method.
- 2 - Raft analysis as a beam on elastic foundation.
- 3 - The grid method where the raft is considered as an assembly of separate beams in the longitudinal and transversal directions.
- 4 - The finite Element method.

The mats may be subjected to any combination of vertical loads and moments. Moreover, the vertical loads may be concentrated or uniformly distributed over a rectangular area of raft. These external loads are applied at the nodes where the displacements of the raft are calculated and the stresses are found.

Using Winkler model, the soil is replaced by individual springs under the nodes with different values according to the modulus of subgrade reaction.

### **5 - Numerical Examples :-**

To compare the different methods of analysis, a raft model is chosen with dimensions and load locations as shown in (Fig. 6) it is 5 x 5m and 0.5 cm thick resting on sandy soil at - 3.0m from the ground level. The soil has modulus of subgrade reaction of 1600 t/m<sup>3</sup> and allowable stress 30t/m<sup>2</sup>. The raft is divided into rows as shown in (Fig. 7) and elements and is analyzed by the different previously mentioned methods.

#### **6-1 - Comparison of Different Methods of Analysis :-**

The previously mentioned methods are considered along with the well known packaged SAP 80. For the case considered the results for the different methods are close as shown in Figs. (8), (9) where the displacements and bending moment in the raft are plotted.

### **6-2- Raft Thickness Effect :-**

The raft thickness effect is considered by considering the previous model with different thickness ranging from 0.25 to 1.0. It is concluded that a moderate value gives better result, small values make the raft very flexible while big thickness make the raft very rigid. The raft displacements and bending moments for the different axes are shown in Fig. (10) and (11). In this case the finite element program is used for analysis.

### **6-3- Raft With Variable Thickness :-**

For economic design, there is to take a constant thickness through the plate raft and only the raft need to be thickened under the different loads to ensure safe punching stresses and in the same time the footing should satisfy the allowable soil pressure. The previous model is used with different thicknesses as shown in Fig. (12).

A comparison of raft displacements and bending moments for constant thickness of 0.5m thick and variable thickness as in Fig. (12) is shown in Fig, (13), (14), (15) and (16<sup>1</sup>). It is clear that the raft with different thicknesses may be more suitable in some cases rather than raft with constant thickness.

### **7- Conclusion :-**

Different methods for analysis of raft foundation are investigated. A computer program has been developed for analysis of raft foundation according to the discussed methods considering the interaction between the raft and the underlying soil strata. The soil is modeled by a series of an infinite number linearly elastic springs. Different numerical examples are presented. The computer program has been proven to be adequate for analysis of raft foundation in an accurate way.

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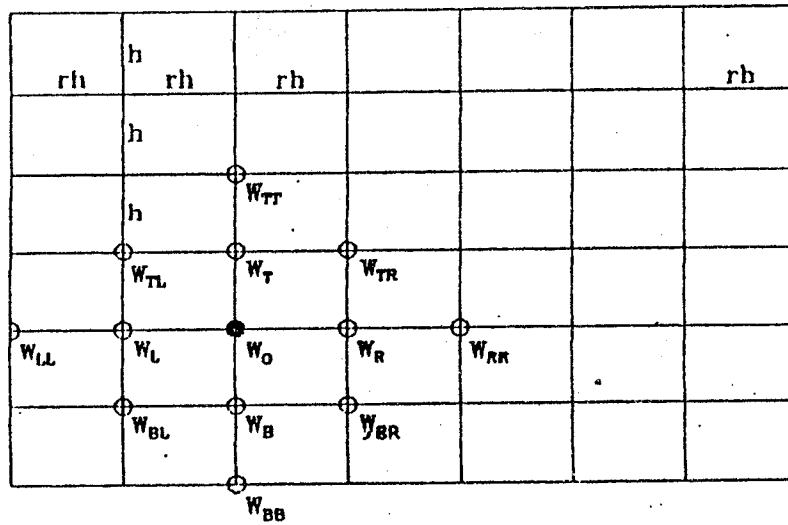


Fig. (1) Finite Difference Model

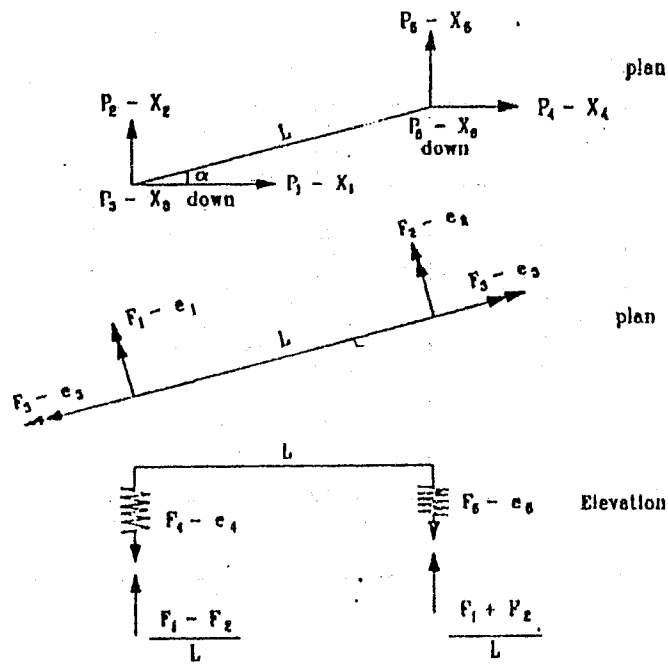


Fig.(2) External (nodal) and Internal (Member) Element Forces with General Orientation.

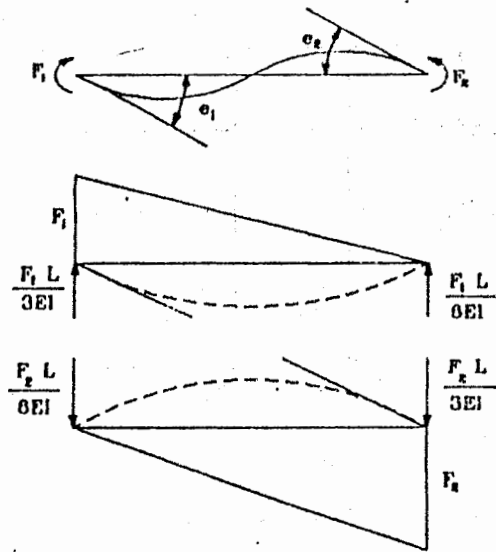


Fig.(3) Conjugate Beam Relationships between End Moments and Beam Rotation.

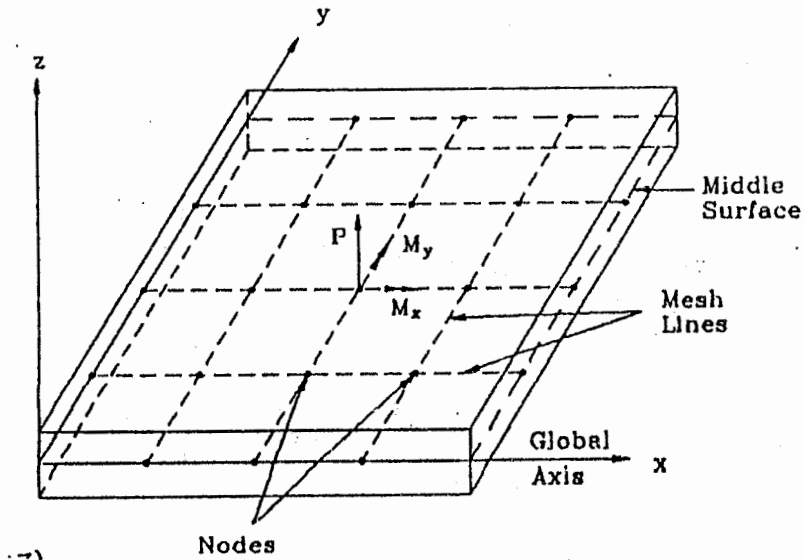


Fig.(4) Plate Discretization, Geometrical Description and Directions For External Loads

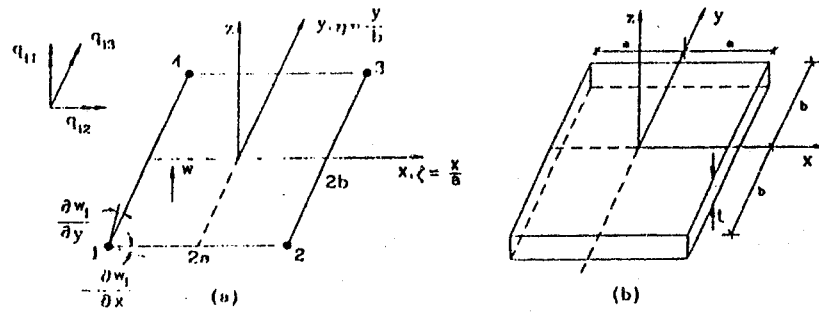


Fig.(5) Finite Element Model and its Degrees of Freedom.

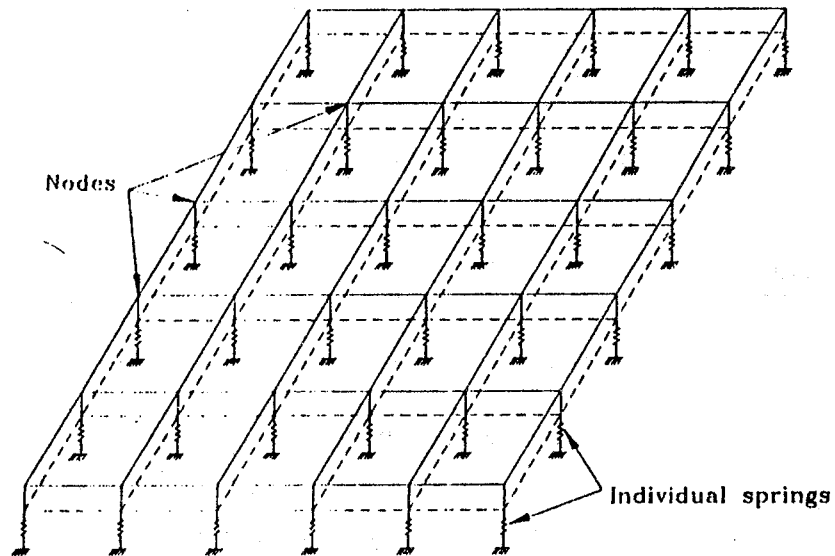


Fig.(6) Structural Idealization of Mat and Supporting Soil.

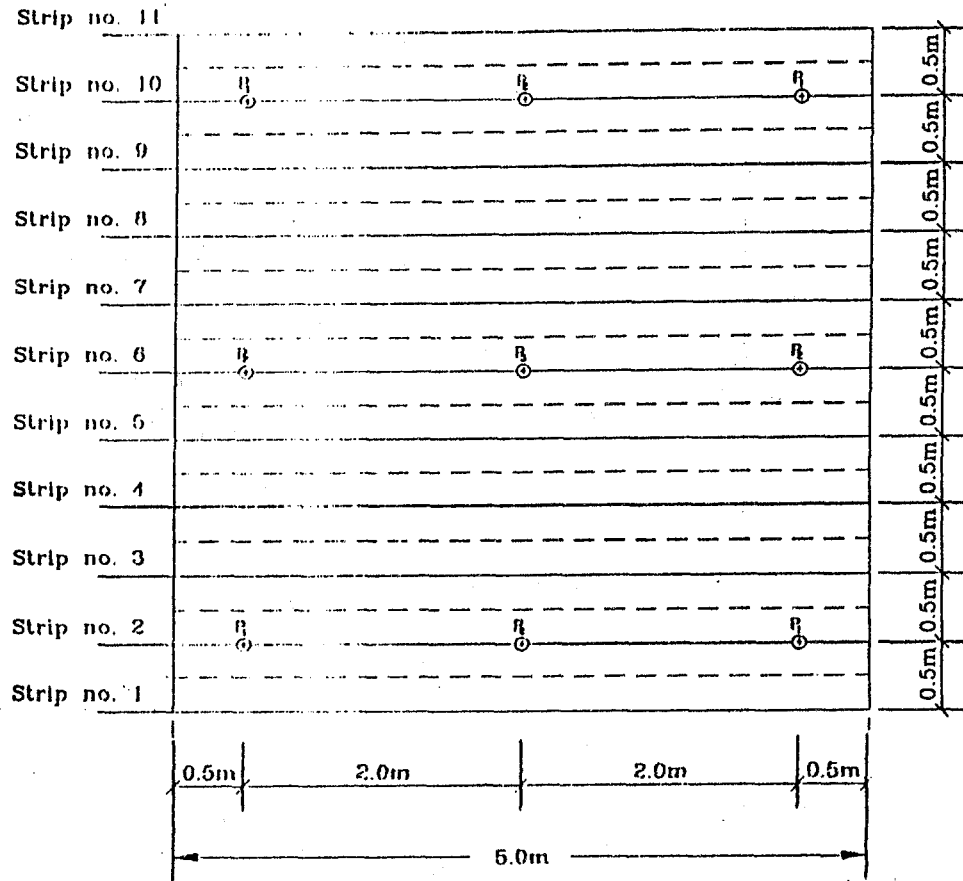


Fig. (7) The Dimensions and Load Locations in the Raft Model.

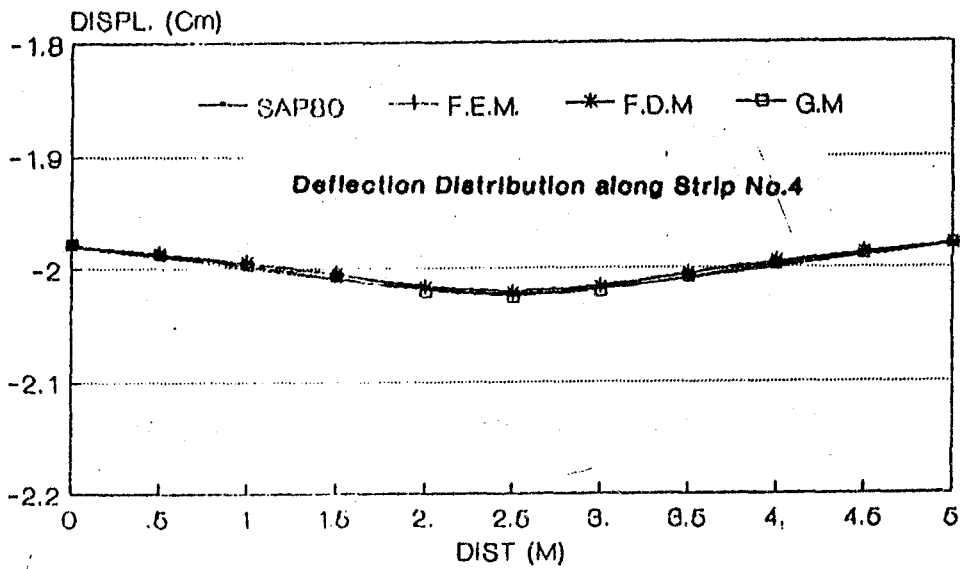


Fig.(8) Comparison between program Results and SAP- 80.



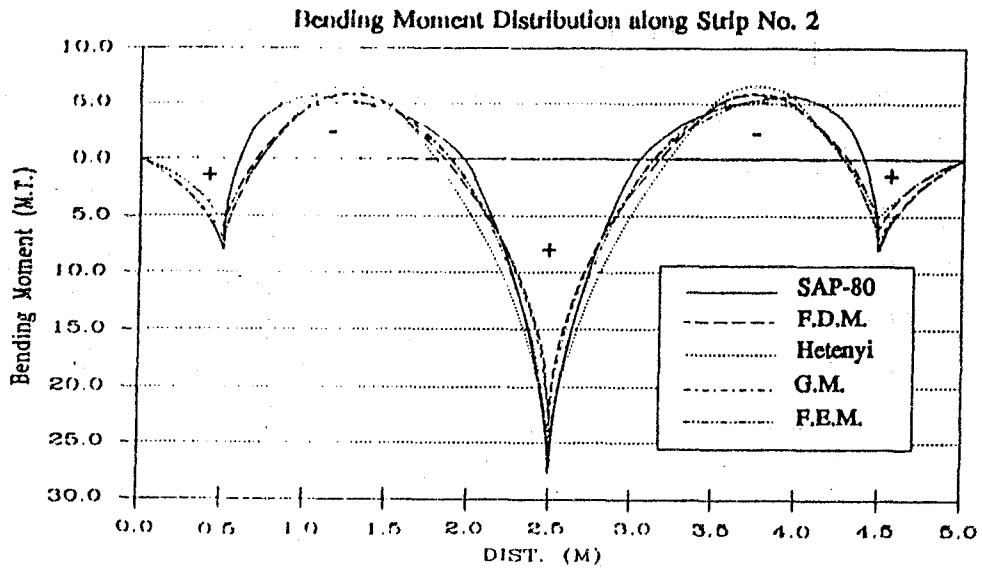


Fig.(9) Comparison between program Results and SAP -80.

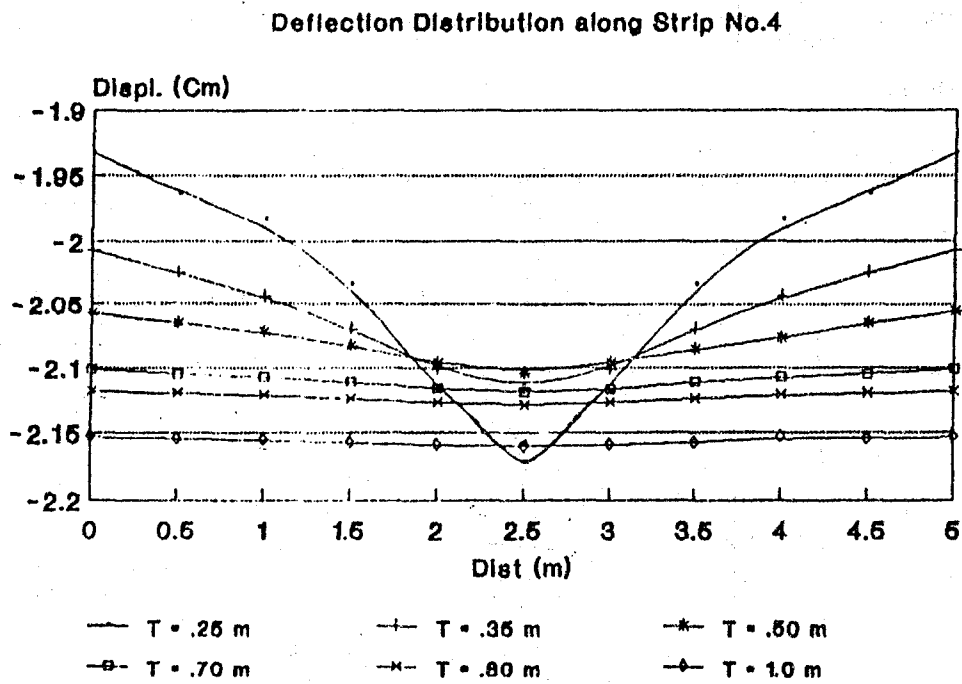


Fig.(10) Comparison For Deflections of Different Raft Thicknesses.

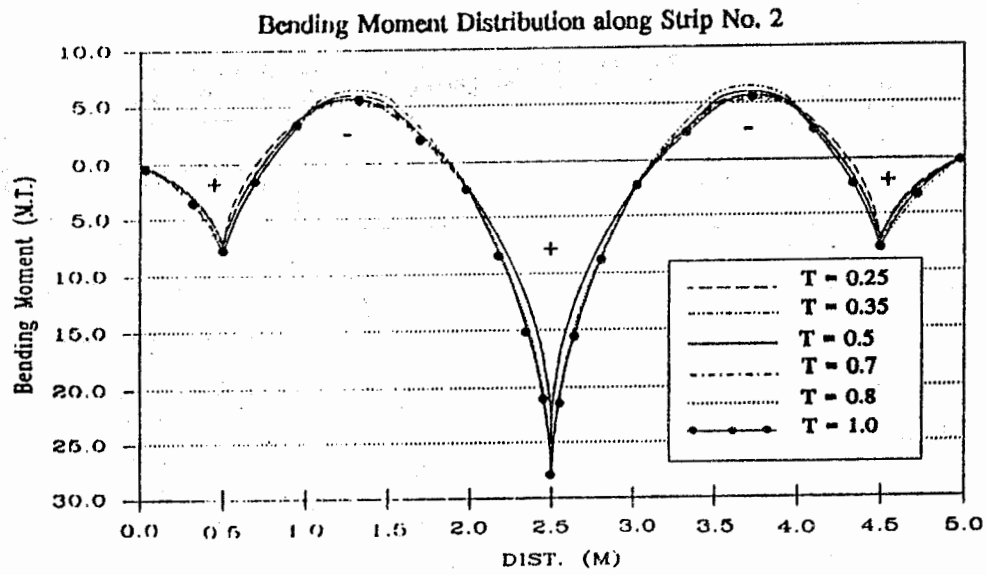


Fig.(11) Bending Moments in Rafts with Different Thicknesses.

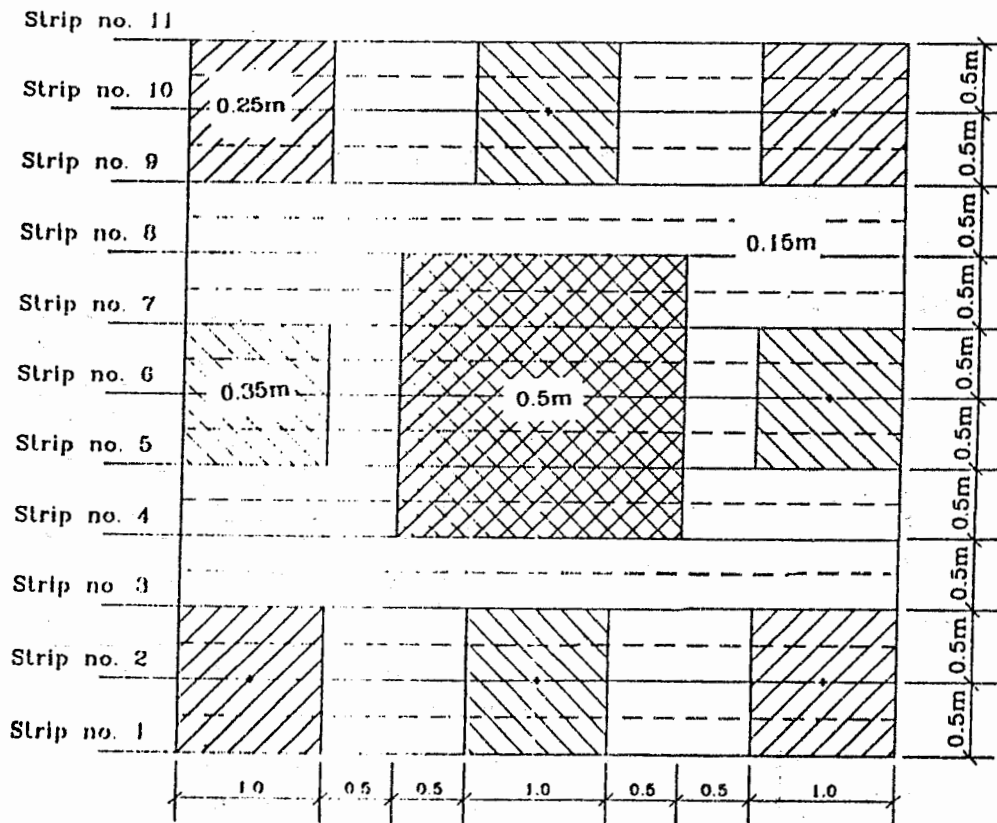


Fig.(12) Raft plate with Variable Thickness.

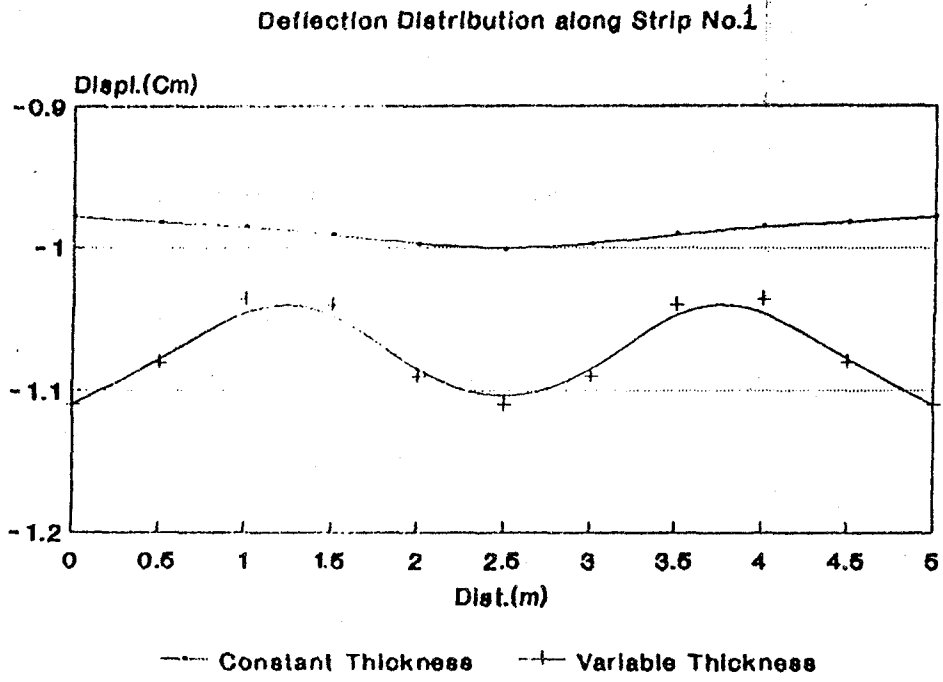


Fig.(13) Comparison of Deflections For Different Rafts.

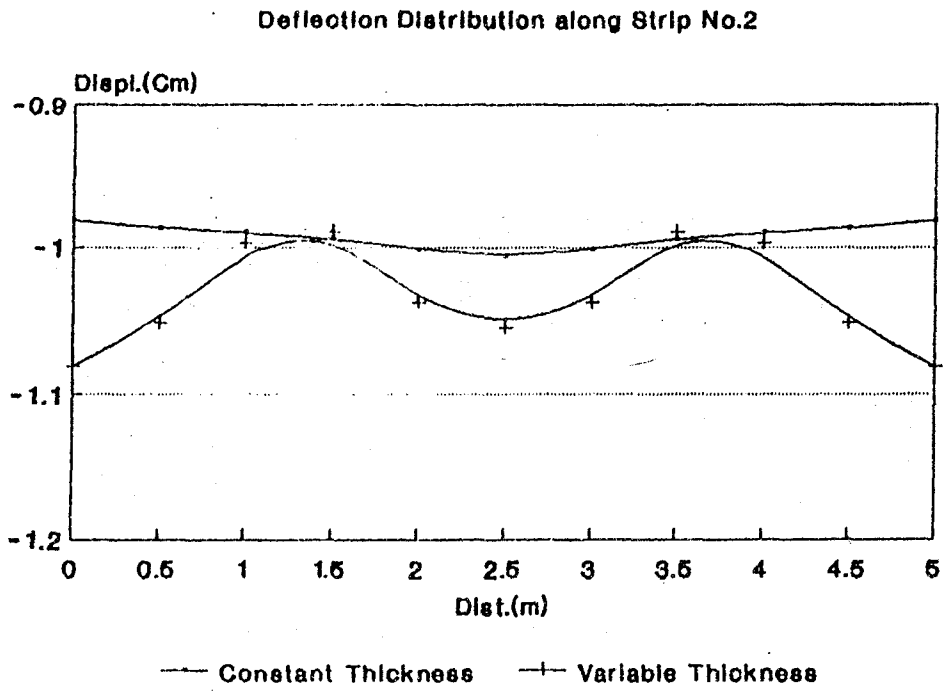


Fig.(14) Comparison of Deflections For Different Rafts.

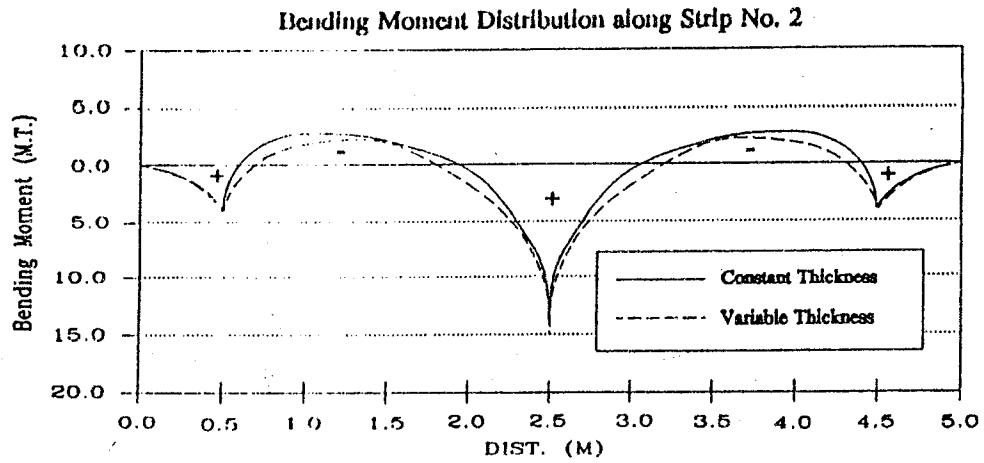


Fig.(15) Bending Moments of Rafts with Different Thicknesses.

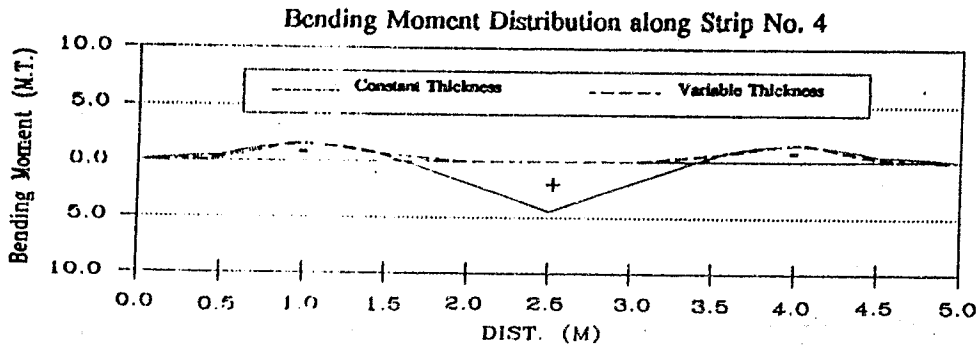


Fig.(16) Bending Moments of Rafts with Different Thicknesses.

## مقارنه طرق تحليل القواعد المستمره فى الاتجاهين

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الملخص العربى :-

يناقش هذا البحث الطرق المستخدمه فى تحليل القواعد المستمره حيث أن هذه القواعد تمثل أهميه كبيرى وتستخدم كثيرًا فى حالة التربيه الضعيفه أو عندما تكون مساحات القواعد المنفصله أكبر من نصف مساحه المنشأ .

والطرق المستخدمه فى تحليل القواعد المستمره هى الطريقة الجاسئسه وهى طريقة تقريبية وتعطى نتائج غالبًا غير دقيقه . أما الطرق الأخرى والأكثر دقه التى تم مناقشتها هى طريقة الفسروق المحدده وطريقه الكميرات الشبكية ( المتقاطععه ) وطريقة الكميرات المرتكزه على قواعد مرنه وأخيرًا طريقة العناصر المحدده .

لمقارنه هذه الطرق ومعرفه مدى تأثير العوامل المختلفه على دقه النتائج تم كتابه برنامج على الحاسب الآلى وتم مقارنه النتائج مع النتائج الناتجه من استخدام البرنامج المعروف SAP 80<sup>®</sup> وبذلك أمكن التأكد من امكانيه استخدام هذا البرنامج بكفاءه . وباستخدام هذا البرنامج يمكن تحليل القواعد المتصله بأى من الطرق المذكوره سابقًا .

وأخيرًا تم استنتاج توصيه لتسهيل تصميم وتحليل القواعد المتصله وذلك بناءً على مخرجات برنامج الكمبيوتر .