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Complex Systems of Marshall-Olkin Right Truncated Fréchet-Inverted Weibull Distribution

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Abstract: In this paper, we will focus a light on complex systems of Marshall-Olkin Right Truncated Fréchet-Inverted Weibull Distribution. There are many complex systems such as series system, parallel system, series- parallel system, parallel-series Systems, k-out-of-n:G system, linear consecutive k-out-of-n:G system, stand by redundancy of parallel systems. We will compare between the Right Truncated Fréchet-Inverted Weibull and Marshall-Olkin Right Truncated Fréchet-Inverted Weibull in all these systems.

keywords: Marshall and Olkin, Complex systems, series system, parallel system.

1.Introduction

Previously, scientists interested on the reversal of univariate distribution. They have stratified the backwards method for some models. For instance, there are numerous model as the reversal of beta distribution [1], the reversal of Rayleigh distribution [2], the reversal of Gaussian distribution [3], the reversal of Weibull distribution [4] and others models. Fréchet [5] introduced the Weibull model. This model is continuous distribution which has the probability density function a random variable x has the reversal Weibull distribution with two operators α and β is given by

$$G(x;\alpha,\beta)=e^{(-\alpha x^{-\beta})}, x\geq 0, \alpha>0, \beta>0. \quad (1.1)$$

The essential goal in this paper is the concentrate how the Marshall and Olkin right truncated Fréchet-Inverted Weibull distribution applied. Afterward, Haq [6] utilized this change to further develop the length of Marshall-Olkin. An ingenious approach was utilized by Marshall and Olkin [7] for adding a new shape boundary to the current model. Along these lines, we observe the cumulative distribution function F(x) of right truncated Fréchet-inverted Weibull distribution (RTFIWD)

$$F(x;\alpha,\beta,b)=\frac{G(x;\alpha,\beta)}{G(b;\alpha,\beta)}=e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}.$$

$$,0<x\leq b, \alpha>0, \beta>0 \quad (1.2)$$

The pdf f(x) of (RTFIWD) is

$$f(x)=\frac{dF(x;\alpha,\beta,b)}{dx}=\frac{\alpha\beta}{x^{\beta+1}}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)},$$

$$0<x\leq b, \alpha>0, \beta>0 \,. \quad (1.3)$$

the survival function $R(x)$ of the Marshall and Olkin model is defined by

$$R(x;\alpha,\beta,b)=\frac{\gamma\bar{F}(x;\alpha,\beta,b)}{1-\bar{\gamma}\,\bar{F}(x;\alpha,\beta,b)}, \gamma>0 \ , \bar{\gamma}=1-\gamma, \quad (1.4)$$

using (1.2) into (1.4) we obtain $R(x;\alpha,\beta,b)$by MORTFIWD as

$$R(x;\alpha,\beta,b)=\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}, \gamma>0. \quad (1.5)$$

2.System Reliability Models

The configurations of the system are different from a simple one which con sisting of one or two combinations for a complex system containing plenty of combinations. Such systems may be anatomized by system decomposing and this decomposed into subsystems of appropriate size such that each of them is performing an explicit function. After system decomposition, the reliability of all system is created by the reliability of subsystems by combing each of them [8].

2.1 Series Systems

In a series system where components were associated serially, the failure of one of its components chiefs to failure of the system. Assuming the system is having n combinations. The functional illustration introduces that for the effective system operation, the careful activity of all the n combinations, then we name that the design of the framework is a series type [9].Such frameworks are indicated, as displayed in Figure (1). The input from the IN end would connect the OUT end only if all the n combinations were operated

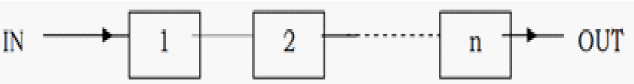


Fig 1: series systems

Let $R_i\left(x;\acute{\alpha}_i,\acute{\beta}_i,b_i\right)$ represent the reliability of combination $i = 1,2,\dots,n$ of and the series system has the reliability function given by the following equation

$$R_s\left(x;\acute{\alpha}_i,\acute{\beta}_i,b_i\right)=\prod_{i=1}^{n}R_i\left(x;\acute{\alpha}_i,\acute{\beta}_i,b_i\right).$$

$$=\prod_{i=1}^{n}\frac{\gamma\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)\right)}$$

$$=\frac{\gamma^n-\gamma^n e^{\sum_{i=1}^{n}-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}}{n-\prod_{i=1}^{n}\left(\overline{\gamma}\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)\right)}$$

$$=\frac{\gamma^n\left(1-e^{\sum_{i=1}^{n}-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)}{n-\overline{\gamma}^n-\overline{\gamma}^n-e^{\sum_{i=1}^{n}-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}}.(2.1)$$

The simplest case of combinations in a series configuration is when the components are independent. All the components have the same failure and repair distributions. In this case $R_i\left(x;\acute{\alpha}_i,\acute{\beta}_i,b_i\right)=R\left(x;\acute{\alpha},\acute{\beta},b\right)$, $i = 1,2,\dots,n$ and the reliability of the series system denoted by the following equation:

$$R_s(x;\alpha_i,\beta_i,b_i)=[R(x;\alpha,\beta,b)]^n$$

$$=\left[\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]^n.(2.2)$$

The following figure for reliability of the series system of five components by using component reliability in 2.1 with $\alpha = 0.5,\beta = 2$, $b = 6$ and $\gamma = 0.8$and Equation when $R(x;\alpha,\beta,b)$ replaced to $\overline{F}(x;\alpha,\beta,b)$ with $\alpha = 0.5$, $\beta = 2$, $b = 6$

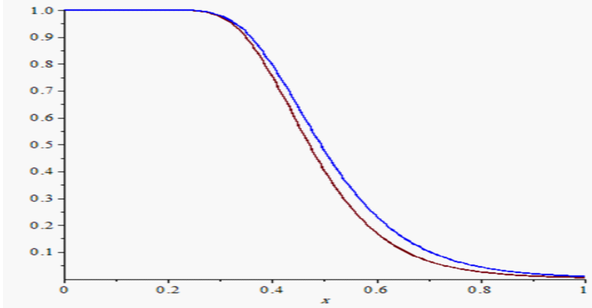


Fig 2: Reliability of the series system

RTFIWD——MORTFIWD——

2.2 Parallel System

The system has numerous combinations to perform the same operation, and the acceptable performance of any of these combinations is sufficient to sure the effective system operation. The combinations for such a system are also linked in a parallel configuration, as shown below [10].

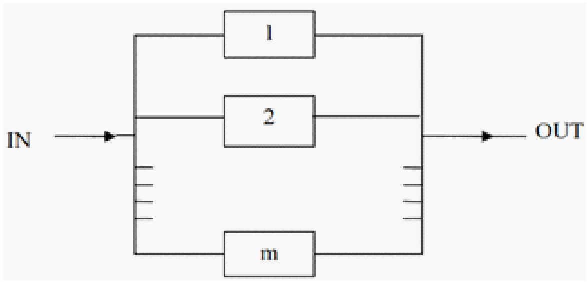


Fig 3 Parallel system

A parallel system will fail only if all its combinations fail. Let $R_i(x;\alpha_i,\beta_i,b_i)$represent the reliability of component $i = 1,2,\dots,m$, and the parallel system has the reliability function given by the following equation:

$$R_p(x;\alpha_i,\beta_i,b_i)=1-\prod_{i=1}^{m}[1-R_i(x;\alpha_i,\beta_i,b_i)]$$

$$=1-\prod_{i=1}^{m}\left[1-\frac{\gamma\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)\right)}\right].(2.3)$$

When all components are independent and identical. The reliability function of the parallel system given by the following equation

$$R_p(x;\alpha_i,\beta_i,b_i)=1-[1-R(x;\alpha_i,\beta_i,b_i)]^m$$

$$=1-\left[1-\frac{\gamma\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)\right)}\right]^m.(2.4)$$

The following figure for reliability of the parallel system of five components by using component reliability in 2.1 with $\alpha = 0.5, \beta = 2$, $b = 6$ and $\gamma = 0.8$ and Equation when

$\mathrm{R}(\mathrm{x}; \alpha, \beta, \mathrm{b})$replaced to $\overline{F}(x;\alpha,\beta,b)$ with $\alpha = 0.5, \beta = 2$, $b = 6$

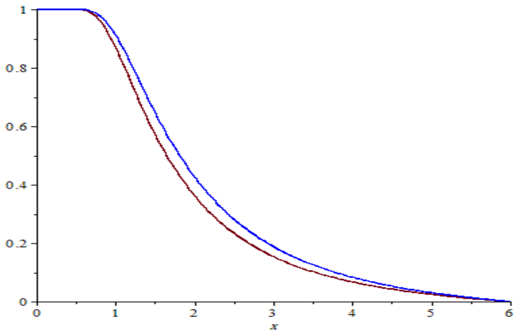
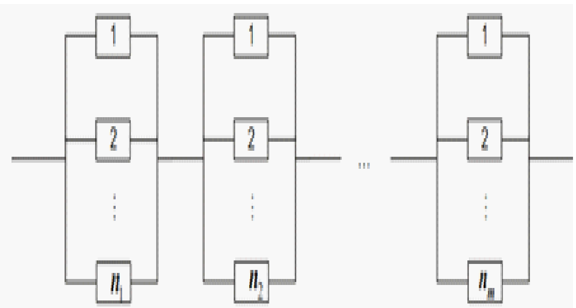


Fig 4 Reliability of the parallel system RTFIWD, MORTFIWD

2.3 Series-Parallel Systems

The standard composition of a series-parallel system is demonstrated in Figure 5 the system is composed of m subsystems with failure dependencies linkedin series, and each subsystem $i = 1, 2, \dots, m$ has $n_i$components connected in parallel. Consider all components are identical in each subsystem. Each parallel subsystem makes if and only if at least one of its combinations make, and the all system makes if and provided that all subsystems make [11].



Considering a subsystem $\mathrm{i} = 1, 2, \dots, \mathrm{m}$ composed of $\mathrm{n_i}$ un-identical component follows Rayleigh failure distribution in parallel, where any combination may be seen as amplitude of another combination. Each subsystem flops if and only if every of the components fail and each repair team may repair only one flopped combination at a time. In this case reliability function given by the following equation

$$R_{sp}(x) = \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n_i} \overline{R}\left(\left(x;\alpha_{ij},\beta_{ij},b_{ij}\right)\right]\right. \quad (2.5)$$

$$= \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n_i}\left[1 - R\left(x;\alpha_{ij},\beta_{ij},b_{ij}\right)\right]\right] \quad (2.6)$$

$$= \prod_{i=1}^{m}\left[1 - \prod_{j=1}^{n_i}\left[1 - \frac{\gamma\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)\right)}\right]\right] . \quad (2.7)$$

Where the unreliability of j^{th} component in the i^{th} subsystem is

$$\overline{R}\left(x;\alpha_{ij},\beta_{ij},b_{ij}\right) = \left[1 - R\left(x;\alpha_{ij},\beta_{ij},b_{ij}\right)\right]$$

When all components are independent and identical

$$\overline{R}\left(x;\alpha_{ij},\beta_{ij},b_{ij}\right) = \overline{R}(x;\alpha,\beta,b)$$

In this case the reliability function of the series- parallel system given by the following equation

$$R_{sp}(x) = \prod_{i=1}^{m}\left[1 - \left(\overline{R}(x;\alpha,\beta,b)\right)^{n_i}\right]$$

$$= \prod_{i=1}^{m}\left[1 - \left[\frac{\gamma\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)}{\left(1-\overline{\gamma}\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)\right)}\right]^{n_i}\right].$$

The following figure for reliability of the series-parallel system of three sub-System $m = 3$ and each subsystem consists of four components $n_1 = n_2 = n_3 = 4$ by using component reliability in Equation (2.7) with $\alpha = 0.5, \beta = 2$, $b = 6$and $\gamma = 0.8$ and Equation(2.6) when $\mathrm{R}(\mathrm{x}; \alpha, \beta, \mathrm{b})$replaced to$\overline{F}(x;\alpha,\beta,b)$ with $\alpha = 0.5, \beta = 2$, $b = 6$

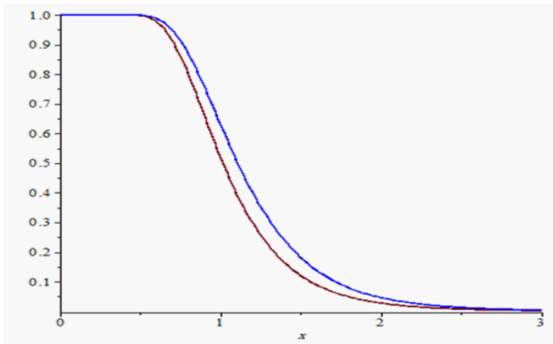


Fig 5 Reliability of the series-parallel system

2.4 Parallel - Series Systems

This framework contains different phases linked in parallel. Each path comprises amount of combinations in series. For the effective running of the system, one way is adequate. Excessive components are combined in parallel-series system to improve the reliability of the phases as both as the system [12]. Excessive may be examined by the stage level with the component level. The reliability of i^{th} a branch is donated by

$$R(x;\alpha_i,\beta_i,b_i) = \prod_{j=1}^{m} R(x;\alpha_{ij},\beta_{ij},b_{ij}),$$

$$i = 1,2,\dots n$$

$$= \prod_{j=1}^{m} \frac{\gamma\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)\right)} . \quad (2.8)$$

The reliability of the system is given by

$$R_{ps}(x) = 1-\prod_{i=1}^{n}[1-R(x;\alpha_i,\beta_i,b_i)],$$

$$= 1-\prod_{i=1}^{n}\left[1-\prod_{j=1}^{m}\frac{\gamma\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)\right)}\right]$$

$$= \prod_{j=1}^{m}\frac{\gamma\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha_{ij}\left(x^{-\beta_{ij}}-b_{ij}^{-\beta_{ij}}\right)}\right)\right)}$$

where $R(x;\alpha_i,\beta_i,b_i)$ is the reliability of the j^{th} component in the i^{th} parallel path.

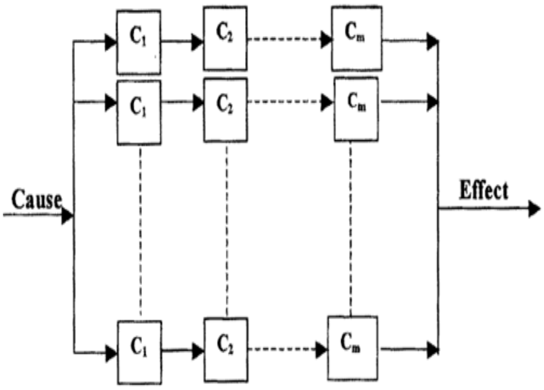


Fig 6 Parallel-series systems

When the components are independent and identical with MORTFIWD distribution. The reliability function of the parallel-series framework given by the following equation

$$R_{ps}(x) = 1-(1-[1-R(x;\alpha,\beta,b)]^m)^n \quad (2.9)$$

$$= 1-\left(1-\left[1-\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]^m\right)^n . \quad (2.10)$$

The following figure for reliability of the parallel -series system of three subsystem $n = 3$ and each subsystem consists of four components $m = 4$ by using component reliability in Equation(6) with $\alpha = 0.5$, $\beta = 2, b = 6$ and $\gamma = 0.8$ and Equation (2.10) when $R(x;\alpha,\beta,b)$replaced to$\bar{F}(x;\alpha,\beta,b)$ with $\alpha = 0.5, \beta = 2$, $b = 6$.

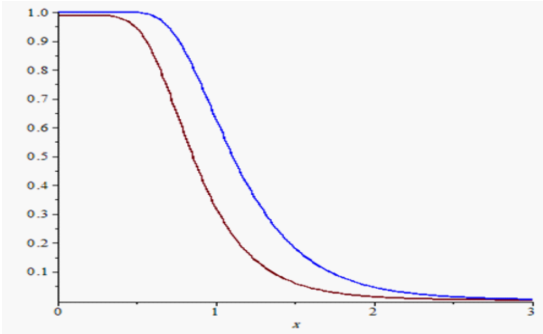


Fig 7 Reliability of the parallel-series system RTFIWD—MORTFIWD —

2.5 K-out-of-n: G System

When the combinations are similar, the probability of precisely i prosperity out of n trials:

We find when we have $X = k$ successes and $n-k$ failures then$R^k(x;\alpha,\beta,b)\left(1-R(x;\alpha,\beta,b)\right)^{n-k}$multiplied by the number of components of n recognized items picked r at a time, that is $n!/(k!\,(n-k)!)$ which is calculated by the binomial theorem. The random variable for the number of cooperative combinations [13].

$$P_{X=k} = \Pr[X = k, n, p]$$

$$= \frac{n!}{k!\,(n-k)!}(R(x;\alpha,\beta,b))^k\left(1 - R(x;\alpha,\beta,b)\right)^{n-k}$$

$$= \frac{n!}{k!\,(n-k)!}\left(\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right)^k \times\left(1-\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right)^{n-k}.$$

The k-out-of-n: G system has the reliability function is

$$R_{k/n}(x;\alpha,\beta,b) = P_{X=k}+P_{X=k+1}+\dots+P_{X=n} \quad (2.13)$$

$$= \sum_{i=k}^{n}\frac{n!}{k!\,(n-k)!}\left(\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right)^k \times\left(1-\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right)^{n-k}.$$

The following figure for reliability of the 3-out-of-5:G system by using component reliability in Equation (6) with α = 0.5, β = 2, b = 6 and γ = 0.8 and Equation (3) when $R(x;\alpha,\beta,b)$replaced to$F(x;\alpha,\beta,b)$ with α = 0.5,β = 2, b = 6.

2.6 Linear circular k-out-of-n:G System

A linear circular k-out-of-n: G system is a concatenation of $k = n$ linearly(circularly) ordered combinations. We find the system tasks if and only if at minimum k sequential combinations work. A sequential k-out-of-n: G system is a series sys- tem when $n = k$ and a parallel system when $k = 1$. The reliability function of a linear consecutive k-out-of-n: G sytem is introduced by Navarro and Eryilmaz in the following equation [14]:

$$R_{line}(x;k,n) = 1 - \prod_{j=1}^{n-k+1}\left[1 - \prod_{i=j}^{j+k-1} R(x;\alpha_i,\beta_i,b_i)\right] \quad (2.14)$$

$$= 1 - \prod_{j=1}^{n-k+1}\left[1 - \prod_{i=j}^{j+k-1}\frac{\gamma\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha_i\left(x^{-\beta_i}-b^{-\beta_i}\right)}\right)\right)}\right] . \quad (2.15)$$

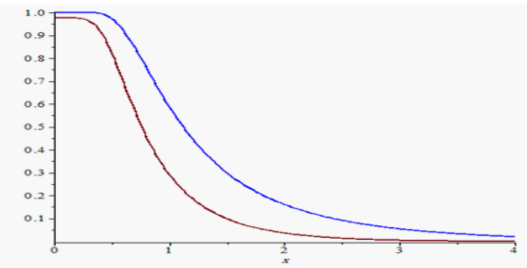


Fig 8:RTFIWD——MORTFIWD——

For example the reliability function of a linear circular 2-out-of-4: G system as follows:

$$R_{line}(x;2,4) = 1 - \prod_{j=1}^{3}\left[1 - \prod_{i=j}^{j+1} R(x;\alpha_i,\beta_i,b_i)\right]$$

$$= 1 - [1 - R(x;\alpha_1,\beta_1,b_1)R(x;\alpha_2,\beta_2,b_2)] \times [1 - R(x;\alpha_2,\beta_2,b_2)R(x;\alpha_3,\beta_3,b_3)] \times [1 - R(x;\alpha_3,\beta_3,b_3)R(x;\alpha_4,\beta_4,b_4)]$$

The following figure for reliability of the linear circular 2-out-of-4: G sys- tem by using un-identical component reliability in (1.3) into (1.5) and when R(x; α, β, b)replaced to$\bar{F}$(x; α, β, b) with α = 0.5, β = 2 , b = 6 with

Component	1	2	3	4
α_i	0.5	0.7	0.9	1.1
β_i	2	1.5	1	0.75
b_i	6	5	4	3
γ_i	0.8	0.7	0.6	0.5

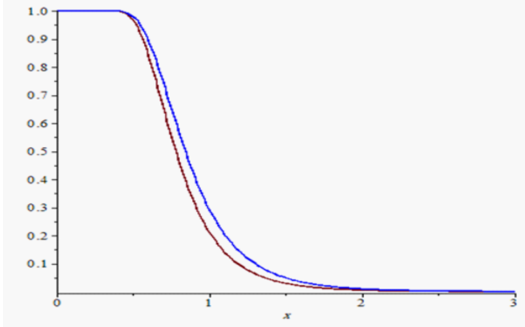


Fig 9 Reliability of the linear consecutive 2-out-of-4:G system

2.7 Stand by Redundancy of Parallel Systems

Redundancy is an effective tool to improve the reliability of a system by adding redundant combination [14].

2.7.1 Hot standby redundancy

This method is considered that several of the system combinations are doubled in parallel [15]. The reliability for each of component improved by hot is given by:

$$R_h(x;\alpha,\beta,b) = [2 - R(x;\alpha,\beta,b)]R(x;\alpha,\beta,b) = \left[2\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]\left[\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]$$

this implies, the reliability of the parallel system of m components improved by n components are hot duplication method is given by:

$$R_H(x) = 1 - [1 - R(x;\alpha,\beta,b)]^{m-n} \times [1 - R_h(x;\alpha,\beta,b)]^n$$

$$= 1 - \left[1 - \frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]^{m-n} \times \left[1 - \left[\left[2 - \frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right] \times \left[\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)}\right]\right]\right]^n$$

2.7.2 Cold duplication method

Some of the system combinations are doubled in paralle. Rade (1989-1) the reliability function with constant failure rates of each combination amended by a cold via perfect switch may be denoted by [16]:

$$R_c(x) = R(x,\alpha,\beta,b) + \int_0^x f(s,\alpha,\beta,b)R(x - s,\alpha,\beta,b)ds,$$

by using (1.3) and (1.5) in the pervious equation:

$$R_c(x) = R(x,\alpha,\beta,b) + \int_0^x f(s,\alpha,\beta,b)R(x - s,\alpha,\beta,b)ds$$

$$= \frac{\gamma\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)} + \int_0^x \left[\frac{\alpha\beta}{s^{\beta+1}} e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} \times \frac{\gamma\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)}\right] ds$$

$$= \frac{\gamma\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)} + \alpha\beta\gamma \int_0^x \left[\frac{e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}}{s^{\beta+1}} \times \frac{\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)}{\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)}\right] ds$$

$$= \frac{\gamma\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)} + \alpha\beta\gamma \int_0^x \left[\frac{e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} - e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)}{s^{\beta+1}\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)}\right] ds$$

$$= \frac{\gamma\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1 - e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)} + \alpha\beta\gamma I_1,$$

we need to compute I_1

$$I_1 = \int_0^x \left[\frac{e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} - e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)}{s^{\beta+1}\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)}\right] ds$$

$$= \int_0^x \frac{e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}}{s^{\beta+1}\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)} ds - \int_0^x \frac{e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)}{s^{\beta+1}\left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)} ds,$$

$$= \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \times \left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)^{-1} ds$$

$$- \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right) s^{-\beta-1} \times \left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)^{-1} ds$$

$$= I_2 - I_3,$$

to compute the value of I_2, I_3 we find

$$I_2 = \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \times \left(1-\bar{\gamma}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)^{-1} ds$$

$$= \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \sum_{j=0}^{\infty} \bar{\gamma}^j \left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)^j ds$$

$$= \sum_{j=0}^{\infty} \bar{\gamma}^j \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)^j ds$$

$$= \sum_{j=0}^{\infty} \bar{\gamma}^j \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \times \sum_{i=0}^{\infty} \frac{\Gamma(-j+1)}{\Gamma(-j)i!} e^{-\alpha i\left((x-s)^{-\beta}-b^{-\beta}\right)} ds$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} \times e^{-\alpha i\left((x-s)^{-\beta}-b^{-\beta}\right)} ds$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} \Psi_{ij},$$

where

$$\Psi_{ij} = \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)} s^{-\beta-1} e^{-\alpha i\left((x-s)^{-\beta}-b^{-\beta}\right)} ds$$

to compute I_3

$$I_3 = \int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1 - e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right) s^{-\beta-1}$$

$$\times\left(1-\bar{\gamma}\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)\right)^{-1} ds$$

$$=\int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)s^{-\beta-1}$$

$$\times\sum_{j=0}^{\infty}\bar{\gamma}^j\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)^{-j} ds$$

$$=\sum_{j=0}^{\infty}\bar{\gamma}^j\int_0^x e^{-\alpha\left(s^{-\beta}-b^{-\beta}\right)}$$

$$\times\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)s^{-\beta-1}$$

$$\times\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)^{-j} ds$$

$$=\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\bar{\gamma}^j\frac{\Gamma(-j+1)}{\Gamma(-j)i!}$$

$$\times\int_0^x\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)s^{-\beta-1}$$

$$\times\left(1-e^{-\alpha(i+1)\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)ds$$

$$=\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\bar{\gamma}^j\frac{\Gamma(-j+1)}{\Gamma(-j)i!}\Phi_{ij},$$

where

$$\Phi_{ij}=\int_0^x\left(1-e^{-\alpha\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)s^{-\beta-1}$$

$$\times\left(1-e^{-\alpha(i+1)\left((x-s)^{-\beta}-b^{-\beta}\right)}\right)ds.$$

Then we find

$$R_c(x)=\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}$$

$$+\alpha\beta\gamma\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\bar{\gamma}^j\frac{\Gamma(-j+1)}{\Gamma(-j)i!}\left[\Psi_{ij}-\Phi_{ij}\right].$$

The reliability of the parallel system of m combinations amended by n combinations are cold duplication method is denoted by:

$$R_H(x)=1-[1-R(x,\alpha,\beta,b)]^{m-n}$$

$$\times[1-R_c(x,\alpha,\beta,b)]^n$$

$$=1-\left[1-\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}\right]^{m-n}$$

$$\times[1-\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}-\alpha\beta\gamma$$

$$\times\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\bar{\gamma}^j\frac{\Gamma(-j+1)}{\Gamma(-j)i!}[\Psi_{ij}-\Phi_{ij}]]^n.$$

3. Stress-Strength Reliability

About $R=P[Y_1>X_1]$,where Y_1 and $X_1$are two independent random variables, is extremely normal in the survival literature. For instance, assuming Y_1 is the strength of a part which is dependent upon a pressure X_1, then, at that point R is a proportion of framework execution and emerges with regards to mechanical dependability of a framework [17]. The framework fizzles if and provided that whenever the applied pressure is more noteworthy than its solidarity. Allow X_1 and Y_1 to be the pressure and the strength random variables, independent of one another, follow respectively. The stress probability density function of RTFIWD(α,β,b) can be gotten as,

$$f_x(x)=\frac{\alpha\beta}{x^{\beta+1}}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}, 0<x\le b, \alpha>0, \beta>0.$$

Also, the strength reliability of the Marshall and Olkin right truncated Fréchet-inverted Weibull distribution MORTFIWD (α,β,γ,b) of Y_1 can be obtained as:

$$R_y(x)=\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}, \gamma>0.$$

Then

$$R=P[Y>X]=\int_0^b f_x(x)R_y(x)dx$$

$$=\int_0^b\frac{\alpha\beta}{x^{\beta+1}}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\frac{\gamma\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}dx,$$

$$=\alpha\beta\gamma\int_0^b x^{-\beta-1}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\frac{\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}dx,$$

$$=\alpha\beta\gamma\left[\int_0^b\frac{x^{-\beta-1}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}dx\right.$$

$$\left.-\int_0^b\frac{x^{-\beta-1}e^{-2\alpha\left(x^{-\beta}-b^{-\beta}\right)}}{1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)}dx\right],$$

$$=\alpha\beta\gamma[I_3-I_4],$$

we find I_3 is given by

$$I_3=\int_0^x x^{-\beta-1}e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}$$

$$\times\left(1-\bar{\gamma}\left(1-e^{-\alpha\left(x^{-\beta}-b^{-\beta}\right)}\right)\right)^{-1}dx$$

$$= \int_0^{x} x^{-\beta-1} e^{-\alpha(x^{-\beta}-b^{-\beta})}$$

$$\times \sum_{j=0}^{\infty} \bar{\gamma}^j \left(1 - e^{-\alpha(x^{-\beta}-b^{-\beta})}\right)^{-j} dx$$

$$= \sum_{j=0}^{\infty} \bar{\gamma}^j \int_0^{x} x^{-\beta-1} e^{-\alpha(x^{-\beta}-b^{-\beta})}$$

$$\times \left(1 - e^{-\alpha(x^{-\beta}-b^{-\beta})}\right)^{-j} dx$$

$$= \sum_{j=0}^{\infty} \bar{\gamma}^j \int_0^{x} x^{-\beta-1} e^{-\alpha(x^{-\beta}-b^{-\beta})}$$

$$\times \sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} e^{-\alpha i(x^{-\beta}-b^{-\beta})} dx$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!}$$

$$\times \int_0^{x} x^{-\beta-1} e^{-\alpha(i+1)(x^{-\beta}-b^{-\beta})} dx$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!}$$

$$\times \int_0^{x} x^{-\beta-1} e^{-\alpha(i+1)x^{-\beta}} e^{-\alpha(i+1)b^{-\beta}} dx$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} e^{-\alpha(i+1)b^{-\beta}}$$

$$\times \int_0^{x} x^{-\beta-1} e^{-\alpha(i+1)x^{-\beta}} dx$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} e^{-\alpha(i+1)b^{-\beta}}$$

$$\times \int_0^{\alpha(i+1)b^{-\beta}} \left(\left(\frac{1}{\alpha(i+1)}\right)^{-\frac{1}{\beta}} t^{-\frac{1}{\beta}}\right)^{-\beta-1} \left(\frac{1}{\alpha(i+1)}\right)^{-\frac{1}{\beta}}$$

$$\times \left(\frac{-1}{\beta}\right) e^{-t} t^{-\frac{1}{\beta}-1} dt$$

$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} e^{-\alpha(i+1)b^{-\beta}} \left(\frac{1}{\alpha(i+1)}\right)^{-\frac{1}{\beta}} \left(\frac{-1}{\beta}\right)$$

$$\times \int_0^{\alpha(i+1)b^{-\beta}} t^{\frac{\beta+1}{\beta}} e^{-t} t^{-\frac{1}{\beta}-1} dt$$

and by the same way we can calculate I_4 then we find the strength reliability of the Marshall and Olkin right truncated Fréchet-inverted Weibull distribution MORTFIWD $(\alpha, \beta, \gamma, b)$ of Y_1 can be obtained as:

$$R = P[Y_1 > X_1]$$

$$= \alpha\beta\gamma \left[\sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} \times e^{-\alpha(i+1)b^{-\beta}} \left(\frac{1}{\alpha(i+1)}\right)^{-\frac{1}{\beta}} \left(\frac{-1}{\beta}\right)\right] \varsigma_i$$

$$- \left[\sum_{j=0}^{\infty}\sum_{i=0}^{\infty} \bar{\gamma}^j \frac{\Gamma(-j+1)}{\Gamma(-j)i!} \times e^{-2\alpha(i+1)b^{-\beta}} \left(\frac{1}{\alpha(i+1)}\right)^{-\frac{1}{\beta}} \left(\frac{-1}{\beta}\right)\right] \varpi_i$$

where

$$\varsigma_i = \int_0^{\alpha(i+1)b^{-\beta}} t^{\frac{\beta+1}{\beta}} e^{-t} t^{-\frac{1}{\beta}-1} dt$$

and

$$\varpi_i = \int_0^{2\alpha(i+1)b^{-\beta}} t^{\frac{\beta+1}{\beta}} e^{-t} t^{-\frac{1}{\beta}-1} dt$$

Conclusion:

In this paper, we introduced some of complex systems and we will shine a light on complex systems of Marshall-Olkin right truncated Fréchet-In-verted Weibull distribution. we compared between the Right truncated Fréchet-Inverted Weibull and Marshall-Olkin right truncated Fréchet-Inverted Weibull in all this systems and me studied the stress and strain of the system.

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