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Rogue wave characteristics in superthermal strongly coupled dusty plasma

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Abstract: The present paper introduces a study on the rogue wave features in strongly coupled dusty plasma in which plasma particles are super-thermally distributed. The nonlinear Schrödinger equation is derived to investigate the modulational instability of dust acoustic wave packets. Then, the carrier wave number, super-thermality of plasma particles, ions to electrons density ratio, coupling parameter and temperature ratio of ions to electrons effects on the rogue wave properties are numerically analyzed and confirmed that they influence strongly on the rogue wave properties. The presented results here predicted nonlinear excitations that may helpful for astrophysical and space environments as well as laboratory experiments where super-thermal particles exist.

Keywords: Rogue waves; Nonlinear Schrödinger equation; Strongly coupled dusty plasma

1. Introduction

Almost everywhere in our universe, dusty plasmas (DPs) can exist [1]. For instance, they can exist in the space environments like interstellar clouds, cometary tails, the earth's mesosphere, earth ionosphere and planetary rings [1-4] as well as in many laboratory experiments such as in coating of thin films and plasma crystals [5,6]. The existence of such particulates will introduce new modes into the plasma such as the dust acoustic (DA) waves in which solitary (SL), shock (SH) and rogue waves (RGWs) are the most important nonlinear structures.

In most laboratory and space environments, DPs are found to be in strongly coupled (SC) state due to large charges on the individual dust particles where the dust kinetic energy will be less than the dust particle interactions' electrostatic energy [7].

The propagation of DA SL, SH and RGWs in SCDPs has been considered by many researchers. For example, the presence of cylindrical and spherical DA RGWs in SCDP has Maxwellian ions and electrons have been numerically analyzed by Almutalk et al. [8]. They derived the modified non-planar nonlinear Schrödinger equation (NLS) and examined the non-planar geometries effects on the DA RGWs features. Kabalan et al. [9] studied the structure and coupling parameters effects on the solitary formation in SCDP has negatively charged dust particulates and thermal plasma particles. They illustrated that structure and coupling parameters had obvious effects on soliton behavior. By deriving the Korteweg-de Vries (KdV)-Burgers equation, Li and Duan [10] in 2021 investigated the propagation of SHWs in a SCDP where ions and electrons follow thermal distributions. Also, the SC quantum DP composed of quantum dust and electrons where ions follow the Maxwellian distribution has been discussed by Chaudhuri and Chowdhury [11]. They reported that their system displays solitons with periodic nature which are called nanopteron solitons.

It is worth noting that the previous investigations focused only on the Maxwellian distribution. However, at the high velocities or energies, many space and astrophysical plasmas [12-14] have particles with non-Maxwellian super-thermal tails that can affect on the nonlinear structures formed in DPs [15]. In 2018, Darweesh et al. [16] examined the two colliding SL/SH Ws properties in SCDP having super-thermal distributed plasma particles. Also, the instability of SLWs in SCDP with super-thermal distributed plasma particles has been discussed by El-Taibany et al [17]. Firstly, they investigated the linear features of DA

SLWs and then they described the DA SLWs behavior. They found that the superthermality of plasma particles, the weakly and SC cases had affected on the DA phase velocity and propagation of DA SLWs as well. For the superthermality effect of plasma particles on the RGWs characteristics in SCDPs, there are no detailed works are reported. Therefore, the main objective in such work is to study the superthermality effect of ions and electrons in SCDP on the RGWs behavior.

2. Strongly Coupled Dusty Plasma Model

Unmagnetized SCDP medium with dust particles of negative charge as well as ions and electrons follow super-thermal distributions has been considered and governed by the following generalized hydrodynamic model [18]

$$\frac{\partial n_d}{\partial t}+\frac{\partial(n_d\, u_d)}{\partial x}=0, \quad (1)$$

$$\frac{\partial u_d}{\partial t}+u_d\frac{\partial u_d}{\partial x}-\frac{\partial \phi}{\partial x}+\frac{\mu_d}{n_d}\frac{\partial n_d}{\partial x}=0, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2}-n_d-n_e+n_i=0\ , \quad (3)$$

where n_d and u_d are the number density and velocity of dust particles, $n_{e,i}$ is the electron/ion number density and ϕ is the electrostatic potential. Following normalizations $x \to x/\lambda_{Dd}$, $t \to t\omega_{pd}$, $n_d \to n_d/n_{d0}$, $u_d \to u_d/C_d$, $\phi \to e\phi/k_B T_i$ and $n_{e,i} \to n_{e,i}/Z_d n_{d0}$ have been applied into $\lambda_{Dd}=[k_B T_i/(4\pi Z_d n_{d0} e^2)]^{1/2}$ is the Debye length of dust where e, n_{d0} and k_B are the electron charge magnitude, unperturbed number density of dust grains and Boltzmann constant. $\omega_{pd}=(4\pi Z_d^2 n_{d0} e^2/m_d)^{1/2}$ is the frequency of DP where m_d is the mass of dust grain and $C_d=(k_B T_i Z_d/m_d)^{1/2}$ is the DA speed. The parameter μ_d in Eq. (2) is defined by the relation $\mu_d=\mu T_d/Z_d T_i$, where Z_d, T_d, T_i are the number of electrons residing on dust grain surface, ion temperature and dust temperature. The compressibility μ appears in μ_d is represented by [22] $\mu=1+\frac{u(\Gamma)}{3}+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial \Gamma}$ where μ and Γ are the compressibility and Coulomb coupling parameter. $u(\Gamma)$ measures the excess internal energy of the system [18].

The normalized superthermal form for electron and ion densities is given as [19]

$$n_e=\mu_e[1-\sigma_i\phi/(\kappa-3/2)]^{(-\kappa+1/2)}, \quad (4)$$

$$n_i=\mu_i\,[1+\phi/(\kappa-3/2)]^{(-\kappa+1/2)}, \quad (5)$$

where $\mu_e=1/(\delta-1)$, $\mu_i=\delta/(\delta-1)$ in which $\delta=n_{i0}/n_{e0}$ and $\sigma_i=T_i/T_e$ with T_e and T_i refer to the temperature of electrons and ions.

4. Rogue Waves Analysis and Discussion

To study the characteristic properties of RGWs produced in our considered SCDP system, the reductive perturbation approach (RPA) [20] will be employed. Accordingly, the expansion of dependent variables follows

$$\psi=\psi_0+\sum_{n=1}^{\infty}\in^n\sum_{l=-\infty}^{\infty}\psi_l^{(n)}(X,T)\exp[il(kx-\omega t)], \quad (6)$$

where $\psi=n_d$, u_d, ϕ, respectively and $\psi_0=1$, 0, 0. k and ω are real variables known as the carrier wave number and the frequency of the nonlinear DA wave, respectively. Here, the independent variables X and T are stretched as

$$X=\in(x-v_g t),\quad T=\in^2 t, \quad (7)$$

where v_g is the group velocity. Assume that all perturbed states depend on the fast scales via the phase $(kx-\omega t)$ only, while the slow scales (X,T) enter the arguments of the l th harmonic amplitude $\psi_l^{(n)}$. The derivative operators $\partial/\partial x$ and $\partial/\partial t$ can be written as follows

$$\partial/\partial x \to \partial/\partial x+\in\partial/\partial X\ , \quad (8a)$$

$$\partial/\partial t \to \partial/\partial t+\in^2\partial/\partial t-\in v_g\partial/\partial X\ . \quad (8b)$$

Using Eqs. (6)-(8) into the fluid model equations (1)-(5), one gets the following relations when $(n,l)=(1,1)$:

$$n_{d1}^{(1)}=[k^2/(\mu_d k^2-\omega^2)]\phi_1^{(1)},$$

$$u_{d1}^{(1)} = [k\omega/(\mu_d k^2 - \omega^2)]\phi_1^{(1)},$$

$$\omega^2 = \mu_d k^2 + k^2/(k^2 - A_3), \quad \text{(9a) with}$$

$$A_0 = -(\kappa - 1/2)/(\kappa - 3/2),$$

$$A_1 = (\kappa - 1/2)(\kappa + 1/2)/[2(\kappa - 3/2)^2],$$

$$A_2 = -(\kappa - 1/2)(\kappa + 1/2)(\kappa + 3/2)/[6(\kappa - 3/2)^3]$$

$$A_3 = (\mu_e \sigma_i + \mu_i)A_0, \qquad (9b)$$

while when $(n, l) = (2, 1)$ the following relations are obtained

$$n_{d1}^{(2)} = [k^2/(\mu_d k^2 - \omega^2)]\phi_1^{(2)} + [2ik\omega(\omega - v_g k)/(\mu_d k^2 - \omega^2)^2]\partial\phi_1^{(1)}/\partial X,$$

$$u_{d1}^{(2)} = [k\omega/(\mu_d k^2 - \omega^2)]\phi_1^{(2)}$$

$$n_{d1}^{(1)} = [k^2/(\mu_d k^2 - \omega^2)]\phi_1^{(1)},$$

$$[\frac{i(\omega^3 - v_g k\omega^2 - \mu_d v_g k^3 + \mu_d k^2\omega)}{(\mu_d k^2 - \omega^2)^2}]\frac{\partial\phi_1^{(1)}}{\partial X},$$

$$v_g = [\omega^2 - (\mu_d k^2 - \omega^2)^2]/(k\omega). \qquad (10)$$

When $(n, l) = (2, 2)$ one gets

$$n_{d2}^{(2)} = A_6 \phi_1^{(1)2},$$

$$u_{d2}^{(2)} = A_7 \phi_1^{(1)2},$$

$$\phi_2^{(2)} = A_5 \phi_1^{(1)2}, \qquad (11a)$$

where

$$A_4 = (-\mu_e \sigma_i^2 + \mu_i)A_1,$$

$$A_5 = \frac{2A_4(\mu_d k^2 - \omega^2)^3 + k^4(\mu_d k^2 + 3\omega^2)}{2(\mu_d k^2 - \omega^2)[k^2 - (A_3 - 4k^2)(\mu_d k^2 - \omega^2)]}$$

$$, A_6 = \frac{2k^2 A_5(\mu_d k^2 - \omega^2)^2 + k^4(\mu_d k^2 - 3\omega^2)}{2(\mu_d k^2 - \omega^2)^3},$$

$$A_7 = \frac{A_6\omega(\mu_d k^2 - \omega^2)^2 - k^4\omega}{k(\mu_d k^2 - \omega^2)^2}, \quad (11b)$$

whereas for $(n, l) = (2, 0)$ one obtains

$$n_{d0}^{(2)} = A_{10}|\phi_1^{(1)}|^2,$$

$$u_{d0}^{(2)} = A_{11}|\phi_1^{(1)}|^2,$$

$$\phi_0^{(2)} = A_9|\phi_1^{(1)}|^2, \qquad (12a)$$

with

$$A_8 = (\mu_e \sigma_i^3 + \mu_i)A_2,$$

$$A_9 = \frac{2A_4(\mu_d k^2 - \omega^2)(\mu_d - v_g^2) + k^2(2v_g k\omega - 1)}{(\mu_d k^2 - \omega^2)[1 - A_3(\mu_d - v_g^2)]}$$

$$A_{10} = \frac{A_9(\mu_d k^2 - \omega^2) - k^2(2v_g k\omega - 1)}{(\mu_d k^2 - \omega^2)(\mu_d - v_g^2)},$$

$$A_{11} = \frac{A_{10}v_g(\mu_d k^2 - \omega^2) - 2k^2\omega}{(\mu_d k^2 - \omega^2)}. \qquad (12b)$$

Proceeding to $(n, l) = (3, 1)$, one can obtain the following NLS equation

$$i\frac{\partial\Phi}{\partial T} + \frac{P}{2}\frac{\partial^2\Phi}{\partial X^2} + Q|\Phi|^2\Phi = 0, \qquad (13)$$

Where, for simplicity, $\Phi \equiv \phi_1^{(1)}$. The dispersion and nonlinear coefficients P and Q are given by

$$P = (\mu_d k^2 - \omega^2)(\omega^2 - \mu_d k^2 - A_{12} - A_{14})/(k^2\omega)$$

$$Q = \frac{1}{(2k^2\omega)}\{-(\mu_d k^2 - \omega^2)(A_{13} + A_{15}) - (\mu_d k^2 - \omega^2)^2[2A_4(A_5 + A_9) + 3A_5]\} \quad (14a)$$

with

$$A_{12} = \frac{\omega^4 - 3v_g k\omega^3 + (\mu_d - 2v_g^2)k^2\omega^2 - \mu_d v_g k^3\omega}{(\mu_d k^2 - \omega^2)^2},$$

$$A_{13} = \frac{k^2\omega^2(A_6 + A_{10}) - k^3\omega(A_7 + A_{11})}{(\mu_d k^2 - \omega^2)},$$

$$A_{14} = \frac{-v_g k\omega^3 + (2\mu_d + v_g^2)k^2\omega^2 - 3\mu_d v_g k^3\omega + \mu_d v_g k^4}{(\mu_d k^2 - \omega^2)^2},$$

$$A_{15} = \frac{[-\mu_d k^4(A_6 + A_{10}) + k^3\omega(A_7 + A_{11})]}{(\mu_d k^2 - \omega^2)} + \frac{\mu_d k^8}{(\mu_d k^2 - \omega^2)^3}. \qquad (14b)$$

The NLS equation (13) describes the nonlinear evolution of an amplitude modulated DA wave carrier.

One of the most important solutions of the NLS equation is known as the rational solution that is located on a nonzero background and localized both in X and T directions and given as follows [22]

$$Q = \sqrt{\frac{P}{Q}}[\frac{4(1 + 2iPT)}{(1 + 4P^2T^2 + 4X^2)} - 1]\exp(iPT). \quad (15)$$

The solution (15) reveals that a significant amount of the DA wave energy is concentrated into a relatively small area in space.

It is noteworthy that for the wave amplitude modulational stability, the product sign of nonlinear and dispersion coefficients must be negative (i.e. $PQ < 0$) while for a random perturbation of the amplitude to grow and hence the creation of the DA RGWs, the product sign must be positive (i.e., $PQ > 0$). Now, we will numerically investigate how the CWN (k) and other plasma parameters like superthermal parameter (κ), density ratio (δ), coupling parameter which including in the parameter (μ_d) and temperature ratio (σ_i) can affect on the wave envelope profile of the DA RGWs. First, we started with investigating the CWN changing effect (k) on the DA RGW profile as depicted in Fig. (1).

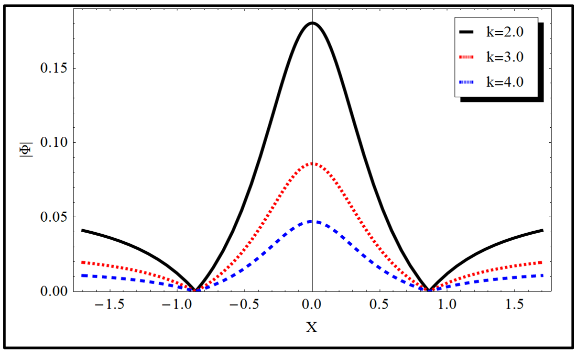

Fig. 1: $|\Phi|$ versus k where $\kappa = 20$, $\delta = 1.3$, $\mu_d = 10$, $\sigma_i = 0.5$.

This figure shows that the RGW amplitude decreases with increasing the CWN value. Then, we investigated other parameters effect on the envelope DA RGW as shown in Figs. (2-5). Figure 2 illustrates the RGW profile with different values for the superthermal parameter (κ). From this figure, one can see that increasing the superthermal parameter value causes enhancing the RGW amplitude.

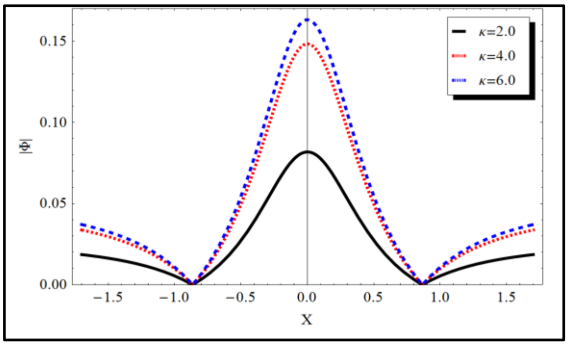

Fig. 2: $|\Phi|$ versus κ where $k = 2$, $\delta = 1.3$, $\mu_d = 10$, $\sigma_i = 0.5$.

On the other side, the role of density ratio (δ) on the RGWs profile can be understandable from Fig. (3), where the RGW amplitude is enhanced with increasing the density ratio.

Figure 4 gives the RGW profile versus the parameter μ_d which includes the coupling parameter. It is clear that the RGW amplitude becomes taller by increasing the coupling parameter. Finally, increasing the temperature ratio (σ_i) is also found to cause a decrement of the RGW amplitude, Fig. (5).

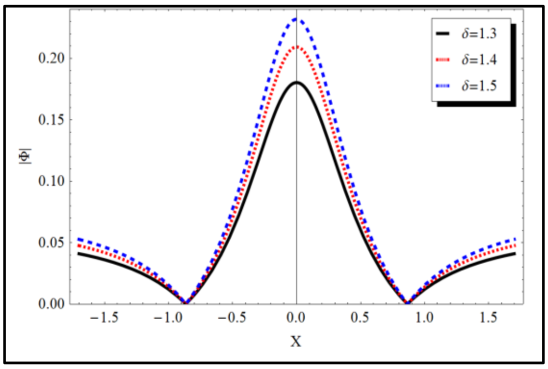

Fig. 3: $|\Phi|$ versus δ where $k = 2$, $\kappa = 20$, $\mu_d = 10$, $\sigma_i = 0.5$.

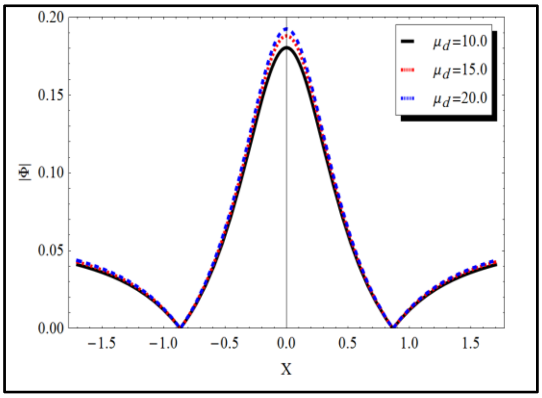

Fig. 4: $|\Phi|$ versus μ_d where $k = 2$, $\kappa = 20$, $\delta = 1.3$, $\sigma_i = 0.5$.

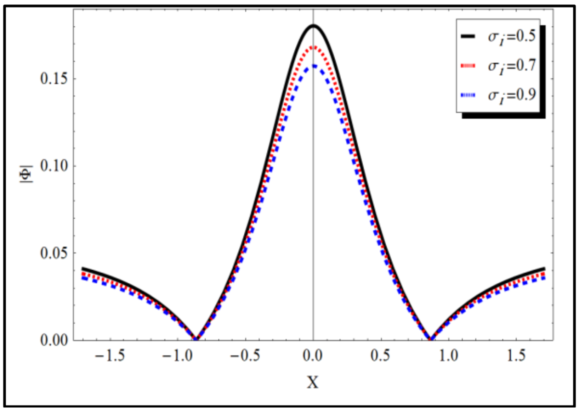

Fig. 5: $|\Phi|$ versus σ_i where $k = 2$, $\kappa = 20$, $\delta = 1.3$, $\mu_d = 10$.

5. Conclusions

In the present paper, the characteristics of the nonlinear DA RGWs in SCDP has negatively charged dust particulates and superthermally distributed plasma particles are discussed. Through the RPA, the NLS equation is derived to describe the characteristics of the nonlinear DA RGWs. The RGW dependence on the CWN (k) and other plasma parameters like superthermal parameter (κ), density ratio (δ), coupling parameter and temperature ratio (σ_i) is illustrated graphically. It is found that increasing the CWN decreases the RGW amplitude. Increasing the superthermality in the present medium enhances the amplitude of the RGW. Increasing both the density ratio and the coupling parameter is also led to enhancing the RGW amplitude whereas increasing the temperature ratio led to energy dissipation and caused in the decrement of the RGW amplitude. The obtained results here may be useful in understanding the basic features of RG DA perturbations in laboratory, space and astrophysical mediums in which superthermal particles can exist.

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