



Answer all the following questions: [100 marks]

1.1 Deduce a first order, first degree, homogeneous partial differential equation which has the general solution $u(x, y) = x^2 f(x-y)$.

1.2 Classify, solve, and check the equation $u_{xx} + u_x = 2(2x^3 - y^2)$. [20 marks]

2.1 Deduce the general solution of the equation $(mz - ny)z_x + (nx - lz)z_y = ly - mx$

2.2 Classify, solve, and check the equation $e^y z_x - e^x z_y = 0$ [20 marks]

3.1 Classify, solve, and check the equation $z_{xx} - 2z_{xy} + z_{yy} = 12xy$.

3.2 Solve and check the equation $(D_x - D_y - 1)(D_x - D_y - 2)z = e^{2x-y}$ [20 marks]

4.1 A flexible string is fixed at two end points π apart and is stretched. The motion of the string takes place by displacing the string in the form $y = \sin x$ from which is released at time $t=0$. If the initial transverse velocity of any point of the string is $y_t(x,0)=0$, determine the unsteady displacement of any point at a distance x from one end at time t .

4.2 A force $F = f_r u_r + f_\theta u_\theta + f_z u_z$ is applied to translate a rigid body of mass m on a cylindrical surface. Deduce Lagrange's equations of motion of the body where: f_r, f_θ, f_z are the force components in r, θ , and z directions. Letting $q_1=r, q_2=\theta, q_3=z$ be the generalized coordinates and $Q_1=f_r, Q_2=rf_\theta, Q_3=f_z$ be the generalized forces. Lagrange's function is $L = 0.5m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - mgz$. Discuss the derived equation of motion. [20 marks]

5.1 Solve the steady state heat equation $\nabla^2 u = -4$ in the annular region $1 < r < 2$ with the boundary conditions: $u(1, \theta) = 0, u(2, \theta) = 1$. Calculate the heat flux from the outer boundary.

5.2 Determine the surface passing through the two lines: $L_1: \{x=0, z=0\}$, and $\{x-y=0, z-1=0\}$ satisfying the differential equation $z_{xx} - 4z_{xy} + 4z_{yy} = 0$. Classify the equation and its solution, then check your solution. [20 marks]

This exam contributes " by measuring in achieving Program Academic Standards according to NARS

Question Number	Q1, Q2, Q3	Q1, Q2, Q3, Q5.2	Q3, Q4, Q5.1
	Knowledge & Understanding Skills	Intellectual Skills	Professional Skills

With my best wishes
 Dr. Bilal Maher